10. Radial Electric Field Dependence of Neoclassical Poloidal and Toroidal Viscosity Coefficients

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We have presented a novel method to obtain the full neoclassical transport matrix for general toroidal plasmas by using the solution of the linearized drift kinetic equation with the pitch-angle-scattering collision operator [1]. This method can also be applied to investigation of the \( \mathbf{E} \times \mathbf{B} \) drift effects on the neoclassical transport coefficients based on the drift kinetic equation given by

\[
V f_a - C^L(\mathbf{f}_a) = -\mathbf{v}_a \cdot \nabla f_a + \frac{e_a}{T_a} \eta \mathbf{v}_a \mathbf{B}(\mathbf{E}^L) + \frac{T_a}{\mathbf{B}^2} \mathbf{f}_a M_a,
\]

where the operator \( V \equiv V_p + V_E \) consists of the parallel motion part

\[
V_p = u \mathbf{\xi} \cdot \nabla - \frac{1}{2} u (1 - \xi^2) (\mathbf{b} \cdot \nabla \ln B) \frac{\partial}{\partial \xi},
\]

and the \( \mathbf{E} \times \mathbf{B} \) drift part

\[
V_E \equiv u \mathbf{\xi} \cdot \nabla = \frac{c E_a}{(B^2)} \nabla s \times \mathbf{B} \cdot \nabla,
\]

with \( \nabla \) taken for \((u, \xi \equiv \eta \eta / v)\) being fixed. The \( \mathbf{E} \times \mathbf{B} \) drift operator \( V_E \) has the same form as employed in the DKES [2]. Here, we assume the incompressibility conditions \( \nabla \cdot u_a = \nabla \cdot q_a = 0 \) and the stellarator symmetry \( B(s, \theta, \zeta) = B(s, -\theta, -\zeta) \).

In helical systems, the poloidal and toroidal viscosities of \([\{B_p \cdot (\nabla \cdot \mathbf{\pi}_a)\}, \{B_p \cdot (\nabla \cdot \Theta_a)\}], \{B_T \cdot (\nabla \cdot \mathbf{\pi}_a)\}, \{B_T \cdot (\nabla \cdot \Theta_a)\}]\) are related to the poloidal and toroidal flows \([\{(u_a^{\|})/\chi, (u_a^{\perp})/\chi, (u_a^\perp)/\psi, (u_a^\perp)/\psi\}]\) by

\[
\begin{bmatrix}
\{B_p \cdot (\nabla \cdot \mathbf{\pi}_a)\} \\
\{B_p \cdot (\nabla \cdot \Theta_a)\} \\
\{B_T \cdot (\nabla \cdot \mathbf{\pi}_a)\} \\
\{B_T \cdot (\nabla \cdot \Theta_a)\}
\end{bmatrix} =
\begin{bmatrix}
M_{a1PP} & M_{a2PP} & M_{a1PT} & M_{a2PT} \\
M_{a2PP} & M_{a3PP} & M_{a2PT} & M_{a3PT} \\
M_{a1PT} & M_{a2PT} & M_{a1TT} & M_{a2TT} \\
M_{a2PT} & M_{a3PT} & M_{a2TT} & M_{a3TT}
\end{bmatrix}
\begin{bmatrix}
\{(u_a^{\|})/\chi\} \\
\{q_a^\perp/\chi\} \\
\{(u_a^\perp)/\psi\} \\
\{q_a^\perp/\psi\}
\end{bmatrix}.
\]

Here, the Onsager-symmetric poloidal and toroidal viscosity coefficients \(M_{a1PP}, M_{a1PT}, \text{ and } M_{a1TP} \) (\(j = 1, 2, 3\)) are also written in the form of the energy integral:

\[
[M_{a1PP}, M_{a1PT}, M_{a1TT}] = n_a \frac{2}{\sqrt{\pi}} \int_0^\infty dK K K e^{-K}
\]

\[
\times \left( K - \frac{5}{2} \right)^{-1} [M_{aPP}(K), M_{aPT}(K), M_{aTT}(K)],
\]

where \(M_{aPP}(K), M_{aPT}(K), \text{ and } M_{aTT}(K)\) represent contributions of monoenergetic particles to \(M_{aPP}, M_{aPT}, \text{ and } M_{aTT}, \) respectively.

Figure 1 shows the normalized monoenergetic neoclassical viscosity coefficients \([M_{1PP}, M_{1PT}, M_{1TT}] \equiv [M_{PP}(K), M_{PT}(K), M_{TT}(K)]/[(4\pi^2/V') n u v T(\psi/\chi)^2 K^3/2]\) as a function of \(c E_a/v\), which are obtained by combining our method with numerical output of the DKES for the magnetic field strength given by

\[
B = B_0 [1 - \epsilon_t \cos \theta_B - \epsilon_h \cos(\theta_B - n \xi_B)]
\]

with \(B_0 = 1 \text{T}, \epsilon_t = 0.1, \epsilon_h = 0.05, l = 2, \text{ and } n = 10.\) Here, \(\nu_D/v = 3 \times 10^{-6}\) is used, which corresponds to the \(1/v\) regime for the case of \(E_a = 0.\) In Fig. 1, \(M_{PP} \approx -M_{PT} \approx M_{TT}\) (which implies small parallel viscosities) and their reduction with increasing \(c E_a/v\) are clearly seen. The \(E_a\)-dependent neoclassical transport coefficients for radial fluxes and parallel currents can be calculated as well.

Reference