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RESEARCH REPORT

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SOME ASPECTS ON TOKAMAKS WITH NON CIRCULAR CROSS SECTION

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Abstract

The main efforst in future Tokamaks experiments appears to be aimed towards three main objectives;

i) Investigation of scaling laws in the collisionless regime.

ii) Reduction of the impurity content.

iii) Optimization of the plasma parameters for better and more economical performance.

In most laboratories large programmes are presently being organized to tackle these goals. Divertors and actions on the friction term between impurities and light ions are the best guide lines for objective ii). Objectives i) and iii) are to be discussed with the 4 free parameters of Tokamaks: Magnetic field, size, aspect ratio and shape. With special emphasis on the importance of the shape we review here some of the basic properties of Tokamaks.

I. Diffusion

I.1. Neoclassical ion energy losses

Let us consider an axisymmetric toroidal plasma with elliptical cross section of the magnetic surfaces. The ellipticity $k = \frac{b}{a}$ is defined to be the ratio between the vertical and horizontal half axis of the ellipse.

It should first be noted[1] that the averaged trapped particle bounce frequency is independent of k. Consequently the transition temperature T_i^* between the plateau and banana diffusion law is the same for circular and non circular tokamaks having the same aspect ratio $\frac{a}{R}$ (where R is the major radius).

$$T_i^* = 2 \times 10^{-6} \left(q R n Z_{eff} \right)^{1/2} \left(\frac{R}{a} \right)^{3/4} \qquad \text{(equ. 1)}$$

This formula (in ev and cm) shows the dependence with respect to the safety factor q and the "effective" plasma atomic number Z_{eff}.[2]

Consider now the ion confinement time τ_i. In the plateau regime we find:

$$\tau_i = \frac{\Omega_{ci}^2 R k^2 a^2}{q \, v_{thi}^3} \qquad \text{"plateau"} \qquad \text{(equ. 2)}$$

In the banana regime τ_i becomes:

$$\tau_i = \frac{\Omega_{ci}^2 k^2 a^2 v_{thi}}{n \, q^2 \left(\frac{R}{a} \right)^{3/2}} \qquad \text{"Banana"} \qquad \text{(equ. 3)}$$

The equations 2 and 3 demonstrate clearly that improved confinement times can be obtained with plasmas elongated in the vertical directions provided stability properties are not worse. Holding fixed n, q, $\frac{R}{a}$, v_{thi} and the total magnetic energy B^2ka^2R we find that the improvement is of the order of k. This result depends however on whether same q values can effectively be obtained in non circular tokamaks and this important point will be discussed in section 3.

Using Artsimovitch's method[3] the ion temperature obtained by ohmic heating can be evaluated by equating the total ion thermal loss flux to the energy flux transferred from electrons to ions. Artsimovitch's formula generalized for elliptical cross sections becomes

$$T_i^3 \propto \frac{nR_a}{q} k^2 a^2 B^2 \qquad \text{"plateau" (equ. 4)}$$

and

$$T_i \propto \frac{k^2 a^2 B^2}{q^2} \left(\frac{a}{R}\right)^{3/2} \qquad \text{"Banana" (equ. 5)}$$

Increased temperatures can therefore be reached for vertical elongations. The gain is specially appreciable in the banana regime and is again on the order of k for equal plasma volumes of non circular and circular plasmas.

Equations 2 to 5 have been obtained with the assumption that k does not depend on the radial coordinate. However k decreases from the boundary to the center when realistic current density profiles are taken into account (for instance $J = \lambda\phi$). Calculations including this effect[4] reveal a small reduction of the values of T_i and τ_i calculated with the value of k at the plasma boundary. The reduction remains negligibly small when $k < 3$.

I.2. Other diffusion laws

In any Bohm-like scaling laws the diffusion depend only on B_T and on the smallest radial dimension. The confinement time of non circular Tokamaks with such a diffusion law is therefore reduced when compared to a circular case of equal volume by a factor of the order of the ellipticity for both vertical or horizontal elongations. Fortunately Bohm diffusion appears unlikely and the ion temperature measured in most Tokamak is in reasonable agreement with neo-classical ion transport.

Transport models including trapped ion modes show a dependence $\tau \propto q^2 a^2 \phi_p^2$ (ϕ_p poloidal flux). ϕ_p being proportional to k, the corresponding improvement of τ for vertical elongation is comparable to the case of the neo-classical theory.

II. Plasma pressure; current density

When the plasma pressure increases a stagnation point approaches from the magnetic axis to the inner plasma boundary. The equilibrium can be maintained when the separatrix remains outside the plamsa. For stability[4] the corresponding $\beta[8\pi\bar{p}/(B_T^2 + B_p^2)]$ must be:

$$\beta \leq \frac{1}{q^2} \frac{a}{R} k^2 \left[\frac{3}{4} + \frac{1}{4k^2}\right] \qquad (\text{equ. } 6)$$

In reactors it is expected that the aspect ratio and the magnetic field will be limited by practical considerations. In such a case a moderate vertical elongation can increase the limiting pressure by an order of magnitude (see figure 1).

