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Critical energetic particle distribution in phase space for the Alfvén eigenmode burst with global resonance overlap

Y. Todo

National Institute for Fusion Science, National Institutes of Natural Sciences, Toki 509-5292, Japan

E-mail: todo@nifs.ac.jp

Abstract. Comprehensive computer simulations of the Alfvén eigenmode burst, which is the synchronized sudden growth of multiple Alfvén eigenmodes interacting with energetic particles, were conducted with continuous neutral beam injection, collisions, and particle losses. It is found that the energetic-particle distribution in phase space reaches a “critical distribution” with a stairway structure where a resonance overlap triggers the Alfvén eigenmode burst. Before the burst, the gradual growth of the Alfvén eigenmodes associated with the beam injection broadens the resonant regions in phase space forming the distribution into a stairway shape. When the distribution reaches the “critical distribution,” a resonance overlap triggers multiple resonance overlaps leading to the synchronized growth of Alfvén eigenmodes and the collapse of the distribution. For another run with the beam deposition power reduced to one-half, the fast ion distribution function just before the Alfvén eigenmode burst is close to that for the original beam power. This result indicates that the critical distribution for the Alfvén eigenmode burst is present.

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1. Introduction

Alfvén eigenmodes (AEs) are magnetohydrodynamic (MHD) oscillations in magnetically confined plasmas [1–4]. AEs are intrinsically stable but can be excited by the resonant interaction with energetic particles. The most important energetic particles that excite AEs in the present magnetic confinement experiments are fast ions generated by neutral beam injection (NBI) and ion-cyclotron-range-of-frequency heating [5–8]. Both types of AE amplitude evolution, steady or intermittent, are observed in the experiments. For the intermittent time evolution, the synchronized sudden growth of multiple AEs sometimes takes place repetitively accompanied by fast ion losses [9–17]. We call these phenomena “Alfvén eigenmode bursts.” Since the fast ions play an important role in the bulk plasma heating to achieve high temperature needed for fusion reaction, AE burst is an important issue for magnetic confinement fusion research.

It was theoretically predicted that the resonance overlap of multiple AEs is the key process for AE burst [18]. Simulations of 1-dimensional bump-on-tail instability with source and sink demonstrated that the bursts of the multiple waves take place repetitively through the resonance overlap [19,20]. Simulations of AE bursts in tokamak plasmas have been also conducted successfully [21–25]. However, the time evolution of the fast ion distribution function and the trigger mechanism of the AE burst have not been clarified yet because huge computational resources are demanded for the analysis of the distribution function in 5-dimensional phase space. The 6-dimensional phase space $(\mathbf{x}, \mathbf{v})$, where $\mathbf{x}$ and $\mathbf{v}$ represent respectively the 3-dimensional position and velocity of particle, is reduced to 5-dimension $(\mathbf{x}, v_{\parallel}, \mu)$ with $v_{\parallel}$ parallel velocity to the magnetic field and μ magnetic moment, because the frequency of AE is so low that magnetic moment is an adiabatic invariant and the dependence on particle gyration phase can be neglected.

The kinetic-MHD hybrid simulation of energetic particles interacting with an MHD fluid is a useful tool to understand and predict the energetic-particle driven AEs [26–34]. The fast ion distributions in the experiments are formed in a four orders of magnitude longer time scale (=collisional slowing-down time for fast ions, $\sim 100\text{ms}$) than the typical AE oscillation period ($\sim 0.01\text{ms}$). We developed the multi-phase simulation, which is a combination of classical simulation and hybrid simulation for fast ions interacting with an MHD fluid, in order to investigate the fast ion distribution formation process with neutral beam injection, collisions (slowing-down, pitch-angle scattering, and energy diffusion), losses, and transport due to the AEs with the fast-ion finite Larmor radius effect and the MHD nonlinearity [35]. In the classical simulation, fast ion orbits are followed in the equilibrium magnetic field with beam injection and collisions while the MHD perturbations are turned off. In the multi-phase simulation, we run alternately the classical simulation without MHD perturbations and the hybrid simulation with MHD perturbations. The multi-phase simulation is a comprehensive simulation, which deals with both the AEs and the fast ion transport as self-consistently and realistically as possible, yet attainable on a tractable timescale. The multi-phase hybrid simulations

using the MEGA code have been validated on DIII-D, JT-60U, and LHD experiments [25, 35–39].

