A new nonlinear electromagnetic gyrokinetic equation is derived for plasmas with large flow velocities on the order of the ion thermal speed [1,2]. The lowest-order flow velocity $V_0$ is related to the electric field $E_0$ by $E_0 + \frac{1}{c} V_0 \times B = 0$. We use the phase variables $(x, w, \mu, \xi)$ in which the particle position $x$ is observed from the laboratory frame while the particle kinetic energy $w$, the magnetic moment $\mu$, and the gyrophase $\xi$ are defined in terms of the velocity $v'$ in the moving frame as $w = \frac{1}{2} m a (v')^2$, $\mu = m a (v')^2 / (2B)$, and $v'_l/v'_l = e_1 \cos \xi + e_2 \sin \xi$, where $(e_1, e_2, \mathbf{b} = B/B)$ are unit vectors which forms a right-handed orthogonal system at each point, and $v' = v'_l \mathbf{b} + v'_n$ with $v'_l = v' \cdot \mathbf{b}$. The lowest-order distribution function $f_{ao}$ for the species $a$ is independent of the gyrophase $\xi$, $\partial f_{ao}/\partial \xi = 0$, and satisfies

$$\overline{C_0 f_{ao}} = \left[ (V_0 + v'_l \mathbf{b}) \cdot \nabla + \frac{(dw/dt)_0}{c} \frac{\partial}{\partial w} \right] f_{ao} = C_a(f_{ao}),$$

where $C_a$ represents the collision operator, and $(dw/dt)_0 = -m a V_0 \cdot \nabla V_0 \cdot \mathbf{b} + e_a E_1 \cdot \mathbf{v}_l - m a (v'_l)^2 \mathbf{b} \cdot \nabla V_0 \cdot \mathbf{b} - \frac{1}{2} m a (v'_l)^2 (\nabla \cdot V_0 - \mathbf{v}_l \cdot \nabla V_0) \cdot \mathbf{b}$. Here, the magnetic moment is conserved along the lowest-order guiding center orbit, $(d\mu/dt)_0 = 0$, where $\mu \equiv f \cdot \mathbf{d} \xi / 2\pi$ denotes the gyrophase average.

The gyrokinetic ordering employed here for the turbulent fluctuations is written in terms of the ordering parameter $\delta \equiv \rho_d/L$ ($\rho_d$: the gyroradius, $L$: the equilibrium gradient scale length) as $\hat{f}_a/f_a \sim e_a \hat{\phi}/T_a \sim e_a v_T a \hat{A}/(c T_a) \sim k_l/k_p \sim (\omega - k_p \cdot V_0) / \Omega_a \sim \delta$, where $(\omega - k_p \cdot V_0)$ denotes the characteristic frequency observed in the moving frame. The characteristic parallel and perpendicular wavenumbers are given by $k_l \sim L^{-1}$ and $k_p \sim \rho_a^{-1}$, respectively. In the presence of the large flow $V_0$ on the order of the thermal speed, the first-order guiding center drift velocity $v_{da}$ is given by

$$v_{da} = \frac{c \mu}{e_a B} (\nabla \times \mathbf{B}) \cdot \mathbf{b} + \frac{c}{e_a B} \mathbf{b} \times [\mu \nabla B + m a (v'_l)^2 \mathbf{b} \cdot \nabla B + e_a \nabla \Phi_1 + m a V_0 \cdot \nabla V_0 + m a v'_l \mathbf{b} \cdot \nabla V_0 + m a v'_l [V_0 \cdot \nabla \mathbf{b}]].$$

The fluctuating part of the distribution function is written as

$$\tilde{f}_a(x) = e_a \left( \hat{\phi}(x) - \frac{1}{c} V_0 \cdot \hat{A}(x) \right) \frac{\partial f_{ao}}{\partial w} + e_a \left[ \left( \hat{\phi}(x) - \frac{1}{c} (V_0 + v'_l \mathbf{b}) \cdot \hat{A}(x) \right) - \hat{\psi}_a(X) \right] \frac{\partial f_{ao}}{B \partial \mu} + \tilde{h}_a(x),$$

where $X \equiv x - \rho_d$ denotes the position of the guiding center, $\hat{\psi}_a(X) = \left< \hat{\phi}(X + \rho_d) - \hat{\psi}_a(X + \rho_d) \right> X$, and $< \cdot > X$ represents the gyrophase average with $X$ fixed. Finally, the nonadiabatic part $\tilde{h}_a$ of the fluctuating distribution function is determined by the gyrokinetic equation,

$$\left[ \frac{\partial}{\partial t} + \overline{Z_0} + \left( v_{da} - \frac{c}{B} \nabla \hat{\psi}_a(X) \times \mathbf{b} \right) \cdot \nabla \right] \tilde{h}_a(X) = \frac{c}{B} \nabla \hat{\psi}_a(X) \times \mathbf{b} \cdot \left[ \nabla - \left( m a V_0 \cdot \nabla V_0 \right) + m a v'_l \mathbf{b} \cdot \left[ \nabla V_0 \right] + \left( m a v'_l \mathbf{b} \right) \cdot \nabla V_0 \right] \cdot \mathbf{b}$$

References

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