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Nonlinear evolution of resistive wall mode in a cylindrical tokamak with poloidal rotation

M. Sato and N. Nakajima

National Institute for Fusion Science, Toki, Gifu 509-5292, Japan

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Nonlinear simulations of resistive wall modes (RWMs) with a Doppler shift dominant equilibrium poloidal rotation have been carried out by using reduced magnetohydrodynamic equations in a low beta cylindrical tokamak, where the core plasma is surrounded by a cold plasma with a high resistivity. When the equilibrium poloidal rotation frequency is small and the Doppler shift is predominant, the *wall mode* becomes unstable, which is one of the RWMs nearly locked to the resistive wall. Since the slowing down torque increases with equilibrium poloidal rotation frequency and the poloidal rotation decreases to almost zero near the plasma surface before the saturation, the nonlinear saturation level does not depend on either the equilibrium poloidal rotation frequency or the density of the cold plasma. When the equilibrium poloidal rotation frequency becomes larger than a critical value, the *plasma mode* rotating to the resistive wall becomes unstable. When the cold plasma has the same density as that in the core plasma, neither the centrifugal force nor the Coriolis force has any effect. In such a case, as the equilibrium poloidal rotation frequency increases, the magnetic flux is so hard to diffuse into the resistive wall that the slowing down torque decreases and the rotation tends to survive in the nonlinear phase, which makes the saturation level decrease.

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I. INTRODUCTION

In order to obtain high beta plasma in tokamaks, stabilization of external kink modes is required. Although the external kink modes can be stabilized perfectly by locating a perfect conducting wall sufficiently close to the plasma surface,¹ the mode cannot be stabilized perfectly when the wall has a finite conductivity, and slowly growing resistive wall modes (RWMs) appear.^{2,3} It is quite important to suppress the slowly growing resistive instabilities such as the RWMs for future steady-state tokamaks like the International Thermonuclear Experimental Reactor (ITER).⁴

Experimentally, it is found that high beta plasmas with $\beta_N > \beta_N^{\text{no wall}}$ are obtained for time periods much longer than the diffusion time of the resistive wall,⁵⁻⁷ where β_N is a normalized beta with respect to the Toroyon limit⁸ and $\beta_N^{\text{no wall}}$ is the β_N limit predicted under no conducting wall. In these experiments, there is a rapid toroidal rotation due to unbalanced neutral beam injection. The experimental results suggest that the plasma rotation is important for stabilizing the RWMs. Theoretically, it is shown by linear analyses that the combination of plasma rotation and some dissipation model can stabilize RWMs.⁹⁻¹⁵ The plasma rotation induces three effects, namely the Doppler shift, the centrifugal force, and the Coriolis force. The Doppler shift is dominant over two forces for a small rotation frequency, and the two forces become dominant over the Doppler shift for a large rotation frequency. Theoretical studies in Refs. 10, 11, 13, and 14 have been done in a straight cylindrical plasma, where the effects of the rigid toroidal rotation are introduced only as a Doppler shift. In Ref. 12, Chu *et al.* included the effect of the Coriolis force into the torus plasma but neglected the effect of the centrifugal force assuming a subsonic toroidal flow.

They found that the effect of the Coriolis force can be neglected. In contrast, Betti studied the effect of the toroidal rotation on the RWM including all effects, i.e., the Doppler shift, the centrifugal force, and the Coriolis force in Ref. 15. He found that the ideal mode is destabilized by the centrifugal force for $\Omega_{\zeta} > 0.07\omega_A$, where ω_A is the Alfvén frequency. Hence, such a fast rotation should be avoided in experimental plasmas. Moreover, in a reactor-sized tokamak such as the ITER, the plasma is expected to be stationary or slowly rotating. Thus, it is considered that the Doppler shift is the most dominant effect induced by the equilibrium rotation in future reactor-sized steady-state tokamaks.

The experimental results on DIII-D⁴⁻⁶ also show that when the plasma rotation decreases below a critical level, the RWM begins to grow. This result indicates that the time evolution of the plasma rotation is an important factor. The slowing down of the plasma rotation mainly results from electromagnetic torque. If the amplitude of the perturbation is sufficiently small, the amplitude of the torque can be obtained approximately in terms of the linear eigenfunction. Then, the equation of time evolution of the rotation frequency is derived by using the approximate torque. There are some studies of RWMs including nonlinear deceleration of the plasma rotation by means of a combination of the equation of time evolution of the rotation and the linear dispersion relation.^{16,17} This method can determine whether the RWM can access a stable region. However, when the RWM finally reaches an unstable region, the final saturation level cannot be obtained from the method. Since the destabilized RWM causes plasma disruption, it is important to investigate the saturation level of the RWM under an equilibrium rotation through the nonlinear analyses. Moreover, the hot core plasmas in experiments are usually surrounded by the cold

plasma. The density of the cold plasma changes according to the operational conditions.

Thus, in the present work, the linear and nonlinear behaviors of the RWMs have been investigated in a Doppler shift predominant rotating plasma, in order to understand the influences of the equilibrium plasma rotation or the Doppler shift and the cold external plasma on the nonlinear saturation level of RWMs. There are many types of reduced magneto-hydrodynamic (MHD) equations including the parallel (toroidal) flow.^{18–21} However, although they include the effect of the poloidal Doppler shift, most of them cannot correctly treat the toroidal Doppler shift since the time evolution of $v_{\parallel} \tilde{\mathbf{b}}$ is not included in their vorticity equation, where $v_{\parallel}$ is parallel velocity, $\tilde{\mathbf{b}} = \tilde{\mathbf{B}}/|\tilde{\mathbf{B}}|$, and $\tilde{\mathbf{B}}$ is the perturbed magnetic field. Although such an effect is included in Izzo's model,¹⁸ the toroidal velocity v_{ζ} is assumed to be $v_{\zeta} \sim \epsilon v_p$, which is not relevant to studying RWMs under the toroidal rotation with a few percent of Alfvén velocity, where v_p is poloidal velocity and ϵ is the inverse aspect ratio. Therefore, a set of reduced MHD equations is used in a cylindrical tokamak plasma with a poloidal rotation, where a core plasma is surrounded by a cold plasma with a high resistivity. Since the cases are considered with the Doppler shift being dominant over the centrifugal force and the Coriolis force, the present results could be used not only to directly understand the nonlinear behaviors under the equilibrium poloidal rotation, but also to qualitatively understand the nonlinear behaviors under the equilibrium toroidal rotation. In fact, in toroidal geometry, poloidal rotation is strongly damped by parallel viscosity.^{22–24} Hence, poloidal rotation large enough to stabilize RWMs hardly ever present in experiments. However, such a large poloidal rotation is used by neglecting the parallel viscosity for studying the effect of the Doppler shift of equilibrium flow in this paper. The RWMs investigated in this paper are the *wall mode* and the *plasma mode*, both of which are different from a resistive plasma RWM or a tearing RWM investigated by the nonlinear simulations under a rotation and a resistive wall in the slab geometry.²⁵ The resistive plasma RWM or the tearing RWM becomes unstable when the core plasma is surrounded by a moderated resistive plasma, and their linear growth rates depend on the resistivity of both the resistive wall and the rational surface. When the resistivity of the cold plasma is sufficiently high, as is the case in this study, the linear growth rate does not depend on the resistivity of the cold plasma.^{26,27}

The rest of the paper is organized as follows. In Sec. II, a set of the reduced MHD equations for simulating RWMs is introduced, together with the numerical model and the numerical scheme. In order to evaluate the effect of the Doppler shift, the centrifugal force, and the Coriolis force brought by the equilibrium poloidal rotation, the linear dispersion relation of the RWMs is analytically derived in Sec. III for a step-like current density profile. From a comparison between the analytical dispersion relation and numerical results, it is confirmed that the simulation code works well. By using a more realistic current density profile, the conditions in which the cold plasma is regarded as the vacuum and the Doppler shift becomes predominant are investigated. In Sec.

IV, the nonlinear properties of RWMs are examined, where single helicity perturbations are assumed. In the MHD equilibria without the rotation, the effects of the density of the cold plasma on the nonlinear evolutions are investigated. The nonlinear evolutions of the *wall mode* and the *plasma mode* are studied in the MHD equilibria under the condition that the effect of the Doppler shift becomes dominant. Section V is devoted to a summary.