We simulated the AE bursts and reproduced many aspects observed in the experiments [24]. However, although the time evolution of fast ion distribution function is the key to demonstrate the resonance overlap, the distribution function during the AE burst has not been clarified yet. In this article, we report the simulations with 16 times increased number of computational particles than the previous work [24] and the analyses of the fast ion distribution function. The number of computational particles is increased to 67 million in the present work from 4.2 million in the previous work. We clarify the time evolution of the fast ion distribution function and the trigger mechanism of the AE burst.

2. Simulation model

We use the MEGA code, in which the bulk plasma is described by the nonlinear MHD equations and the fast ions are simulated with the gyrokinetic particle-in-cell (PIC) method [40]. The effects of fast ions on the MHD fluid are taken into account through the fast-ion current density without $E \times B$ drift in the MHD momentum equation with an assumption that the fast-ion density is much less than the bulk plasma density. The electromagnetic fluctuations are averaged over the fast ion gyro orbit for the fast ion dynamics. We use the standard MHD equations with the fast ion effects. The equations employed in this work are given in Ref. [24]. MEGA code participated in the code benchmark of the Energetic Particle Physics Topical Group of the International Tokamak Physics Activity. Good agreements were found in the spatial profile, frequency, and growth rate of a toroidal Alfvén eigenmode (TAE) among the nine simulation codes [41].

We employ the tokamak equilibrium and the physical condition used in Ref. [24]. Cylindrical coordinates (R, φ, z) are used in the simulations. For the purpose of the data analysis, magnetic flux coordinates (r, φ, ϑ) were constructed for the MHD equilibrium where r is the radial coordinate with $r = 0$ at the plasma center and $r = a$ at the plasma edge, and ϑ is the poloidal angle. The physical parameters are similar to the Tokamak Fusion Test Reactor (TFTR) experiment [9] with minor radius $a = 0.75\text{m}$, major radius $R_0 = 2.4\text{m}$, magnetic field at the plasma center $B_0 = 1\text{T}$, and beam injection energy 110keV . The half-energy and the third-energy components are neglected for simplicity. Both the bulk and beam ions are deuterium. The bulk ion density is $2.8 \times 10^{19}\text{m}^{-3}$, and the bulk plasma beta value, which is the ratio of bulk plasma pressure to the magnetic pressure at the plasma center, is assumed to be 1%. The beam injection velocity corresponds to $v_b = 1.1v_A$, where v_A is the Alfvén velocity at the plasma center. The safety factor profile is assumed to be $q(r) = 1.2 + 1.8(r/a)^2$. A balanced beam deposition profile is assumed, which is in proportion to $\exp[-(r/0.4a)^2] \times \exp\{-(v - v_b)/\Delta v\}^2\} \times \exp\{-[(|\lambda| - \lambda_0)/\Delta\lambda]^2\}$, where v is total velocity, $v_{\parallel}$ is parallel velocity to the magnetic field, $\lambda = v_{\parallel}/v$, $\Delta v = 0.1v_b$,

Figure 1. Time evolution of (a) stored fast ion energy for co-going and counter-going particles to the plasma current and (b) MHD kinetic energy for $P_{\text{NBI}}=10\text{MW}$ and $\tau_s=100\text{ms}$. Hybrid simulation is continuously run after $t=30\text{ms}$.

$\lambda_0 = 0.7$, and $\Delta\lambda = 0.3$. The numbers of grid points are (128, 128, 128) for (R, φ, z) coordinates, respectively, and the number of computational particles is 6.7×10^7 . The number of computational particles is increased by a factor of 16 from the previous work [24]. This improves the numerical resolution in the analysis of fast ion distribution function. Collisions of fast ions with thermal electrons and ions (slowing-down, pitch-angle scattering, and energy diffusion) are implemented in the MEGA code. A fast ion loss condition, where particles are removed when they reach the plasma edge located at $r/a = 1$, is imposed. The fast-ion finite Larmor radius effect is taken into account for the interaction with the Alfvén eigenmodes.