In conventional Tokamak plasmas the limiting β value given by equation 6 cannot be obtained because the input power from ohmic heating is insufficient. The "practical" limit β_{pr} is presently such that

$$\beta_{pr} \propto n j^2 \tau_E \qquad \text{(equ. 7)}$$

Since the current density J is limited by[4]

$$J = \frac{B_T}{qR}\left(\frac{1}{k} + k\right) \qquad \text{(equ. 8)}$$

vertical elongation allows a larger current density; for instance if $k = 2, J(2) = 1.5\ J(1)$. This effect combined with the increase of τ_E gives a significant overall improvement and the practical limit of β increases as k^2.

III. Stability

Improvements in confinement time and plasma pressure will be granted only if stability properties of elongated plasmas does not depend much on the values of the safety factor q. Lower limits of q may be set either by interchange modes or kink modes.

III.1. Interchange modes

Localized perturbations around the magnetic axis have been studied using the Mercier criteria.[5] Both elliptical and triangular deformation have been considered with magnetic surfaces given by (see Fig.2 for definitions):

$$1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{2c}{R}\,\frac{x}{a}\,\frac{y^2}{b^2} = cte$$

This model provides a stability condition:

$$q\,(\text{on axis}) \geq \frac{(3k^2+1)^{\frac{1}{2}}}{2}\;\frac{1}{\left[1 + \frac{3(k^2-1)}{4(k^2+1)}\,\frac{2c}{R}\,\frac{R}{a}\right]^{\frac{1}{2}}} \qquad (\text{equ. } 9)$$

For circular cross section this provides the usual condition $q \geq 1$. For horizontal elongations (figure 3) an important gain can be obtained, the gain is further increased for triangular shapes pointing towards the magnetic axis (Fig.3). However it is not probably possible to take advantage of this low q value on the axis since kink instabilities set a stringent condition on q of the plasma boundary.

For pure vertical ellipse q is increased by about a factor k. This difficulty way be partly or completely removed with a triangular deformation pointing outwards. Triangular deformation appears most effective for small aspect ratios (Fig.3). However fat toruses are now favored on the basis of economical considerations; in such a case q(axis) is expected to be larger than in the circular case. For instance for the shape of Fig.2 (R/a = 2.5, k = 2, c/a = 0.3) q(axis) = 1.4. Using the ralation between q(axis) and q(boundary) calculated for the parabolic current density profile, this corresponds to q(boundary) = 4.7.

Such a large value reduces significantly the advantages of vertical ellipses. Qualitative arguments show that q_b/q_{axis} would be lower in a racetrack where the curvature of the magnetic surfaces is more evenly distributed. Moreover a kidney shape can also provide a reduced value of q_{axis}.

III.2. Kink modes

MHD kink modes appear to be very dangerous since there always exists unstable domains such that

$$m\varepsilon(m) \leq nq_b < m$$

where m and n are integers representinq the number of periods of the perturbation along the θ and ϕ directions respectively.

For that current profile the size of the unstable domain $\varepsilon(m)$ is already large for circular cross section (Fig.4a) it becomes catastrophically large when k increases so that all stability domains disappear when k > 3. It should also be noted that $\varepsilon(m)$ remains for "parabolic" current profile which is a more realistic assumption in ohmically heated plasmas the situation is strickingly different. Numerical calculation[4] show that $\varepsilon(m)$ is considerably reduced and remains unchanged when k is increased (Fig.4b). It also appears that $\varepsilon(m)$ decreases exponentially with m. This stabilizing effect is related to the increased shear appearing with peaked current distribution and elongated plasmas.

III.3. Translational instability

Elliptic cross sections are always unstable with respect to the m = 1 mode which corresponds to a translation along the major axis of the ellipse. However this new instability can be stabilized[6] with a shell. Stabilization is always achieved when the shell is entirely inside the surface joining the locus of purely vertical (if $k > 1$) poloidal field (see dotted line on Figure 5). Stabilization can also been obtained with only elements of shell correctly located. For instance it is effective to introduce 4 passive rods on each side of the stagnation points at a place where a translational motion induces a large variation of the poloidal flux through their surfaces.