II. NUMERICAL MODEL

For studying nonlinear behavior of the RWMs, the following reduced MHD equations are solved numerically in a cylindrical geometry (r, φ, z) :

$$\frac{\partial \psi}{\partial t} = -\mathbf{B} \cdot \nabla \phi + \eta J - E, \quad (1)$$

$$\begin{aligned} \frac{\partial U}{\partial t} = & -\mathbf{v}_{\perp} \cdot \nabla U - \frac{1}{2} \nabla \rho \times \nabla v_{\perp}^2 \cdot \mathbf{e}_z - \mathbf{B} \cdot \nabla J - \mathbf{e}_z \cdot \nabla \\ & \times \nabla \cdot \boldsymbol{\pi} + S_m, \end{aligned} \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}_{\perp} \cdot \nabla T = 0, \quad (3)$$

$$\frac{\partial \rho}{\partial t} + \mathbf{v}_{\perp} \cdot \nabla \rho = 0, \quad (4)$$

$$\mu_0 J = \nabla_{\perp}^2 \psi, \quad (5)$$

$$U = \nabla \cdot (\rho \nabla_{\perp} \phi), \quad (6)$$

$$\eta \propto T^{-3/2}, \quad (7)$$

where $\mathbf{e}_z$ is a unit vector in the axial direction, and indicates the parallel direction. The magnetic field and the perpendicular velocity are expressed as $\mathbf{B} = -\nabla \psi \times \mathbf{e}_z + B_0 \mathbf{e}_z$ and $\mathbf{v}_{\perp} = \nabla \phi \times \mathbf{e}_z$ by using the poloidal magnetic flux ψ and the stream function ϕ , respectively. In Eq. (1), E is the electric field to maintain the toroidal current J . Equations (1)–(7) are an extended version of the standard two-field reduced MHD equations, including temperature and density effects. The equations are obtained by neglecting the diamagnetic effect from the reduced MHD equations in Ref. 28. When the density is assumed to be constant and time evolution of the temperature is neglected, Eqs. (1), (2), (5), and (6) agree with the well-known two-field reduced MHD equations.²⁹

In Eq. (2), the viscosity tensor $\boldsymbol{\pi}$ is assumed to be shear viscosity and is defined as $\boldsymbol{\pi} = \nu \rho (\nabla \mathbf{v}_{\perp} + \nabla \mathbf{v}_{\perp}^t)$. The momentum source term S_m is chosen to satisfy $S_m = \mathbf{e}_z \cdot \nabla \times \nabla \cdot \boldsymbol{\pi}_{\text{eq}}$, where $\boldsymbol{\pi}_{\text{eq}}$ is a viscosity tensor at an equilibrium state. In this paper, we assume a rigid equilibrium poloidal rotation and a free-slip boundary condition, so that $S_m = 0$ is satisfied. In order to determine the extent of the stability window and a critical rotation frequency for stabilizing the RWM, some dissipation mechanisms such as viscosity are introduced. However, since our purpose is not to examine the dissipation

effects but to examine the effect of the rotation, a small viscosity is introduced in order to suppress numerical oscillations.

For treating external MHD modes, a cold plasma with extremely high resistivity is introduced around a hot core plasma with low resistivity.^{26,27,30} The ratio of the resistivity in the cold plasma η_v to the resistivity at the plasma center η_0 is chosen as $\eta_v/\eta_0=10^7$. The magnetic Reynolds number $S=\tau_\eta/\tau_a$ in the core plasma is set to be $S=10^7$, where $\tau_\eta=\mu_0 a^2/\eta_0$ and $\tau_a=R/(B_0/\sqrt{\mu_0 \rho_0})$ are the resistive diffusion time and the Alfvén time in the core plasma, respectively. Here a is the minor radius of the core plasma, ρ_0 is the density at the plasma center, and R is the major radius. In this situation, the current in the cold plasma is almost zero and the linear growth rate of the external mode does not depend on η_v .

The cold plasma is surrounded by a resistive wall located at $r=r_w (>a)$. The resistivity of the resistive wall η_w is assumed to be $\eta_w/\eta_0=10^3$. Another cold plasma representing the vacuum is assumed to be located between the resistive wall $r=r_w$ and a perfect conducting wall at $r=r_d (>r_w)$. Inside the resistive wall and the cold plasma outside the resistive wall, the resistivity is assumed to be uniform and constant. Since the plasma does not penetrate in both regions, only the diffusion equation of the poloidal magnetic flux ψ is solved,

$$\frac{\partial \psi}{\partial t} = \frac{\eta}{\mu_0} \nabla_\perp^2 \psi, \quad \eta = \begin{cases} \eta_w: r_w < r < r_w + d \\ \eta_v: r_w + d < r < r_d. \end{cases}$$

Since the single helicity is assumed for studying the linear and nonlinear evolutions of RWM, we solve the reduced MHD equations in two-dimensional space by imposing the condition of helical symmetry. For solving Eqs. (1) and (2) in our numerical code, the radial derivatives are replaced with standard difference approximations, and the derivatives with respect to the angle are treated with Fourier expansions. The time advancement is made with a predictor-corrector method. The diffusion term in Eq. (1) is treated implicitly. The equation of the time evolution of the temperature should be solved with high accuracy, otherwise the numerical calculation cannot be carried out when a temperature becomes negative due to numerical oscillation near the plasma edge where the temperature is very small. Extremely large numbers of Fourier modes and radial meshes are required to avoid the numerical oscillation if the Fourier-expansion method is used. To settle the problem, the RCIP (Rational Constrained Interpolation Profile or Rational Cubic-Interpolated Pseudo-Particle) method³¹ is used to solve Eqs. (3) and (4). In the CIP (Constrained Interpolation Profile or Cubic-Interpolated Pseudo-Particle) method,³² not only the variables T and ρ but also their first-order derivatives $\partial T/\partial r$, $\partial T/\partial \theta$, $\partial \rho/\partial r$, and $\partial \rho/\partial \theta$ are solved simultaneously. While the standard CIP method uses a cubic polynomial function, the RCIP method uses a rational function to construct the interpolating function so as to suppress the numerical oscillation near a discontinuity. In order to solve Eqs. (1) and (2), the numbers of radial meshes and Fourier components are

500 and 20, respectively. For Eqs. (3) and (4), 1000 radial meshes and 120 angle meshes are used.

The boundary condition for ψ is $\psi(r_d)=0$ at the perfect conducting wall ($r=r_d$). At the magnetic axis ($r=0$), the radial derivative of $\psi_{0,0}$ is zero for $m=0$, and the amplitude of $\psi_{m,n}$ is zero for $m>0$, where $m(\geq 0)$ and $n(\geq 0)$ are the poloidal and toroidal mode numbers, respectively. For ϕ and U , the boundary condition at $r=0$ is the same one for ψ , i.e., the radial derivatives of $\phi_{0,0}$ and $U_{0,0}$ are zero for $m=0$, and the amplitudes of $\phi_{m,n}$ and $U_{m,n}$ are zero for $m>0$. For the boundary condition at the resistive wall ($r=r_w$), a free-slip condition is imposed and it is assumed that no force acts on the plasma there. Since the radial velocity must vanish at the resistive wall, the condition of the vorticity at the resistive wall, U_w , is given by

$$\frac{\partial U_w}{\partial t} + v_\theta \frac{1}{r} \frac{\partial U_w}{\partial \theta} = 0.$$

This implies that the vorticity at the resistive wall does not depend on time and angle when the initial vorticity at the resistive wall does not depend on poloidal angle. Therefore, the boundary condition for perturbed vorticity is $\tilde{U}(r=r_w)=0$ at the resistive wall by assuming the initial vorticity at $r=r_w$ is constant. For the stream function, $\phi_{m,n}(r=r_w)=0$ for $m>0$ since the radial velocity must vanish at the resistive wall. For $(m,n)=(0,0)$ mode, we can set $\phi_{0,0}(r=r_w)$ to be any value since the derivative of $\phi_{0,0}$ is zero at the plasma center. Thus, we choose $\phi_{0,0}(r=r_w)=0$ and then the plasma can flow along the resistive wall since the radial derivative of ϕ can have a finite value at $r=r_w$. The boundary conditions for T and ρ are that the perturbative components of T , $\partial T/\partial r$, $\partial T/\partial \theta$, ρ , $\partial \rho/\partial r$, and $\partial \rho/\partial \theta$ vanish at $r=r_w$.

III. LINEAR PROPERTIES

In order to clarify the properties of the *wall mode* and the *plasma mode* and their relation to the Doppler shift, centrifugal force, and Coriolis force brought by the rigid poloidal rotation, the linear dispersion relation is examined analytically and compared to the corresponding numerical calculations by using the step-like current density profile in Sec. III A. In Sec. III B, the numerical calculations have been done for the nonuniform current profile. From those investigations, it is clarified for the nonuniform current profile that influences of the cold plasma density on the linear growth rates of the *wall mode* and the *plasma mode* become strong.