3. Simulation results

3.1. Alfvén eigenmode bursts

We performed the multi-phase hybrid simulations for various beam deposition power (P_{NBI}) and slowing-down time (τ_s) in the tokamak plasmas. Figure 1 shows the time evolution of stored fast ion energy and MHD kinetic energy in the simulation for $P_{\text{NBI}}=10\text{MW}$ and $\tau_s=100\text{ms}$, which are similar to the TFTR experiment. The stored fast ion energy evolution is shown for co-going and counter-going particles to the plasma current. In this simulation, a combination of classical simulation for 2ms and hybrid simulation for 0.5ms was run until $t = 30\text{ms}$, and the hybrid simulation was continuously run after $t = 30\text{ms}$ until $t = 45\text{ms}$. The stored fast ion energy increases at the beginning due to the beam injection and is saturated at $t = 40\text{ms}$. We see that the AE bursts take place during the continuous hybrid simulation after $t = 30\text{ms}$ with a time interval of 2.8ms which is close to the TFTR experiment [9]. The stored fast ion energy drops at the AE bursts both in the multi-phase simulation ($t < 30\text{ms}$) and the continuous hybrid

Figure 2. (a) Frequency spectra for toroidal mode numbers $n=1-5$, and toroidal electric field for the dominant frequency peaks in a poloidal plane for (b) $n=1$, $f=37\text{kHz}$, (c) $n=2$, $f=42\text{kHz}$, (d) $n=2$, $f=56\text{kHz}$, (e) $n=3$, $f=56\text{kHz}$, and (f) $n=3$, $f=64\text{kHz}$. Magnetic flux surfaces are shown for $r/a=0.2, 0.4, 0.6, 0.8$ and 1.0 in panels (b)-(f).

simulation ($t \geq 30\text{ms}$). The drop in fast ion stored energy at each burst is attributed to fast ion losses. The loss of counter-going fast ions is larger than that of co-going fast ions. This property was observed in the previous simulations [22, 24] and is explained in Ref. [42]. Counter-going particles are easier to lose than co-going particles because counter-going particles reach the plasma edge with less energy transfer to the AEs.

The frequency spectra of toroidal mode numbers $n=1-5$ and the toroidal electric field for the dominant frequency peaks are shown for $t=30-45\text{ms}$ in Fig. 2. The most dominant modes in the simulation are the $n=2$ and $n=3$ modes with frequency 56kHz shown in Fig. 2(d) and (e). The time evolution of the primary harmonics of the dominant modes is shown in Fig. 3(b) and (c). The saturation amplitude of radial MHD velocity fluctuation is $v_r/v_A \sim 3 \times 10^{-3}$, which is consistent with the TFTR experiment [22, 43]. We see in the figure that the growth and the decay of the dominant modes are synchronized. We analyzed the fast ion energy transport flux in the radially outward direction. The fast ion energy transport flux is the energy transport flux brought about by the electromagnetic field of the AEs. The definition is given in Appendix of Ref. [37]. The time evolution of the fast ion energy transport flux profile is shown in Fig. 3(a). We see in the figure that global transport takes place at each AE burst. It is interesting to note that the maximum energy transport flux reaches the

Figure 3. (a) Radial profile evolution of fast ion energy transport flux in the radially outward direction, and amplitude evolution of radial MHD velocity fluctuation for (b) the dominant $n=2$ (red) and $n=3$ (blue) AEs and (c) the other AEs with $n=1-3$. The unit of color bar is MW.

huge level of 60MW, which is 6 times larger than the beam deposition power.

3.2. Fast ion distribution function in phase space

For the understanding of the synchronized growth, we have analyzed the fast ion distribution function in phase space. It is convenient to express the distribution as a function of constants of motion in the axisymmetric equilibrium magnetic field. The constants of motion in the axisymmetric equilibrium magnetic field are particle kinetic energy (E), toroidal canonical momentum (P_φ), and magnetic moment (μ). Magnetic moment is an adiabatic invariant for the interaction with the AEs whose frequency is sufficiently lower than the Larmor frequency. Then, the analysis of the distribution function $f(P_\varphi, E)$ which is integrated in poloidal and toroidal angles for a constant μ is illuminative to understand the interaction between energetic particles and AEs. Toroidal canonical momentum is a radius-like coordinate in phase space and is given by $P_\varphi = e_h \Psi + m_h R b_\varphi v_\parallel$ where e_h and m_h are energetic particle charge and mass, respectively, Ψ is poloidal magnetic flux, and b_φ is the φ component of the unit vector of the magnetic field. The poloidal magnetic flux is $\Psi = \Psi_0$ at the plasma center and $\Psi = 0$

at the plasma edge. If the kinetic momentum $m_h R b_\varphi v_\parallel$ is neglected, $P_\varphi/(e_h \Psi_0) = 1$ at the plasma center and $P_\varphi/(e_h \Psi_0) = 0$ at the plasma edge. However, since the kinetic momentum is not negligible for fast ions, $P_\varphi/(e_h \Psi_0)$ deviates from 0 and 1 at the edge and at the center, respectively.