Conclusion

The advantages and drawbacks of Tokamak with non circular cross section are summarized on Table 1. Vertical elongation of the plasma cross section is expected to provide a substantial overall improvement of the plasma parameters. In such configurations ohmic heating will allow to reach higher β and T_i values. The configuration appears most approximate to investigate scaling laws in the collisionless regime. Let us recall that the ratio between the ion temperature obtained by ohmic heating with the banana diffusion law to the transition temperature between the plateau and banana regime is:

$$\frac{T_i}{T_i^{x}} = \left(B^2 a^2 R k\right)\left(\frac{a}{R}\right)^{9/4} \frac{k}{R^{3/2}} \frac{1}{q^{3/2}\sqrt{n Z_{eff}}}$$

This result constitutes the main motivation of the modern trend towards fatter toruses with vertical elongation.

Improvement of the ion confinement time will make more effective the additional heating methods, in particular those which require confinement of a group of high energy particles.

These improvements are obtained mainly at the cost of building shaping coils driven by a large current (about twice the plasma current). These coils may turn out to be expensive since, being of multipole nature, they will have to be inserted between the liner and the coils producing the toroidal field. However the use of shaping coils allows to vary the plasma shape by changing the current distribution. This flexibility will be used to check the relevance of theory concerning interchange modes.

Interchange modes localized in the vicinity of the magnetic axis appear indeed to be the most dangerous drawback for the fat vertically elongated plasmas which are generally favored for future Tokamaks (Table 2). However the effect of interchange modes on the magnetic axis may not lead to serious losses but only to flattening of the density and current density gradients. The optimum shape of the plasma is not an ellipse. It seems preferable to choose a racetrack which has a lower q_b/q_{axis} or a kidney shape which has a lower limiting q_{axis} and would be the best choice if the interchange modes are the limiting phenomena.

The main experiments and projects of low β Tokamaks with non circular cross section are reviewed in Table 2. Among the existing experiments Doublet II A has already demonstrated the larger current density in an elongated plasma. Preliminary measurements in Finger Ring at low current level have been shown to be compatible with the theoretical prediction of the shape of the plasma inside a D shaped copper shell.

Ambitious new experiments are presently being projected to investigate Doublet configurations (Doublet III), horizontal elongations[7] (JFT 2A), rectangular shapes which are stable with respect to the translational instability (PDX) and vertical elongations (TESEE, JFT4). The european project JET takes also advantage of a moderate vertical elongation which, in addition to the improvements already discussed, allows to create the toroidal field with pure tension D shaped coils.

Another group of experiments makes use of fast compression heating to study the high β properties of elongated Tokamak plasmas. In these experiments transient plasmas with high initial β are produced and stability can be investigated during the free plasma decay. Among the facilities concerning this approach we may mentioned ISAK TX[10] and TENQ.[11]

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	Advantages	Draw-backs
Arguments of Physical Origin	Diffusion ($k > 1$)	Interchange modes $k > 1$
	Limiting β ($k > 1$)	Translational instability
	Modification of Stability	Unknown physics (tearing modes)
	Increase current density ($k > 1$)	
Arguments of Economical or Practical Origin	Natural shape of magnets ($k > 1$)	Large current in shaping coil
	Compatible with axisymmetric divertors	Lower inductance in primary windings

TABLE 1

Plasma parameters	Finger Ring	Doublet 2A	JFT 2A	Doublet III	JFT 4	TESEE	PDX	JET	PLT
R (m)	0.36	0.6	0.6	1.4	1.2	1.3	1.4	2.72	1.3
b (m)	0.11	0.45	0.08	1.4 (?)	0.8	1.1	0.7	1.98	0.45
a (m)	0.055	0.15	0.11	0.45	0.27	0.5	0.35	1.28	0.45
k	2	3	0.7	3	3	2.2	2	1.55	1
I_p (q=3) (MA)	0.14	0.18	0.1	1.7	0.6	1.5		3	1.6
B_T (kG)	15	10	10	26	20	20	25	30	50

TABLE II

FIGURE CAPTIONS

Fig.1. High β limit of Tokamak with elliptical cross section.

Fig.2. Plasma cross section with combined elliptical and triangular deformations.

Fig.3. Minimum value of q on axis for stability against localized MHD modes in the vicinity of the magnetic axis.

Fig.4a Unstable domains for kink modes in the case of uniform current distribution.

Fig.4b Unstable domains for kink modes in the case of parabolic current distribution.[4] $\bar{J}/J_o$ is the averaged current density compared to the circular case. A confocal elliptical shell is assumed to have the vertical dimension b'.