A. On the equilibrium with a step-like current density profile

By using the Fourier-decomposed perturbation form, $\tilde{f}(r)\exp(-i\omega t + im\theta - in\zeta)$, Eqs. (1)–(7) are linearized, where θ and $\zeta=-z/R$ are the poloidal and toroidal angle, respectively, and $\tilde{f}(r)$ describes a radial mode structure corresponding to $\tilde{\psi}$ and $\tilde{\phi}$, etc. The linearized reduced MHD equations are given as

$$-i\omega\tilde{\psi} = -im\Omega_0\tilde{\psi} + i\left(\frac{m}{q} - n\right)\frac{B_0}{R_0}\tilde{\phi} + \eta_{\text{eq}}\tilde{J} + \tilde{\eta}J_{\text{eq}}, \quad (8)$$

$$-i\omega\tilde{U} = -im\Omega_0\tilde{U} - \frac{im}{r}U'_{\text{eq}}\tilde{\phi} + i\left(\frac{m}{q} - n\right)\frac{B_0}{R_0}\tilde{J} + \frac{im}{r}J'_{\text{eq}}\tilde{\psi} + im\Omega_0(\Omega_0\tilde{\rho} + \rho'_{\text{eq}}\tilde{\phi}'), \quad (9)$$

$$-i\omega\tilde{T} = -im\Omega_0\tilde{T} - \frac{im}{r}T'_{\text{eq}}\tilde{\phi}, \quad (10)$$

$$-i\omega\tilde{\rho} = -im\Omega_0\tilde{\rho} - \frac{im}{r}\rho'_{\text{eq}}\tilde{\phi}, \quad (11)$$

$$\mu_0\tilde{J} = \nabla_{\perp}^2\tilde{\psi}, \quad (12)$$

$$\tilde{U} = \nabla \cdot (\rho_{\text{eq}}\nabla_{\perp}\tilde{\phi}) + \nabla \cdot (\tilde{\rho}\nabla_{\perp}\phi_{\text{eq}}), \quad (13)$$

$$\tilde{\eta} \propto -\frac{3}{2}\eta_{\text{eq}}\frac{\tilde{T}}{T_{\text{eq}}}, \quad (14)$$

where Ω_0 is the equilibrium rigid poloidal rotation frequency, the subscript eq denotes equilibrium quantities, and the viscosity is neglected in Eq. (9). The equilibrium current density profile is assumed to be a step function, in order to obtain the dispersion relation analytically. The profiles of the current density, the safety factor, and the density are chosen as

$$J_{\text{eq}}(r) = J_0, \quad q(r) = q_a, \quad \rho_{\text{eq}}(r) = \rho_0 \quad \text{for } r \leq a, \quad (15)$$

$$J_{\text{eq}}(r) = 0, \quad q(r) = q_a\left(\frac{r}{a}\right)^2, \quad \rho_{\text{eq}}(r) = \rho_v \quad \text{for } r > a, \quad (16)$$

where q_a is the safety factor at the edge of the hot core plasma ($r=a$). Note that equilibrium quantities of the current density, the safety factor, and the density are constant for $r < a$. For simplicity, the plasma is assumed to be ideal in the core plasma ($r < a$).

By combining Eqs. (8), (9), and (11), we obtain

$$\left\{(\omega - m\Omega_0)^2 - \left(\frac{m}{q_a} - n\right)^2\left(\frac{B_0}{R_0}\right)^2\right\}\nabla_{\perp}^2\tilde{\psi} = 0. \quad (17)$$

By using the boundary condition at the plasma center, the profiles of $\tilde{\psi}$ and $\tilde{\phi}$ for $r < a$ are given as

$$\tilde{\psi} = C_0\left(\frac{r}{a}\right)^m, \quad (18)$$

$$\tilde{\phi} = -(\omega - m\Omega_0)\left(\frac{m}{q_a} - n\right)^{-1}\frac{R_0}{B_0}C_0\left(\frac{r}{a}\right)^m. \quad (19)$$

At the edge of the core plasma ($r=a$), integration of the vorticity equation across $r=a$ gives

$$-(\omega - m\Omega_0)(\rho_0\tilde{\phi}' - \rho_v\tilde{\phi}'_+) = \frac{m}{a}(\tilde{U}_{0+} - \tilde{U}_{0-})\phi + \frac{m}{a}J_0\tilde{\psi} - \left(\frac{m}{q_a} - n\right)\frac{B_0}{R_0}(\tilde{\psi}'_+ - \tilde{\psi}'_-) + \frac{m}{a}\frac{m\Omega_0^2}{\omega - m\Omega_0}(\rho_0 - \rho_v)\tilde{\phi}, \quad (20)$$

where $\tilde{f}_{\pm} \equiv \tilde{f}(a \pm 0)$. The second term on the right-hand side of Eq. (20) corresponds to the current driven term; the third term is the stabilizing term associated with field line bending. When the density of the cold plasma is lower than that of the core plasma ($\rho_0 > \rho_v$) and poloidal rotation exists, the fourth term potentially becomes a destabilizing term through the coupling between density gradient and the rotation acting as centrifugal force.

In the inner cold plasma region ($a < r < r_w$), the resistivity is so extremely large that the current density almost vanishes. Therefore, the profile of $\tilde{\psi}$ for $a \leq r < r_w$ is given by

$$\tilde{\psi} = C_1\left(\frac{r}{a}\right)^m + C_2\left(\frac{r}{a}\right)^{-m}. \quad (21)$$

Since the electromagnetic force $\mathbf{J} \times \mathbf{B}$ can be neglected, the vorticity equation becomes $\nabla_{\perp}^2\phi \approx 0$ in the cold plasma, so that the stream function for $a < r < r_w$ is given as

$$\tilde{\phi} = C_4\left[\left(\frac{r}{r_w}\right)^m - \left(\frac{r}{r_w}\right)^{-m}\right], \quad (22)$$

by using the boundary condition for the stream function on the resistive wall at $r=r_w$.

In the resistive wall ($r_w \leq r \leq r+d$), the poloidal flux is diffused according to the following equation:

$$\omega\tilde{\psi} = i\frac{\eta_w}{\mu_0}\nabla_{\perp}^2\tilde{\psi}. \quad (23)$$

For the limit of a thin resistive wall, integration across the resistive wall gives

$$\omega\tau_w = ir_w\Delta'_w, \quad (24)$$

where $\Delta'_w = [\psi'(r_w+0) - \psi'(r_w-0)]/\psi(r_w)$ is the discontinuity of the logarithmic derivative at the resistive wall, and $\tau_w = \mu_0 r_w d / \eta_w$ is the diffusion time of the resistive wall.

In the external vacuum ($r+d < r \leq r_d$), $\tilde{J}=0$ is satisfied, so that the solution of the poloidal flux is given by

$$\tilde{\psi} = C_3\left[\left(\frac{r}{r_d}\right)^m - \left(\frac{r}{r_d}\right)^{-m}\right], \quad (25)$$

by taking into account the boundary condition at $r=r_d$.

Substituting Eqs. (18), (19), (21), and (22) into Eq. (20) and using the condition of the continuity of $\tilde{\psi}$ and $\tilde{\phi}$ at $r=a$, we obtain

$$\begin{aligned}
& (\omega - m\Omega_0)^2 \left(\frac{B_0}{R_0} \right)^2 \left[\rho_0 + \rho_v \frac{1 + (a/r_w)^{2m}}{1 - (a/r_w)^{2m}} \right] \\
&= -\frac{2}{q_a} \left(\frac{m}{q_a} - n \right) \frac{B_0}{R_0} + 2 \left(\frac{m}{q_a} - n \right)^2 \left(\frac{B_0}{R_0} \right)^2 \frac{C_2}{C_0} \\
&\quad - 2(\rho_0 - \rho_v)(\omega - m\Omega_0)\Omega_0 - (\rho_0 - \rho_v)m\Omega_0^2. \quad (26)
\end{aligned}$$

In the same way, by using the condition of the continuity of the poloidal flux at the resistive wall, substitution of Eqs. (21) and (25) into Eq. (24) gives

$$m\tau_w = i \frac{1 - [1 - (a/r_d)^{2m}]C_2/C_0}{1 - [1 - (a/r_w)^{2m}]C_2/C_0} \frac{2m}{(r_w/r_d)^{2m} - 1}. \quad (27)$$

Eliminating C_2/C_0 from Eqs. (26) and (27), the dispersion relation of the RWM is obtained as

$$\begin{aligned}
& -(\omega - m\Omega_0)^2 \left(1 + \alpha \frac{\rho_v}{\rho_0} \right) \\
&= \frac{\gamma_d^2 - i\gamma_w^2 \omega \tau_w^*}{1 - i\omega \tau_w^*} - 2 \left(1 - \frac{\rho_v}{\rho_0} \right) (m\Omega_0 - \omega)\Omega_0 \\
&\quad + \left(1 - \frac{\rho_v}{\rho_0} \right) m\Omega_0^2, \quad (28)
\end{aligned}$$

where $\alpha (>1)$ and τ_w^* are defined as

$$\begin{aligned}
\alpha &= \frac{1 + (a/r_w)^{2m}}{1 - (a/r_w)^{2m}}, \\
\tau_w^* &= \frac{1 - (r_w/r_d)^{2m}}{1 - (a/r_d)^{2m}} \left[1 - \left(\frac{a}{r_w} \right)^{2m} \right] \frac{\tau_w}{2m}.
\end{aligned}$$