Figure 4(a) and (b) show the fast ion distribution functions for co-going and counter-going particles to the plasma current, respectively, in (P_φ, E) space for a constant μ and $t = 43.5\text{ms}$ just before the AE burst. The moment $t = 43.5\text{ms}$ is denoted by vertical lines in Fig. 3. We assumed the balanced beam injection, and μ is chosen close to the peak of the beam distribution. We see in Fig. 4(b) that a stairway structure is formed in P_φ direction, which is roughly the radial direction. In Fig. 4(d), we compare the 1-dimensional fast ion distribution function $f(P_\varphi)$ for a constant E along the horizontal line drawn in Fig. 4(b) with that of a classical simulation which started with the data just after the previous AE burst ($t = 41.5\text{ms}$). We can confirm the stairway structure of the distribution function in the hybrid simulation. Since we see in Fig. 3(b) and (c) that the AEs have grown to low amplitude at $t = 43.5\text{ms}$, the stairway distribution is formed by the local flattening at the resonances with the low-amplitude AEs. For co-going particles, we see also the locally flattened regions in Fig. 4(c), but the flattening is weaker than that for counter-going particles shown in Fig. 4(d).

The time evolution of fast ion distribution function $f(P_\varphi)$ with a constant energy $E = 0.55 m_h v_A^2$ and the constant magnetic moment μ are shown in Fig. 5. We see in Fig. 5(a) and (b) the stairway structures with the continuous rise and the collapse starting at $t = 43.9\text{ms}$ for both the co-going and the counter-going particles. The time evolution of the distribution function is also plotted with a constant interval in P_φ in Fig. 5(c) and (d). We see sparse and dense regions of lines, which indicates steep and flat gradient, respectively. The dense regions of lines with colors red, orange, and purple in Fig. 5(d) are locally flattened regions before the AE burst.

In order to clarify the triggering process of the AE burst, the fast ion distribution differences between the hybrid and the classical simulations in (P_φ, E) space for the constant μ are shown in Fig. 6 for various moments. The red (blue) region corresponds to the positive (negative) value, where the distribution of the hybrid simulation is greater (less) than that of the classical simulation. From the plasma center to the plasma edge, a combination of blue and red regions in this order indicates flattening of the distribution, while a combination of red and blue regions in this order indicates steepening. We see in Fig. 6(b) that 4 combinations of blue and red regions are created in this order from the plasma center to the edge, which indicates flattening of the distribution, at $t = 43.5\text{ms}$ just before the burst. We can confirm in this figure that the stairway structure is formed by the interaction with the AEs after the previous burst. At $t = 43.7\text{ms}$ shown in Fig. 6(c), the merging of the two red regions is starting near the plasma center. The merging point is highlighted with a circle in the figure. This indicates the overlap of the two resonances. At $t = 43.9\text{ms}$ shown in Fig. 6(d), one big flattened region (=a combination of blue and red regions) is created near the plasma center by the overlap of the two resonances. At $t = 44.05\text{ms}$ shown in Fig. 6(e), one globally flattened region

Figure 4. Fast ion distribution functions in (P_ϕ, E) space for (a) co-going and (b) counter-going particles with a constant magnetic moment μ at $t = 43.5$ ms just before the AE burst. The right and left edges of the distribution are plasma center and edge, respectively. The distribution functions for a constant energy denoted by the horizontal lines in panels (a) and (b) are compared between the hybrid and the classical simulations for (c) co-going and (d) counter-going particles.

Figure 5. Time evolution of fast ion distribution function $f(P_\phi)$ with a constant energy $E = 0.55m_h v_A^2$ and the constant magnetic moment μ for (a) co-going and (b) counter-going particles to the plasma current. Color represents P_ϕ . The time evolution of the distribution functions is plotted with a constant interval in P_ϕ for (c) co-going and (d) counter-going particles.

Figure 6. Fast ion distribution differences between the hybrid and the classical simulations in (P_φ, E) space for counter-going particles with a constant magnetic moment μ at (a) $t=42.5\text{ms}$, (b) 43.5ms , (c) 43.7ms , (d) 43.9ms , (e) 44.05ms , and (f) 44.2ms . Contours represent the fast ion distributions in the hybrid simulation.

is formed by the overlap of the multiple resonances. At $t=44.2\text{ms}$ shown in Fig. 6(f), the blue region where the distribution is reduced from the classical simulation becomes dominant because the total number of particles decreases due to the particle losses at the plasma edge.