Fig.5. Magnetic surfaces and shape of the shell (dotted line) which provides marginal stability with respect to the translational instability.[6]

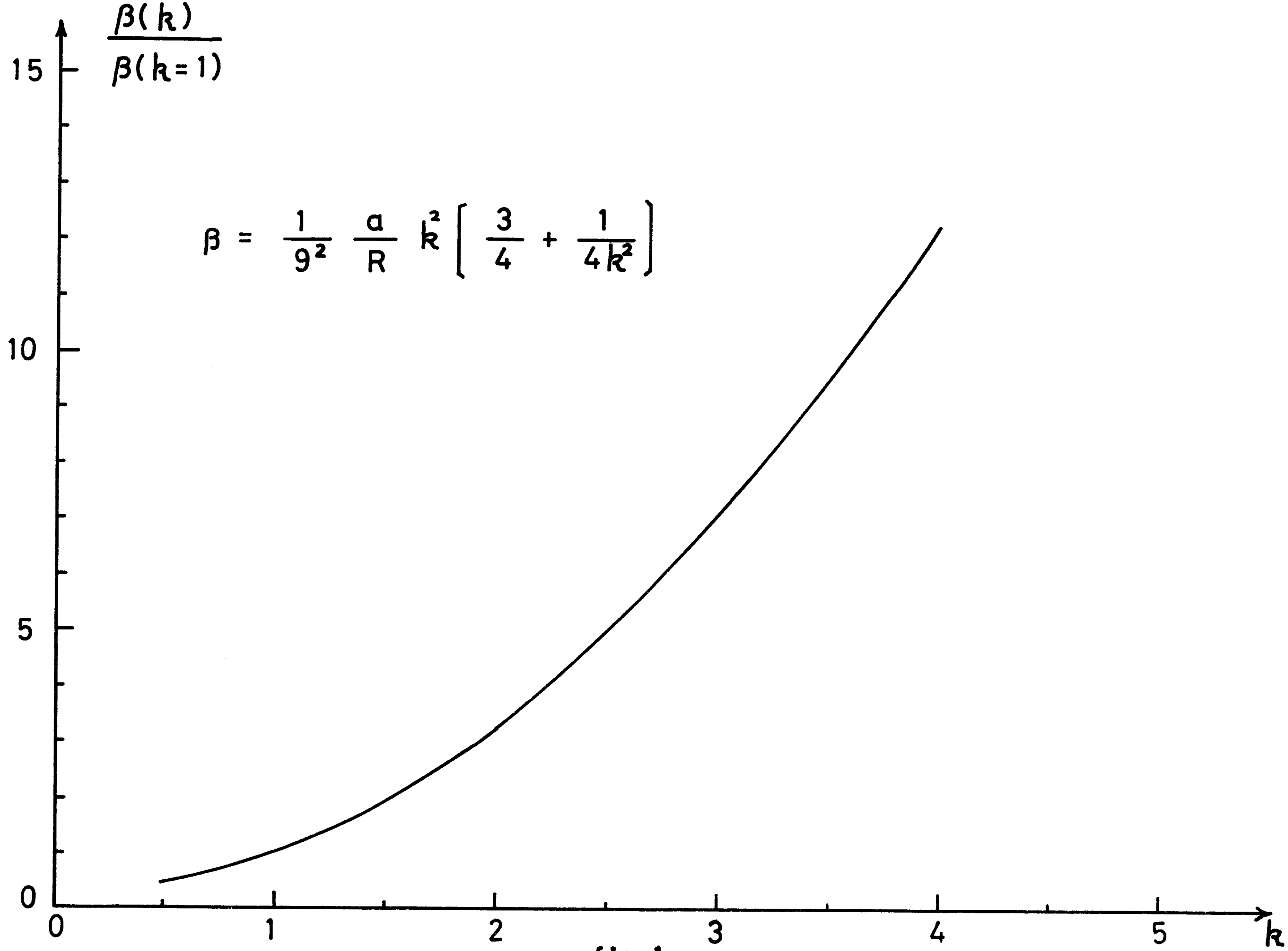
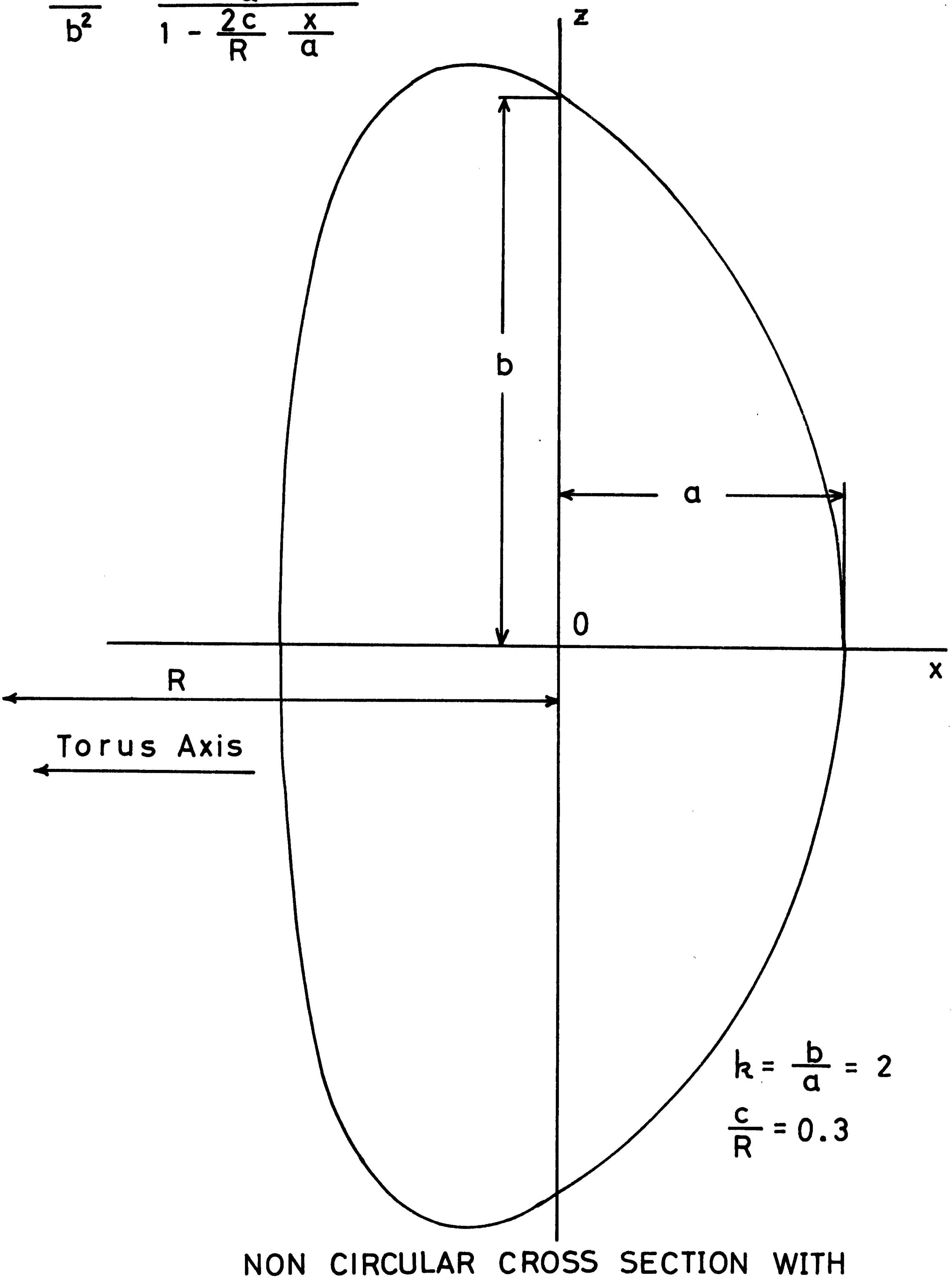