Also, γ_d and γ_w are linear growth rates of ideal kink modes when the conducting wall is located at $r=r_d$ and r_w , respectively. Both are defined as

$$\begin{aligned}
\gamma_d^2 \tau_a^2 &= -2 \left(\frac{m}{q_a} - n \right)^2 \left(\frac{B_0}{R_0} \right)^2 \left[1 - \left(\frac{a}{r_d} \right)^{2m} \right]^{-1} + 2 \left(\frac{m}{q_a} - n \right) \\
&\quad \times \left(\frac{B_0}{R_0} \right) \frac{1}{q}, \\
\gamma_w^2 \tau_a^2 &= -2 \left(\frac{m}{q_a} - n \right)^2 \left(\frac{B_0}{R_0} \right)^2 \left[1 - \left(\frac{a}{r_w} \right)^{2m} \right]^{-1} + 2 \left(\frac{m}{q_a} - n \right) \\
&\quad \times \left(\frac{B_0}{R_0} \right) \frac{1}{q}.
\end{aligned}$$

Since the resistive wall is located between the core plasma and the perfect conducting wall, i.e., $r_d > r_w$, the relation $\gamma_d^2 > \gamma_w^2$ is satisfied. The left-hand side of the dispersion relation given by Eq. (28) includes the term $\alpha\rho_v/\rho_0$, which results from the inertia term in the cold plasma. The boundary condition for $\tilde{\phi}=0$ at the resistive wall ($r=r_w$) forces the plasma not to penetrate into $r > r_w$ and increases the effective mass of the cold plasma. The third and the fourth terms of the right-hand side of the dispersion relation correspond to the Coriolis force and the centrifugal force, respectively.

Since the dispersion relation given by Eq. (28) is a cubic equation in ω , it is well known that there are two potentially

unstable roots and one always stable root of RWMs.^{9,10,12,13,17} The two potentially unstable roots are called *wall mode*, which is nearly locked to the resistive wall, and *plasma mode*, which is rotating to the resistive wall, respectively. They can be approximately obtained by means of the perturbation method, where the eigenvalue $\omega \equiv \omega_r + i\gamma$ is expanded as $\omega = \omega_0 + \epsilon\omega_1 + \dots$, by using an expansion parameter $\epsilon = \tau_d/\tau_w \ll 1$.^{13,17,33,34} From the perturbation method, the growth rate γ and the real frequency ω_r of the *wall mode* can be written as

$$\begin{aligned}
\gamma &\approx -\frac{\frac{\gamma_d^2}{1 + \alpha \frac{\rho_v}{\rho_0}} + m[m-f]\Omega_0^2}{\frac{\gamma_w^2}{1 + \alpha \frac{\rho_v}{\rho_0}} + m[m-f]\Omega_0^2} \frac{1}{\tau_w^*}, \quad (29)
\end{aligned}$$

$$\omega_r \approx 0, \quad (30)$$

and those of the *plasma mode* become

$$\gamma \approx \frac{\gamma_d^2 - \gamma_w^2}{2 \left(1 + \alpha \frac{\rho_v}{\rho_0} \right) \omega_r \{ [m-f]\Omega_0 - \omega_r \}} \frac{1}{\tau_w^*}, \quad (31)$$

$$\omega_r \approx [m-f]\Omega_0 - \sqrt{-\frac{\gamma_w^2}{1 + \alpha \frac{\rho_v}{\rho_0}} - f[m-f]\Omega_0^2}, \quad (32)$$

where $f \equiv (1 - \rho_v/\rho_0)/(1 + \alpha\rho_v/\rho_0)$ indicates the effective density gradient at the boundary between the core and cold plasmas, so that f is closely related to the centrifugal force and the Coriolis force of the poloidal rotation. The factor f satisfies $0 \leq f \leq 1$ for $0 \leq \rho_v \leq \rho_0$, and so $m-f \geq m-1 \geq 1$ for $m \geq 2$.

When the resistive wall is very close to the plasma surface, i.e., $\gamma_w^2 < 0 < \gamma_d^2$, the *wall mode* becomes unstable for $\Omega_0 < \Omega_{c1}$, where Ω_{c1} is determined by the divergence of the linear growth rate given by Eq. (29) or the condition that the real frequency of *plasma mode* given by Eq. (32) vanishes, and is given as

$$\Omega_{c1}^2 = -\frac{\gamma_w^2}{\left(1 + \alpha \frac{\rho_v}{\rho_0} \right) m[m-f]}, \quad (33)$$

which does not depend on τ_w . The ratio of the real frequency of the *plasma mode* with $\Omega_0=0$, $\omega_r|_{\Omega_0=0}$, to $m\Omega_{c1}$ is $(\omega_r|_{\Omega_0=0}/m\Omega_{c1})^2 = 1 - f/m \geq 1 - 1/m$. Both frequencies become the same order for $m \geq 2$. Also, when the density of the cold plasma is the same as that of the core plasma, namely $\rho_v = \rho_0$, $f=0$, and so $m\Omega_{c1}$ is equal to $\omega_r|_{\Omega_0=0}$.³⁴ Thus, Ω_{c1} , corresponding to the real frequency of the *plasma mode* without equilibrium poloidal rotation divided by the poloidal mode number m , is recognized as the critical rotation frequency distinguishing *wall mode* from *plasma mode*, namely the *wall mode* becomes unstable for $\Omega_0 < \Omega_{c1}$, and *plasma mode* becomes unstable for $\Omega_0 > \Omega_{c1}$.

Since $\omega_r \sim 0$ for the *wall mode*, in the dispersion relation (28) divided by $1 + \alpha\rho_v/\rho_0$ the stabilizing Coriolis force term and the destabilizing centrifugal term become $-2fm\Omega_0^2$ and $fm\Omega_0^2$, respectively. Thus, the sum of both forces acts as the stabilizing force, which appears as $-fm\Omega_0^2$ in Eq. (29). On the other hand, the effect of the Doppler shift appears as $m^2\Omega_0^2$ in Eq. (29). All the effects of the poloidal rotation appear as $[m-f]m\Omega_0^2$ in Eq. (29). Since $m-f \geq m-1 \geq 1$ for $m \geq 2$, the destabilizing effect of the Doppler shift overcomes the stabilizing effects coming from the Coriolis force, leading to the destabilization of the *wall mode*. The effect of the Doppler shift becomes dominant, as m increases and/or the density of the cold plasma reaches that of the core plasma ($f \rightarrow 0$). In the present case with $m=3$, the poloidal rotation dependence of the linear growth rate of the *wall mode* is considered to be mainly determined by the Doppler shift. Note that the effects of the Coriolis force and the centrifugal force disappear when the density of the cold plasma becomes the same as that of the core plasma ($f=0$).

In the case of *plasma mode*, the first term on the right-hand side of the real frequency given by Eq. (32); $[m-f]\Omega_0$ consists of the effects of the Doppler shift $m\Omega_0$ and of the Coriolis force $-f\Omega_0$. On the other hand, the second term in the square root, namely $[m-f]\Omega_0^2$, consists of the centrifugal force $m\Omega_0^2$ and the Coriolis force $-f\Omega_0^2$. Since $m-f \geq m-1 \geq 1$ for $m \geq 2$, the effects of the poloidal rotation mainly appear as the Doppler shift and the centrifugal force in the linear growth rate γ through the real frequency ω_r , as the poloidal mode number m increases. Moreover, as the density of the cold plasma reaches that of the core plasma ($f \rightarrow 0$), the effect of the centrifugal force disappears as well as the *wall mode* and the Doppler shift becomes dominant and destabilizes the *plasma mode* for $\Omega_0 > \Omega_{c1}$.