The time evolution of 1-dimensional fast ion distribution function is shown in Fig. 7(a) for counter-going particles with the constant $E = 0.55m_h v_A^2$ and μ . The energy E is denoted by the horizontal line in Fig. 6(b). The top of the figure is the plasma center and the bottom is the plasma edge. We see in Fig. 7(a) a continuous rise in the distribution near the plasma center until $t = 43.9\text{ms}$ and the collapse at $t = 44.05\text{ms}$. The time evolution of the distribution difference between the hybrid simulation and the classical simulation which started at $t = 41.5\text{ms}$ is shown in Fig. 7(b). At $t = 43.9\text{ms}$ the two red regions overlap, which indicates the overlap of the two resonances. The overlap is highlighted by a circle in Fig. 7(b). After $t = 43.9\text{ms}$ the global flattening or the collapse of the distribution takes place. The 1-dimensional distribution functions $f(P_\varphi)$ for $t = 43.5\text{ms}$, 43.9ms , 44.05ms , and 44.2ms , which are denoted by vertical lines in Fig. 7(a) and (b), are shown in Fig. 7(c). We see in Fig. 7(c) that locally flattened regions are formed around $P_\varphi/(e_h \Psi_0) = -0.30, -0.15, 0.05$, and 0.20 at $t = 43.5\text{ms}$. The flattened regions around $P_\varphi/(e_h \Psi_0) = -0.15$ and 0.05 disappear at $t = 43.9\text{ms}$. This indicates the resonance overlap. We see that the distribution at $t = 44.05\text{ms}$

Figure 7. Time evolution of (a) 1-dimensional fast ion distribution function $f(P_\phi)$ and (b) the distribution difference between the hybrid simulation and the classical simulation for counter-going particles for the constant E and μ . The top of the figure is the plasma center and the bottom is the plasma edge. Contours represent the time evolution of $f(P_\phi)$ in panel (a). Panel (c) shows $f(P_\phi)$ for various moments denoted by vertical lines in panels (a) and (b).

Figure 8. Time evolution of (a) stored fast ion energy for co-going and counter-going particles to the plasma current and (b) MHD kinetic energy for $P_{\text{NBI}}=5\text{MW}$ and $\tau_s=100\text{ms}$. Hybrid simulation is continuously run after $t=34.5\text{ms}$.

Figure 9. Comparison of (a) fast ion pressure profiles averaged in the last 10ms of the simulations and (b) fast ion distribution functions just before the AE bursts between $P_{\text{NBI}} = 5\text{MW}$ and 10MW .

decreases near the plasma center $P_{\varphi}/(e_h\Psi_0) > -0.07$ and increases near the plasma edge $P_{\varphi}/(e_h\Psi_0) < -0.07$ from that at $t = 43.9\text{ms}$. This indicates the global flattening or the collapse of the distribution. The distribution totally decreases at $t = 44.2\text{ms}$ because the particle losses occur at the plasma edge.

We conducted another simulation where the beam deposition power is reduced to one-half ($P_{\text{NBI}} = 5\text{MW}$). In this simulation, a combination of classical simulation for 2ms and hybrid simulation for 0.5ms was run until $t = 34.5\text{ms}$, and the hybrid simulation was continuously run after $t = 34.5\text{ms}$ until $t = 50\text{ms}$. Figure 8 shows the time evolution of stored fast ion energy and MHD kinetic energy in the simulation. We see that the AE bursts take place during the continuous hybrid simulation after $t = 34.5\text{ms}$ with a time interval 6ms, which is roughly double that for $P_{\text{NBI}} = 10\text{MW}$. The fast ion pressure profiles averaged in the last 10ms are compared in Fig. 9(a) between $P_{\text{NBI}} = 5\text{MW}$ and 10MW . The central pressure increases only by 15% with the increase of beam power from 5MW to 10MW. The pressure profiles are very close to each other, which is similar to the result presented in Ref. [24]. We call this saturation of fast ion pressure profile “profile resiliency.” Figure 9(b) compares the fast ion distributions just before the AE bursts between $P_{\text{NBI}} = 5\text{MW}$ and 10MW . We see the profile resiliency for the fast ion distribution function. This demonstrates that the distribution is a “critical distribution” for the AE burst to occur leading to the saturation of the fast ion distribution.