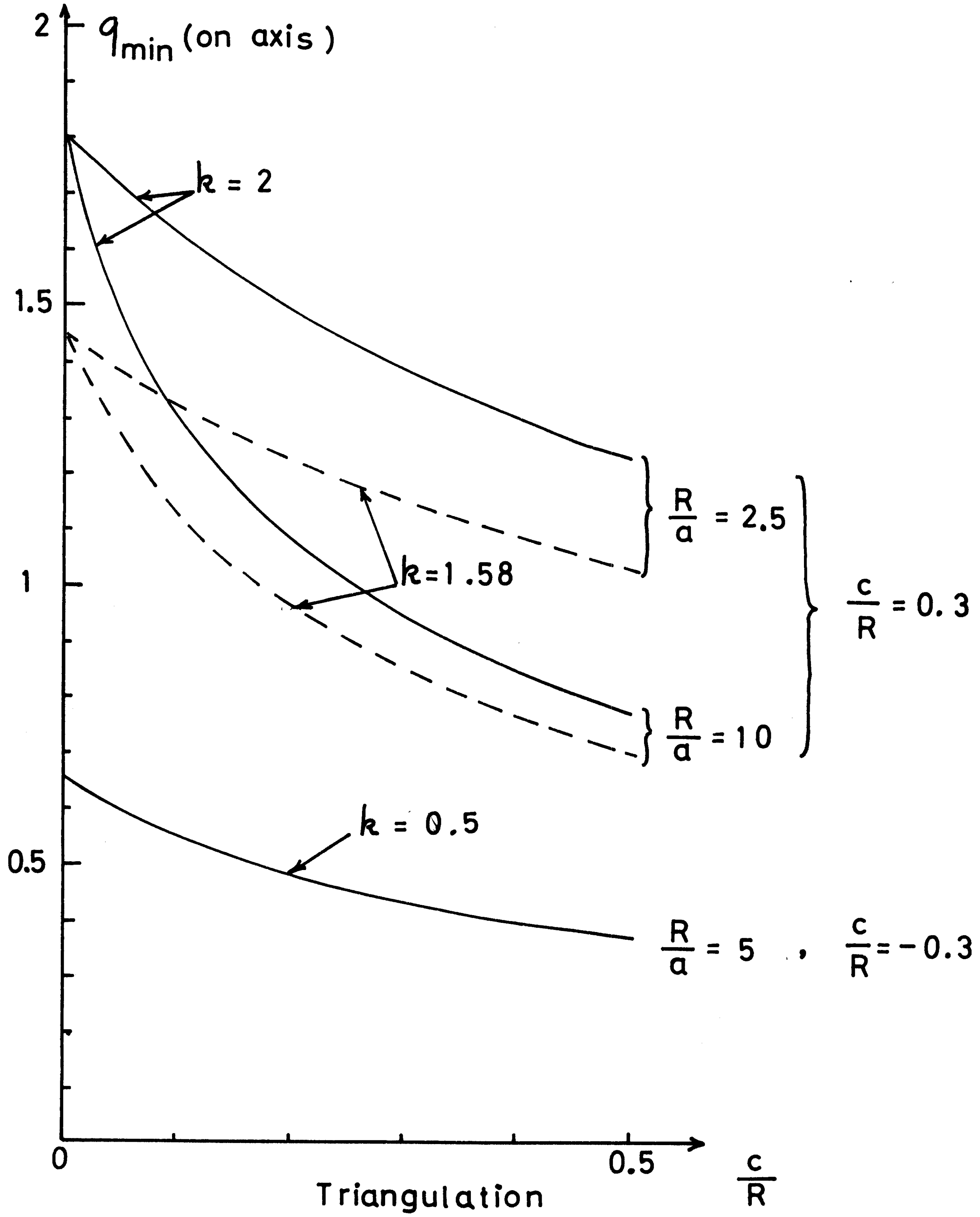
fig 1

$$\frac{z^2}{b^2} \quad \frac{1-\frac{x^2}{a^2}}{1-\frac{2c}{R}\,\frac{x}{a}}$$



NON CIRCULAR CROSS SECTION WITH TRIANGULATION