When the density of the cold plasma is exactly the same as that of the core plasma ($f=0$), only the Doppler shift influences the linear growth rate γ of the *plasma mode* and γ monotonically decreases with Ω_0 . In contrast, when the density of the cold plasma becomes smaller than that of the core plasma ($\rho_v < \rho_0$), or $0 < f < 1$, the effective gravity of the poloidal rotation brought by the centrifugal force and the Coriolis force drives an instability like the Rayleigh-Taylor instability. A critical rotation frequency Ω_{c2} , beyond which a mode like the Rayleigh-Taylor instability occurs, is determined by the divergence condition of the linear growth rate γ given by Eq. (31),

$$\Omega_{c2}^2 = - \frac{\gamma_w^2}{\left(1 + \alpha \frac{\rho_v}{\rho_0}\right) f [m-f]}. \quad (34)$$

In the case with $\Omega_0 > \Omega_{c2}$, under the condition $\gamma\tau_w^* \gg 1$ and $\gamma\tau_w^* |\gamma_w^2| \gg \gamma_{dt}^2$, the dispersion relation becomes

$$-(\omega - m\Omega_0)^2 \left(1 + \alpha \frac{\rho_v}{\rho_0}\right) = \gamma_w^2 + \left(1 - \frac{\rho_v}{\rho_0}\right) (2\omega - m\Omega_0) \Omega_0. \quad (35)$$

In this case, the mode grows faster than the diffusion time of the resistive wall so that the resistive wall behaves like a

perfect conducting wall. From Eq. (35), we obtain the unstable root,

$$\gamma \approx \sqrt{f[m-f]\Omega_0^2 + \frac{\gamma_w^2}{1 + \alpha \frac{\rho_v}{\rho_0}}} = \sqrt{f[m-f](\Omega_0^2 - \Omega_{c2}^2)}, \quad (36)$$

$$\omega_r \approx [m-f]\Omega_0. \quad (37)$$

In this high rotation case with $\Omega_0 > \Omega_{c2}$, although the real frequency ω_r given by Eq. (37) is determined by the combination between the Doppler shift $m\Omega_0$ and the Coriolis force $-f\Omega_0$, the linear growth rate is mainly determined by the centrifugal force $m\Omega_0^2$ and the Coriolis force $-f\Omega_0^2$, and the Doppler shift does not contribute to the destabilization as is understood from Eq. (36). Since $m-f \geq m-1 \geq 1$ for $m \geq 2$, the effects of the poloidal rotation, expressed as $[m-f]\Omega_0^2$, mainly appear as the centrifugal force in the linear growth rate γ given by Eq. (36). The first factor f in $f[m-f]\Omega_0^2$ of Eq. (36) indicates the effect of the density gradient at the plasma boundary [see $f \equiv (1 - \rho_v/\rho_0)/(1 + \alpha\rho_v/\rho_0)$]. When the density of the cold plasma is lower than that of the core plasma, namely $\rho_v < \rho_0$ or $0 < f < 1$, the direction of the density gradient is opposite to that of the centrifugal force of the poloidal rotation. In such a situation, when the poloidal rotation frequency is larger than Ω_{c2} , the destabilizing effect of the centrifugal force of the poloidal rotation becomes larger than the stabilized effect of the magnetic field line bending, so that the mode like the Rayleigh-Taylor instability becomes unstable with the linear growth rate γ given by Eq. (36).

The above analytical results are summarized as follows. When the cold plasma surrounding the core plasma has the same density as that of the core plasma ($\rho_v = \rho_0$) or ($f=0$), as is directly understood from the dispersion relation, the effects of the poloidal rotation appear only as the Doppler shift, and the mode like the Rayleigh-Taylor instability does not appear. On the other hand, when the cold plasma is treated as the vacuum ($f=1$), the effects of the centrifugal force and the Coriolis force brought by the poloidal rotation appear most significantly, and the mode like the Rayleigh-Taylor instability appears for high poloidal rotation. Although the effects of the Doppler shift, the centrifugal force, and the Coriolis force coexist in a realistic situation with a cold plasma with a relatively low density ($0 < f < 1$), the effect of the Doppler shift is more dominant than both the Coriolis force and the centrifugal force in the range of a low rotation frequency, which is our main interest. This tendency becomes strong as the poloidal mode number m increases.

In the remainder of this subsection, the above analytical consideration is compared with the numerical results. It is important to suppress RWMs with low toroidal mode number. The pressure-driven RWMs with $n=1$ limit attainable beta in several tokamaks. For low beta plasma, RWMs are driven by a current gradient. In JT-60U, the current-driven RWMs with $(m,n)=(3,1)$ mode have been observed.³⁵ Thus, we study RWMs with $(m,n)=(3,1)$ mode in this paper. Figure 1 shows the dependence of linear growth rate γ of the $(m,n)=(3,1)$ mode on the resistive wall position r_w in a

FIG. 1. Dependence of the linear growth rate of $(m,n)=(3,1)$ mode on wall position for $\Omega_0=0$.

nonrotating equilibrium with $\Omega_0=0$, where a perfect conducting wall is located at $r_d=4a$ and the safety factor at the plasma surface $q_a=q(r=a)$ is chosen as $q_a=2.05$. The thickness of the resistive wall d is assumed to be $d=0.04a$. For calculating the linear growth rate using our simulation code, the current profile is assumed to be

$$J_{eq} = \frac{1}{2}(J_0 - J_v) \left\{ 1 - \tanh \left[200 \left(\frac{r}{a} - 1 \right) \right] \right\} + J_v, \quad (38)$$

where $J_0 \gg J_v$ and the profile of density is chosen as the same functional form as that in Eq. (38). The resistivity is assumed to be inversely proportional to the current density.

The simulation results agree well with the analytically obtained dispersion relation given by Eq. (28) as shown in Fig. 1. For $r_w > 1.5a$, the external kink mode becomes unstable and the linear growth rate depends on the density of the cold plasma. The linear growth rate γ decreases as the density of the cold plasma ρ_v increases, since the released magnetic energy is used to move not only the core plasma but also the cold plasma for $\rho_v \neq 0$. When the resistive wall is sufficiently close to the plasma surface ($r_w < 1.5a$), the wall mode becomes unstable. Since the inertia term can be neglected for the slowly growing wall mode, the linear growth rate of the wall mode does not depend on ρ_v for $\Omega_0=0$.

Figure 2 shows the dependence of the linear growth rate γ on the poloidal rotation frequency Ω_0 for various densities of the cold plasma ρ_v for $r_w=1.3a$. The numerical results show good agreement with analytical ones. According to Eq. (33), $\Omega_{c1}\tau_a \approx 0.14$ for $\rho_v=0$ and $\Omega_{c1}\tau_a \approx 0.06$ for $\rho_v/\rho_0=1$, respectively. The range of the equilibrium rotation frequency giving the wall mode increases as the density of the cold plasma decreases. The linear growth rate of the wall mode increases with poloidal rotation frequency for $\Omega_0 \leq \Omega_{c1}$. Even for a rigid equilibrium rotation, there is a difference between the equilibrium rotation frequency and the real mode frequency due to the resistive wall. Thus, the plasma rotation provides the energy source and the mode is driven by not only the current gradient but also the rotation energy

FIG. 2. Dependence of (a) linear growth rate and (b) real mode frequency of $(m,n)=(3,1)$ mode on poloidal rotation frequency. Here the resistive wall is located at $r_w=1.3a$. The approximate solutions for $\rho_v=0$ are also plotted.

through the Doppler shift term, as is understood from Eq. (29). For $\rho_v \neq 0$, the wall mode can grow using the rotation energy in the cold plasma as well as that in the core plasma. Consequently, the linear growth rate of the wall mode becomes larger as the density of the cold plasma increases for the same equilibrium rotation frequency in contrast to the external kink mode.

The plasma mode is stable for $\Omega_0 < \Omega_{c1}$. If the perfect conducting wall is located at $r=r_w$ instead of the resistive wall, i.e., $\tau_w^* \rightarrow \infty$, the mode is a pure wave with real mode frequency $\omega_r = -\sqrt{\gamma_w^2/(1 + \alpha\rho_c/\rho_0)}$ (<0) for $\Omega_0=0$. When the resistive wall is located at $r=r_w$, the wave is attenuated due to the resistivity of the resistive wall for $\Omega_0=0$. As the poloidal rotation increases, the real mode frequency of the plasma mode also increases. When the poloidal rotation fre-

FIG. 3. Nonuniform profiles of safety factor q , plasma current density J , resistivity η , and density ρ at equilibrium state. For the density, the profiles for $\Omega_0=0$ and $\Omega_0\tau_0=0.16$ are plotted, where $\rho_v/\rho_0=10^{-3}$ is assumed. They are almost the same profile.

quency is larger than Ω_{c1} , the real mode frequency becomes positive and the *plasma mode* becomes unstable. The linear growth rate of the *plasma mode* monotonically decreases as the equilibrium rotation frequency increases for $\Omega_0 > \Omega_{c1}$. The linear growth rate of the *plasma mode* becomes small as the density of the cold plasma increases for the same equilibrium rotation frequency in contrast to the *wall mode*, because the density gradient at the plasma boundary at $r=a$ decreases. The linear growth rate for $\rho_v/\rho_0 < 1$ increases again according to Eq. (36) for $\Omega_0 > \Omega_{c2}$ as a mode like the Rayleigh-Taylor instability.

$$J_{\text{eq}}(r) = \begin{cases} (J_0 - J_v) \left\{ 0.48 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + 0.52 \right\} \left[1 - \left(\frac{r}{a} \right)^{30} \right]^{10} + J_v & \text{for } 0 \leq r \leq a \\ J_v (\ll J_0) & \text{for } a < r \end{cases} \quad (39)$$

The profiles of q , J_{eq} , and η_{eq} are shown in Fig. 3. The resistivity profile in Eq. (1) is assumed to be inversely proportional to $J_{\text{eq}}(r)$ for $r < r_w$. Hence, the temperature is determined so as to satisfy $T \propto J^{2/3}$ by using Eq. (7). The density profile for $r < r_L = 0.95a$ is determined from the equilibrium equation

$$\rho(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \mathbf{J} \times \mathbf{B}, \quad (40)$$

where $P = nT$. The density profile is chosen as a monotonically decreasing function for $r_L < r < a$ and $\rho_{\text{eq}}(r) = \rho_v$ for $r > a$. Thus, the equilibrium equation (40) is not satisfied near the plasma surface ($r_L < r < a$). However, since the initial profile depends on only minor radius r , it satisfies equilib-

FIG. 4. Dependence of linear growth rate of $(m,n)=(3,1)$ mode on resistive wall position for $\Omega_0=0$ with nonuniform current profile as shown in Fig. 3.