4. Discussion and summary

In this paper, we presented the comprehensive simulation results of the AE bursts and the time evolution of the fast ion distribution function. We found that a critical distribution of fast ions exists where the distribution collapses with the synchronized sudden growth of multiple AEs. Before the collapse, the fast ion distribution increases with the continuous beam injection destabilizing the AEs to low amplitude. The low-amplitude AEs trap the resonant fast ions, and the fast ion distribution is locally flattened by the AEs in the trapped regions in phase space. The gradual growth of the AEs broadens the trapped regions forming the distribution into a stairway shape. When the fast ion distribution reaches the critical distribution, a resonance overlap triggers the overlap of multiple resonances leading to the synchronized growth of multiple AEs and the collapse of the distribution. We found the resiliency of the fast ion distribution function between the two runs with the beam deposition power 5MW and 10MW, which demonstrates that the distribution is the critical distribution where the AE burst occurs.

When we consider the interaction between fast ions and an AE, the fast ion distribution is relaxed locally at the resonances in phase space. The relaxation of the distribution function is limited around the resonances. The global stability of the distribution is not determined by the linear stability of the AEs due to the locality of the interaction or the inhomogeneity of the phase space structure. The global stability depends on the width of the trapped regions by the AEs. The width of the trapped region is basically in proportion to the square root of the AE amplitude. With increasing AE amplitude, resonance overlap may take place between two resonances merging them into a broader flattened region. This process remains a local relaxation if the merged flattened region does not overlap any other resonance. However, if the resonance overlap takes place successively for multiple resonances and the overlapped region reaches the plasma edge where the particles are lost, the distribution entirely collapses with significant particle losses. This is the AE burst which causes the saturation and the resiliency of the fast ion distribution.

The fast ion transport brought about by resonance overlap of multiple AEs has been pointed out for DIII-D experiments where the AEs were observed with steady amplitude [37, 44–46]. One may ask a question what causes the different types of time evolution, AE bursts and steady amplitude. It was theoretically predicted that collisions of fast ions affect the time evolution of AEs [47]. High collision frequency leads to the steady evolution while low collision frequency leads to the intermittent evolution. Micro-turbulence in fusion plasmas may work as an effective collision, and may affect the time evolution of AEs. The effect of micro turbulence on the AE evolution has been theoretically predicted to be larger than that of collisions for present tokamaks and ITER ([48, 49]). The different types of time evolution may be caused by the difference in the effective collision frequency. We can point out other differences between the AE bursts presented in this paper and the DIII-D experiments. We assumed the balanced injection for the AE bursts while the co-beam is dominant in the DIII-D experiments.

We saw in this paper the stronger interaction of counter-going particles than that of co-going particles with the AEs. This may partially explain the different types of time evolution. Another difference is that the beam injection velocity is higher than the Alfvén velocity for the AE bursts presented in this paper while it is sub-Alfvénic in the DIII-D experiments. The phase space structure of fast ions interacting with the AEs might be different between the phase space region close to the Alfvén velocity and that for the side-band resonance. These aspects discussed here are possible reasons for the different types of time evolution.

The critical distribution found in this work is based on nonlinear physics of energetic particles and AEs. Many efforts have been devoted to the modeling of energetic particle distribution with AEs such as resonance broadened quasi-linear (RBQ) model [46], kick model [50], and critical gradient model for AE stability [51]. The RBQ model constructs fast ion distribution with the quasi-linear diffusion based on the broadening of the resonance region with the finite AE amplitude. The kick model provides a kick matrix for kinetic energy and toroidal canonical momentum brought about by the interaction with the finite-amplitude AE. Both the RBQ model and the kick model take into account the width of resonance region depending on the AE amplitude and can simulate the AE amplitude self-consistently. The critical gradient model for AE stability predicts the spatial profile of fast ion distribution where the linear AE growth rate is assumed to be the same as the linear growth rate of ion-temperature-gradient mode and trapped electron mode. We believe that nonlinear physics with the finite-amplitude AEs and the relaxation in both kinetic energy and toroidal canonical momentum should be considered in the energetic particle distribution modeling because we found in this paper that the energetic particle distribution is saturated by the AE burst with the global resonance overlap. The critical distribution for the AE burst leading to the collapse of the distribution is critical knowledge for the modeling and the prediction of the energetic alpha particle distribution in burning plasmas.

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