fig 2



Triangulation

fig 3

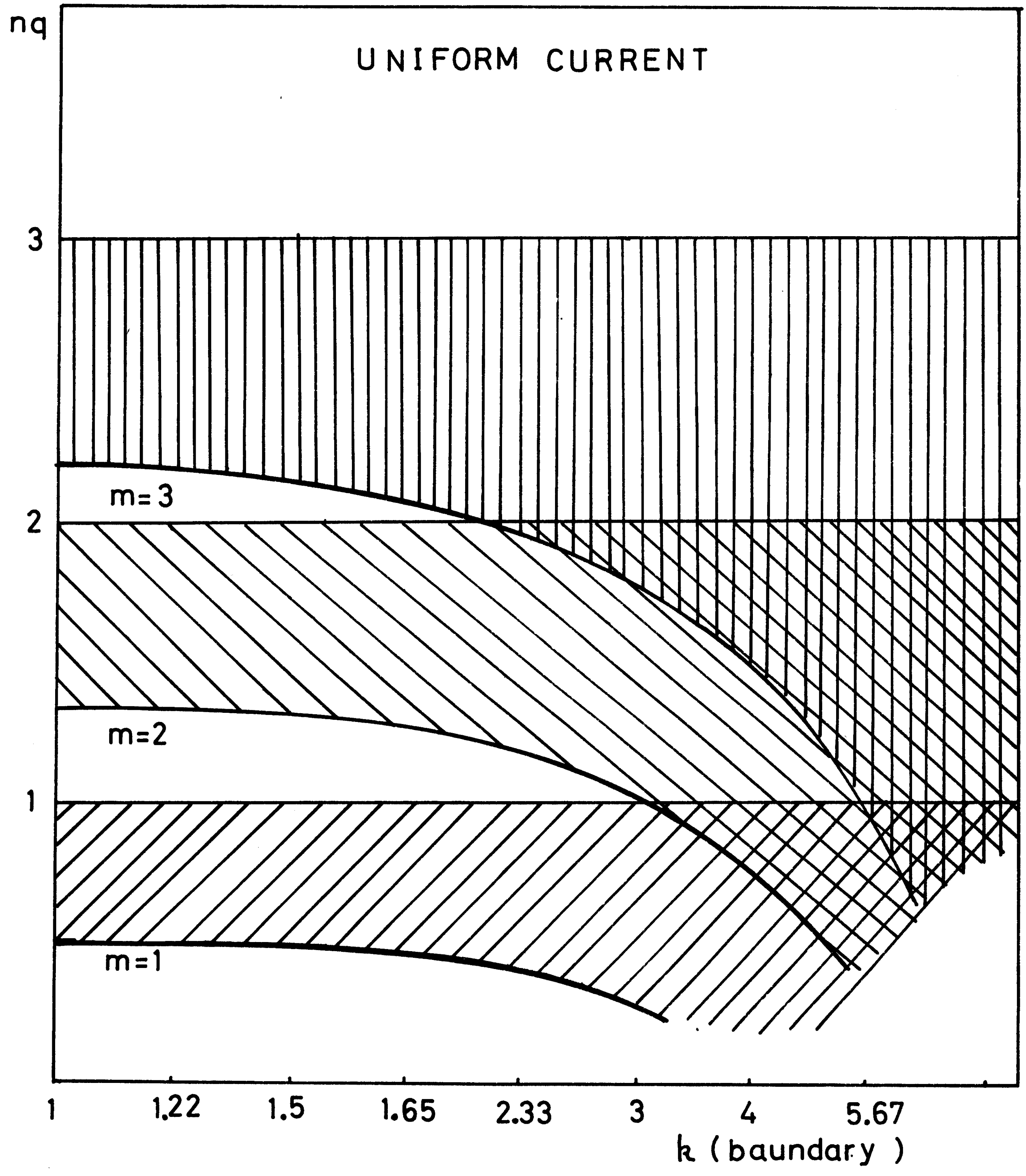

nq
UNIFORM CURRENT
3
m= 3
2
m=2
1
m=1
1
1.22
1.5
1.65
2.33
3
4
5.67
k (baundary)