The linear results obtained from our simulation code agree well with the solutions of the analytic dispersion relation in Fig. 2. The cold plasma for $\rho_v/\rho_0 = 10^{-3}$ may be regarded as the vacuum, since the results for $\rho_v/\rho_0 = 10^{-3}$ are almost the same as the analytic ones for $\rho_v = 0$ for a uniform current density profile as is shown in Figs. 1 and 2.

B. On the equilibrium with a nonuniform current density profile

The step-like current profile is a highly idealized case. In general, the current profile is smooth and magnetic shear is finite in the core plasma. In this section, we use a nonuniform current profile in order to consider the effects of the current profile on the linear growth rate of the RWMs with $(m,n)=(3,1)$. The current profile at equilibrium is chosen as

rium conditions of our numerical model equations (1)–(7). Note that the density profile n or the mass density ρ_{eq} changes according to the poloidal rotation frequency Ω_0 . However, the density profile largely does not change in our range of interest of the rotation as shown in Fig. 3. The equilibrium profile discussed in this subsection is used for nonlinear simulation in the next section. When the perfect conducting wall is located at $r=1.3a$, the mode is stabilized perfectly for this equilibrium profile. When the wall has a finite resistivity, the RWM becomes unstable, $r_w < 1.3a$, as shown in Fig. 4. Figure 5 shows the dependence of the linear growth rate of the $(m,n)=(3,1)$ mode on the poloidal rotation frequency for various densities of the cold plasma,

FIG. 5. Dependence of (a) linear growth rate and (b) real mode frequency of $(m,n)=(3,1)$ mode on poloidal rotation frequency in plasma with nonuniform current profile as shown in Fig. 3.

where the resistive wall is located at $r_w=1.2a$ and the perfect conducting wall is located at $r_d=2a$. The thickness of the resistive wall is assumed to be $d=0.02a$. As well as in the case of the step-like current density profile, the linear growth rate for $\rho_v/\rho_0 \leq 10^{-3}$ does not depend on ρ_v . Thus, we can regard the cold plasma with $\rho_v/\rho_0=10^{-3}$ as a good model of the vacuum. By using the consistent MHD equilibria shown in Fig. 5, it is found that (i) the range of the *wall mode*, $\Omega_0 < \Omega_{c1}$ increases as the density of the cold plasma ρ_v decreases, (ii) the linear growth rate of the *wall mode* increases as the density of the cold plasma ρ_v decreases, (iii) the linear growth rate of the *plasma mode* decreases as the density of the cold plasma ρ_v decreases, and (iv) when the density of the cold plasma is the same as that of the core plasma, the

mode like the Rayleigh-Taylor instability does not appear. The second result is opposite to that in the step-like current density plasma as shown in Fig. 2. It might be considered to come from the inertia effect. As shown in Fig. 4, the dependence of the linear growth rate without rotation on ρ_v becomes stronger as r_w increases. For $r_w=1.2a$, the linear growth rates for $\rho_v/\rho_0=10^{-3}$ and $\rho_v/\rho_0=1$ without rotation are $\gamma\tau_a=3.4 \times 10^{-2}$ and 2.3×10^{-2} , respectively. Thus, there is about a 30% difference between them. The difference results from the effect of the inertia term. Although the effect of the inertia term for the RWM with $r_w=1.2a$ is weaker than for the kink mode with $r_w > 1.3a$, a part of the released magnetic energy is used to move not only the core plasma but also the cold plasma for $\rho_v \neq 0$ by small but non-negligible inertia effect. Hence, the linear growth rate decreases as the density of the cold plasma increases like the kink mode. On the other hand, when poloidal rotation is applied, the density of the cold plasma has a destabilizing effect since the rotation of the cold plasma provides the energy source as mentioned in Sec. III A. Therefore, the density of the cold plasma has both a stabilizing and a destabilizing effect. For $r_w=1.2a$, since the stabilizing effect of the inertia effect is superior to the destabilizing effect of the rotation, the linear growth rate of *wall mode* decreases as the density of the cold plasma increases, as shown in Fig. 5. As the resistive wall becomes close to the plasma surface, the stabilizing effect of the inertia becomes small. For $r_w=1.1a$, we have confirmed that the linear growth rate of the *wall mode* increases with the density of the cold plasma, which is consistent with that in the step-like current density plasma as shown in Fig. 2. Since we are interested in the effects of the Doppler shift dominant brought by the low equilibrium poloidal rotation frequency on the *wall mode* and the *plasma mode*, such situations are treated in the following nonlinear simulations.

IV. NONLINEAR PROPERTIES

In Sec. IV A, the effects of the density of the cold plasma on nonlinear evolutions of the external kink mode and the *wall mode* are investigated in equilibria without poloidal rotation. Also, the energy conversion among the kinetic energy, magnetic energy, Joule heating, and viscous energy is discussed. The nonlinear evolutions of the *wall mode* and the *plasma mode* with poloidal rotation are investigated in Sec. IV B. The emphasis is put on the relation of the saturation level to the equilibrium poloidal rotation frequency.

A. Nonlinear behaviors of external kink modes and wall modes in equilibria without poloidal rotation

Figure 6 shows the time evolution of magnetic energy of the $(m,n)=(3,1)$ Fourier component for (a) the kink mode without the resistive wall and (b) the *wall mode* with $r_w=1.2a$, where the equilibrium in Sec. III B is used with the perfect conducting wall located at $r_d=2a$. The thickness of the resistive wall is assumed to be $d=0.02a$. Although the linear growth rate of the kink mode depends on the ratio of the density ρ_v/ρ_0 , the saturation levels are almost the same independent of the ratio of the density. For the reduced MHD

FIG. 6. Time evolution of magnetic energy of $(m, n) = (3, 1)$ component with various ρ_v (a) for the kink mode without the resistive wall and (b) for the wall mode with the resistive wall located at $r_w = 1.2a$.

equations used in this paper, the energy conservation equation is given by

$$\frac{dE_m}{dt} + \frac{dE_k}{dt} + \frac{dQ_j}{dt} + \frac{dQ_v}{dt} = 0, \quad (41)$$

where

$$E_m = \int_0^{r_d} dr \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta r R |\nabla_{\perp} \psi|^2 / 2,$$

$$E_k = \int_0^{r_w} dr \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta \rho r R |\nabla_{\perp} \phi|^2 / 2,$$

$$\frac{dQ_j}{dt} = r R \int_0^{r_d} dr \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta (\eta J^2 - JE),$$

FIG. 7. Time evolution of various energies of the external kink mode for $\rho_v / \rho_0 = 10^{-3}$.

$$\frac{dQ_v}{dt} = r R \int_0^{r_w} dr \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta (-\phi \mathbf{e}_z \cdot \nabla \times \nabla \cdot \boldsymbol{\pi} + \phi S_m).$$

The time evolution of the various energies of the kink mode for $\rho_v / \rho_0 = 10^{-3}$ is shown in Fig. 7, where the change of magnetic energy $\Delta E_m = E_m(t) - E_m(0)$, the change of kinetic energy $\Delta E_k = E_k(t) - E_k(0)$, the dissipative energy due to Joule heating $Q_j = \int_0^t dQ_j / dt' dt'$, and the dissipative energy due to viscosity $Q_v = \int_0^t dQ_v / dt' dt'$ are plotted separately. In the linear phase, the magnetic energy decreases and most of the released magnetic energy is converted into the kinetic energy, since the kink mode is a current driven mode. In the nonlinear phase, the kinetic energy decreases and the released magnetic energy dissipates due to Joule heating. Consequently, a new helical equilibrium profile without flow is obtained and the inertia term is not important at the saturation state, leading to the fact that the saturation level almost does not depend on the density of the cold plasma.