fig 4a

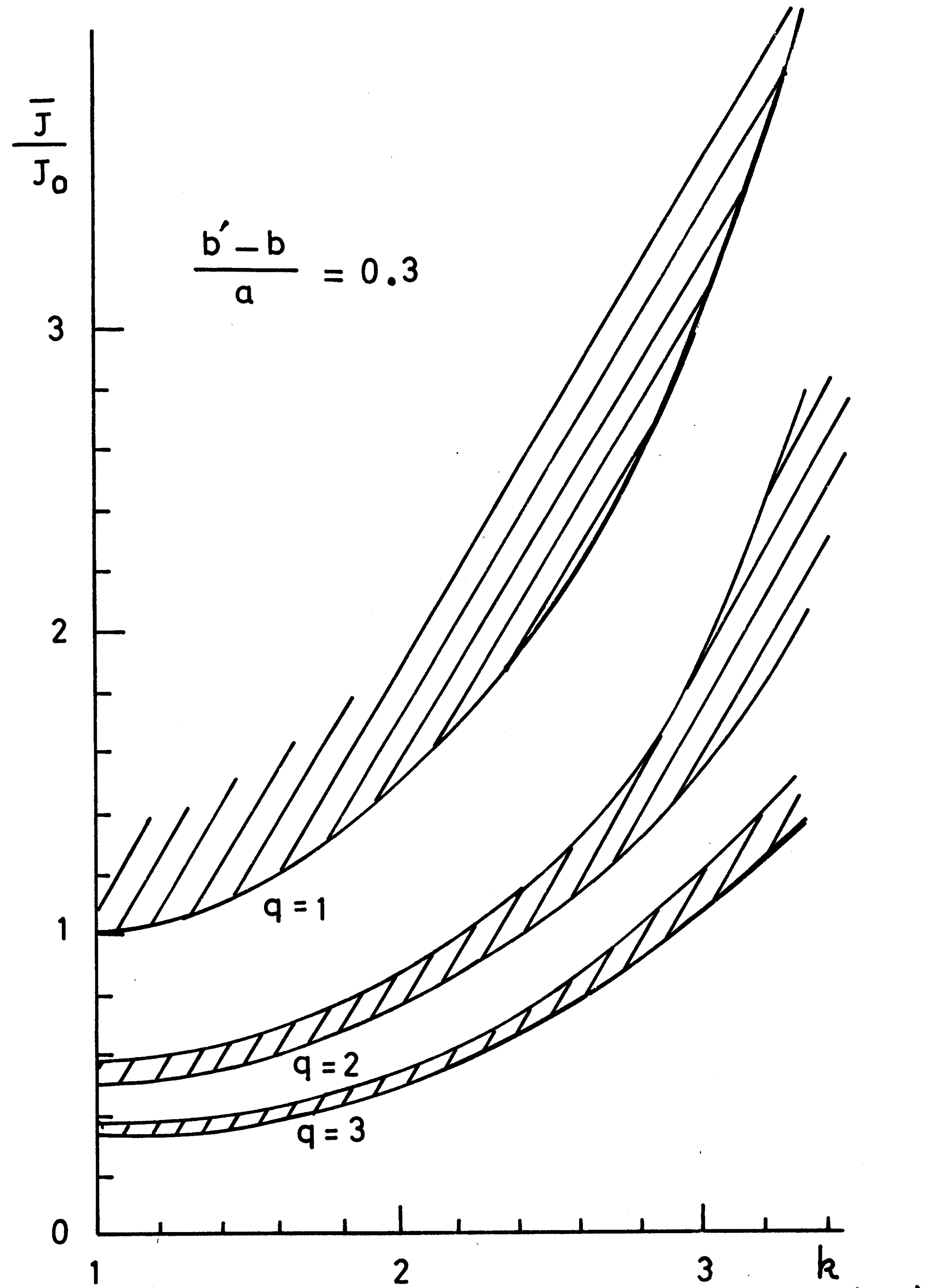


fig 4b

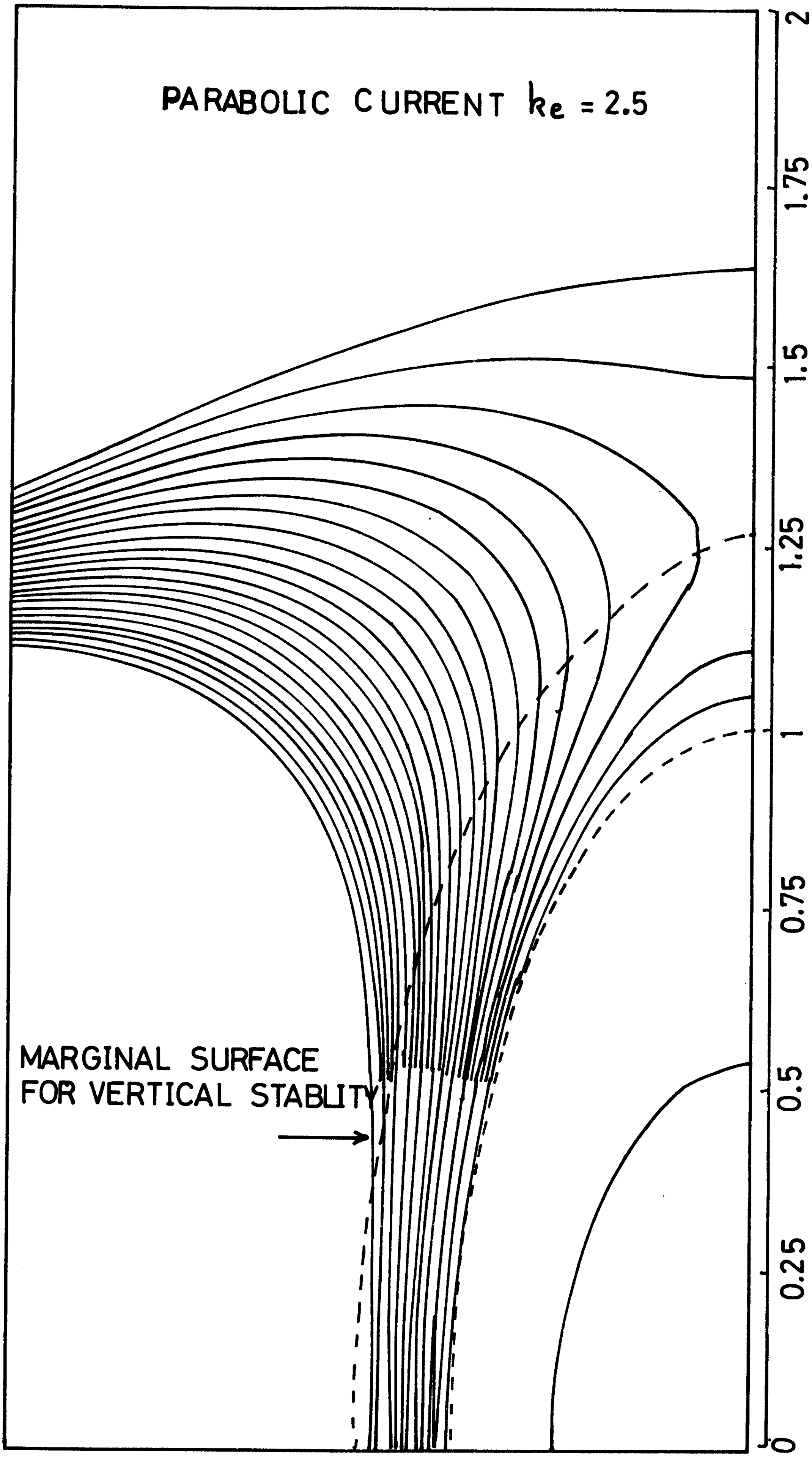

PARABOLIC CURRENT ke = 2.5
MARGINAL SURFACE
FOR VERTICAL STABLITY
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2

fig 5