As is shown in Fig. 6(b), the saturation level of the wall mode also does not depend on the ratio of the density ρ_v / ρ_0 . The time evolution of the various energies of the wall mode for $\rho_v / \rho_0 = 10^{-3}$ is shown in Fig. 8. The wall mode grows so slowly that the inertia term can be neglected in the linear and the nonlinear phase, and most of the released magnetic energy dissipates due to Joule heating in the linear and the nonlinear phase, which leads to the fact that the saturation level is almost independent of the density of the cold plasma. Figure 9 shows where the energy of the wall mode dissipates due to the Joule heating. In Fig. 9, $dQ_j / dt|_{\text{core}} = \int_0^a F dr$, $dQ_j / dt|_{\text{cold}} = \int_a^{r_w} F dr$, $dQ_j / dt|_{\text{wall}} = \int_{r_w}^{r_w+\delta} F dr$, and $dQ_j / dt|_{\text{vacuum}} = \int_{r_w+\delta}^{r_d} F dr$ are time derivatives of dissipative energy due to Joule heating in the core plasma ($r < a$), in the cold plasma ($a \leq r < r_w$), in the resistive wall ($r_w \leq r \leq r_w + \delta$), and in the external pseudovacuum ($r_w + \delta < r < r_d$), respectively, where $F \equiv \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta r R (\eta J^2 - JE)$. The energy dissipates in the resistive wall in the linear phase. As the amplitude of the wall mode becomes close to the saturation

FIG. 8. Time evolution of various energies of the *wall mode* for $\rho_v/\rho_0 = 10^{-3}$, where the resistive wall is located at $r_w = 1.2a$.

level, the *wall mode* grows more slowly and the current flowing in the resistive wall becomes smaller. Thus, the energy dissipates at the plasma rather than the resistive wall in the nonlinear phase. From the viewpoint of the mode structure, the mode structure of the $(m,n)=(3,1)$ Fourier component of the poloidal magnetic flux $\tilde{\psi}_{3,1}$ becomes similar to that of the kink mode, since the effect of the resistive wall becomes weak in the nonlinear phase.

B. Nonlinear behaviors of wall modes and plasma modes in equilibria with poloidal rotation

In Sec. IV B 1, the effects of the poloidal rotation; mainly the effects of the Doppler shift on the *wall mode*, are investigated in rotating equilibria surrounded by the vacuum, where the cold plasma with $\rho_v/\rho_0 = 10^{-3}$ is used instead of the vacuum. The effects of the Doppler shift on both the *wall*

FIG. 9. Time evolution of dissipative energy $dQ_j/dt|_{\text{core}}$, $dQ_j/dt|_{\text{cold}}$, $dQ_j/dt|_{\text{wall}}$, and $dQ_j/dt|_{\text{vacuum}}$ due to Joule heating for the *wall mode* corresponding to Fig. 8.

FIG. 10. Time evolution of magnetic energy of $(m,n)=(3,1)$ component for various poloidal rotation frequencies Ω_0 . Here the density ratio is assumed to be $\rho_v/\rho_0 = 10^{-3}$.

mode and the *plasma mode* are investigated in rotating equilibria surrounded by the dense cold plasma in Sec. IV B 2, where the cold plasma with $\rho_v/\rho_0 = 1$ is used. The nonlinear behaviors of the equilibrium poloidal rotation are examined in Sec. IV B 3 in terms of the torque.

1. Wall modes in rotating equilibria surrounded by the vacuum

In order to examine the effects of the equilibrium rigid poloidal rotation on the *wall mode* in rotating equilibria surrounded by the vacuum, the nonlinear evolution of the *wall mode* is investigated. In such equilibria surrounded by the vacuum, although the effects of the equilibrium poloidal rotation appear as the Doppler shift, the centrifugal force, and the Coriolis force, the Doppler shift becomes the dominant effect for the *wall mode*. Time evolution of magnetic energy of the $(m,n)=(3,1)$ Fourier component of the *wall mode* is shown in Fig. 10, where the cold plasma with $\rho_v/\rho_0 = 10^{-3}$ is used instead of the vacuum, because such a low-density cold plasma acts as the vacuum as is discussed in Sec. III B. The equilibria with a nonuniform current density profile discussed in Sec. III B are used for various initial rigid poloidal rotation frequencies $\Omega_0\tau_a (< \Omega_{c1}\tau_a \approx 0.14)$, where the resistive wall and the perfect conducting wall are located at $r_w = 1.2a$ and $r_d = 2a$, respectively. The thickness of the resistive wall is assumed to be $d = 0.02a$. It is found from Fig. 10 that the final saturation levels of the *wall modes* do not depend on the initial rotation frequency. Figure 11 shows the time evolution of the radial profile of the initial poloidal rotation frequency with $\Omega_0\tau_a = 0.08$. Although the poloidal rotation still remains in the linear phase ($t < 70\tau_a$), the rotation frequency almost vanishes near the plasma surface in the nonlinear phase ($t > 100\tau_a$), leading to the fact that the saturation level of the *wall mode* does not depend on the equilibrium poloidal rotation.

FIG. 11. Time evolution of radial profile of poloidal rotation frequency for $\Omega_0 \tau_a = 0.08$ corresponding to Fig. 10.

Figure 12 shows the time evolution of various energies in equilibrium with $\Omega_0 \tau_a = 0.12$. In the rotating equilibria, the *wall mode* is driven by the rotation energy rather than the magnetic energy in the linear phase, so that the kinetic energy E_k decreases and is converted into the Joule heating Q_j in contrast to the external kink modes or the *wall mode* in equilibria without the poloidal rotation. In the nonlinear phase, the rotation decreases near the plasma surface and the magnetic energy E_m begins to decrease.

When the equilibrium plasma rotation exists, the mode structure of the poloidal magnetic flux for the $(m, n) = (3, 1)$ Fourier component can be described as

FIG. 12. Time evolution of energy of the *wall mode* for $\Omega_0 \tau_a = 0.12$ corresponding to Fig. 10.

FIG. 13. Time evolution of radial profiles of (a) ψ^* and (b) δ of $(m, n) = (3, 1)$ mode for $\Omega_0 \tau_a = 0.12$ and $\rho_v / \rho_0 = 10^{-3}$. The radial profiles of ψ^* for the *wall mode* and the external kink mode for $\Omega_0 = 0$ at the saturation are also shown in (a).

$$\tilde{\psi}^*(r) \cos[3\theta - \delta(r)] = \tilde{\psi}_{3,1}^c(r) \cos(3\theta) + \tilde{\psi}_{3,1}^s(r) \sin(3\theta) \quad (42)$$

on the poloidal section with $\zeta = 0$. In Fig. 13, the amplitude $\tilde{\psi}^*(r)$ and the relative phase shift $\delta(r) - \delta(0)$ are shown in both equilibria with $\Omega_0 \tau_a = 0.12$ and with $\Omega_0 = 0$. When the equilibrium plasma rotation exists, both the amplitude and the phase have kink structures in the initial phase on the wall position at $r = r_w$. However, such kink structures disappear in the nonlinear phase because the poloidal rotation almost vanishes near the plasma boundary. Consequently, the mode structure changes into that in the equilibrium without rotation.

FIG. 14. Time evolution of magnetic energy of $(m,n)=(3,1)$ component for various poloidal rotation frequencies Ω_0 for a constant density profile, i.e., $\rho_v = \rho_0$.

2. Wall modes and plasma modes in rotating equilibria surrounded by the dense cold plasma

The effects of the Doppler shift on both the *wall mode* and the *plasma mode* are investigated in rotating equilibria surrounded by the dense cold plasma. For this purpose, the cold plasma with $\rho_v/\rho_0=1$ is used, because the effects of the centrifugal force and the Coriolis force disappear when the density discontinuity vanishes at the plasma boundary, as is understood from Eq. (26) or Fig. 5. In this case with $\rho_v/\rho_0=1$, the *wall mode* becomes unstable for $\Omega_0\tau_a < \Omega_{c1}\tau_a \approx 0.025$, and the *plasma mode* becomes unstable for $\Omega_0\tau_a > \Omega_{c1}\tau_a \approx 0.025$, as is understood from Fig. 5. The difference of the nonlinear behavior between the *wall mode* and the *plasma mode* is studied. By comparing the results between $\rho_v/\rho_0=10^{-3}$ and 1, the effect of the density of the cold plasma on the *wall mode* is also investigated.

Figure 14 shows the time evolution of magnetic energy of the $(m,n)=(3,1)$ Fourier component for various poloidal rotation frequencies. The time evolutions for $\Omega_0\tau_a \leq \Omega_{c1}\tau_a \approx 0.02$ correspond to the *wall mode*, and the time evolutions for $\Omega_0\tau_a \geq 0.04 > \Omega_{c1}\tau_a$ correspond to the *plasma mode*. The final saturation level of the *wall mode* does not depend on the initial rotation level, a fact that is considered to be due to the reduction of the rotation near the plasma boundary $r=a$. From the comparison between Figs. 10 and 14, the nonlinear saturation level of the *wall mode* is considered not to depend on the density of the cold plasma. On the other hand, the saturation level of the *plasma mode* decreases as the initial rotation frequency increases, because the reduction of the poloidal rotation near the plasma boundary decreases as the initial poloidal rotation increases, as is shown in Fig. 15. Figure 16 shows the profiles of the amplitude $\psi^*(r)$ and the relative phase $\delta(r) - \delta(0)$ given by Eq. (42) for the *plasma mode* with $\Omega_0\tau_a=0.04$ and 0.1 in the linear phase and at the saturation state. When the initial rotation frequency becomes larger, the mode amplitude $\psi^*(r)$ and the relative phase shift $\delta(r) - \delta(0)$ are more kinked around $r=r_w$. When

FIG. 15. Radial profile of poloidal rotation frequency at saturation for $\Omega_0\tau_a=0.04, 0.08$, and 0.1.

the initial poloidal rotation is small ($\Omega_0\tau_a=0.04$), such kinked structures are reduced in the nonlinear phase as well as the *wall mode*, because the poloidal rotation is reduced. In contrast, the mode structure does not change at saturation when the initial poloidal rotation is large ($\Omega_0\tau_a=0.1$). Thus, the saturation level of the *plasma mode* becomes smaller due to the stabilizing effect of field line bending as the initial poloidal rotation frequency increases.

3. Reduction of poloidal rotation

The simulation results suggest that the saturation level of the RWM depends on the reduction of the rotation, especially near the plasma boundary. The time evolution of poloidal rotation is obtained from the average of the poloidal component of the momentum equation with respect to the angles or a volume integral of the vorticity equation (2),

$$\frac{\partial}{\partial t} \langle \rho v_\theta \rangle = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle \rho v_r v_\theta \rangle + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle B_r B_\theta \rangle - \langle (\nabla \cdot \pi)_\theta \rangle, \quad (43)$$

where $\langle f \rangle \equiv \int_0^{2\pi} \int_0^{2\pi} f d\theta d\zeta / 4\pi^2$. From Eq. (43), the electromagnetic torque T_m and the kinetic torque T_v are defined as

$$T_m = 4\pi^2 R \int_0^{r_w} \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle B_r B_\theta \rangle, \\ T_v = -4\pi^2 R \int_0^{r_w} \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle \rho v_r v_\theta \rangle.$$

Figure 17 shows the dependence of two types of torque $T_m/\max[|\tilde{\psi}|^2]$ and $T_v/\max[|\tilde{\psi}|^2]$ on the poloidal rotation frequency, where each torque is calculated by using a linear eigenmode structure. Since the T_m is negative and T_v is positive, the electromagnetic torque T_m acts as the slowing down torque, and the kinetic torque T_v acts as the accelerating torque. The equilibrium poloidal rotation decreases mainly due to the electromagnetic torque T_m . For the *wall mode* with

FIG. 16. The time evolution of radial profiles of (a) ψ^* and (b) δ for $\Omega_0 \tau_a = 0.04$ and 0.1 . Here the results at $t = 200 \tau_a$ and at $t = 680 \tau_a$ for $\Omega_0 \tau_a = 0.04$ correspond to the linear phase and the saturation, respectively. For $\Omega_0 \tau_a = 0.1$, the results at $t = 1000 \tau_a$ and at $t = 4680 \tau_a$ correspond to the linear phase and the saturation, respectively. The profiles of ψ^* and δ for $\Omega_0 \tau_a = 0.1$ almost do not change with time.

$\Omega_0 \leq \Omega_{c1}$, the linear growth rate and the slowing down electromagnetic torque increase with the rotation frequency, where $\Omega_{c1} \tau_a \approx 0.14$ for $\rho_v/\rho_0 = 10^{-3}$ and $\Omega_{c1} \tau_a \approx 0.02$ for $\rho_v = \rho_0$, respectively. In contrast, for the *plasma mode* with $\Omega_0 > \Omega_{c1}$, the slowing down electromagnetic torque decreases as the initial rotation increases.

The electromagnetic torque acting inside the plasma results from the reaction of the torque acting inside the resistive wall; $T_w \propto \tilde{\psi}_w^2 [\delta']_{r_w^+}^{r_w^-}$, where $\tilde{\psi}_w$ and δ are the amplitude and the phase of the poloidal magnetic flux given by Eq. (42) at the resistive wall.³⁶ The electromagnetic torque depends on the effect of both the jump of the radial derivative of the phase, $[\delta']_{r_w^+}^{r_w^-}$, and the amplitude of the flux at the resistive wall, $\tilde{\psi}_w$. As the initial poloidal rotation frequency increases,

FIG. 17. Dependence of torque acting inside the plasma for $\rho_v = \rho_0$ and $\rho_v/\rho_0 = 10^{-3}$ on poloidal rotation frequency.

the kink of the mode becomes larger and such a kink has the effect of increasing the slowing down torque T_m . The other effect of the rotation is to prevent the magnetic flux from diffusing into the resistive wall. This effect decreases the slowing down torque. When the *wall mode* is unstable or the initial poloidal rotation frequency is small, the kink effect of the mode is larger than the effect of preventing the magnetic flux from diffusing into the wall. Thus, the slowing down torque T_m increases as the initial rotation frequency increases. When the *plasma mode* is unstable or the initial poloidal rotation frequency is larger than Ω_{c1} , the effect of preventing the flux from diffusing into the wall becomes larger. Thus, as the initial rotation frequency ($> \Omega_{c1}$) increases, the electromagnetic slowing down torque decreases and the rotation tends to survive in the nonlinear phase, as shown in Fig. 15.

V. SUMMARY

In a reactor-sized tokamak such as the ITER, the plasma is expected to be almost stationary or slowly rotating, and the remaining resistive wall modes (RWMs) might lead to the plasma disruption. Also, core hot plasmas are usually surrounded by cold plasmas with various levels of density compared to that of the core plasma, according to the experimental conditions. Thus, the linear and nonlinear behaviors of RWMs have been mainly investigated in a Doppler shift predominant rotating plasma, in order to understand the influences of both the equilibrium plasma rotation or the Doppler shift and the external cold plasma on the nonlinear saturation level of RWMs. A set of well-known reduced MHD equations is used in a cylindrical tokamak geometry, where a core plasma is surrounded by a cold plasma with an extremely high resistivity, and the rotation of the equilibrium plasma is in the poloidal direction.

The linear growth rates of RWMs investigated in this work do not depend on the resistivity of the cold plasma, and so they are ideal RWMs and not resistive RWMs. When the equilibrium rotation does not exist, the density of the cold plasma does not affect the linear growth rate and nonlinear behavior of the RWMs. On the other hand, when the equilibrium poloidal rotation exists, the linear growth rates of RWMs depend on the ratio of the density of the cold plasma ρ_v to the density of the core plasma ρ_0 . In the present situation, the cold plasma with the density $\rho_v \sim 10^{-3} \rho_0$ is regarded as the vacuum.

Since the linear dispersion relation of RWMs is a cubic equation, there are three roots. One of them is always stable, and the others are two potentially unstable roots, which are called the *wall mode* nearly locked to the resistive wall and the *plasma mode* rotating to the resistive wall. When the poloidal rotation frequency Ω_0 is fairly small ($\Omega_0 < \Omega_{c1}$), the *wall mode* becomes unstable by the Doppler shift, and the linear growth rate increases with rotation frequency, where Ω_{c1} is approximately the real frequency of the *plasma mode* in the stationary plasma with $\Omega_0 = 0$. For a higher rotation frequency with $\Omega_0 > \Omega_{c1}$, the plasma mode becomes unstable by the Doppler shift and/or the centrifugal force, and the linear growth rate decreases as the rotation frequency increases. When the density of the cold plasma is fairly low compared to that of the hot plasma, an instability such as the Rayleigh-Taylor instability appears for $\Omega_0 > \Omega_{c2}$ by driving the coupling of the inward density gradient to the outward centrifugal force caused by the poloidal rotation. When the cold plasma is regarded as the vacuum ($\rho_v = 0$), Ω_{c2} is given by $\Omega_{c2}/\Omega_{c1} \approx \sqrt{m}$, where m is the poloidal mode number. Note that the strong destabilization due to the centrifugal force occurs also for toroidal rotation.¹⁵ For the *wall mode*, the Doppler shift is predominant over the centrifugal force and the Coriolis force is independent of the density of the cold plasma. On the other hand, the *plasma mode* is mainly dominated by the Doppler shift, when the density of the cold plasma is similar to the density of the core plasma ($\rho_v/\rho_0 \approx 1$).

Nonlinear behaviors are investigated under the condition that the Doppler shift is predominant. The nonlinear saturation level of the *wall mode* does not depend on either the equilibrium poloidal rotation frequency or the density of the cold plasma, since the rotation decreases to almost zero near the plasma surface. In the case of the *plasma mode*, the saturation level becomes smaller as the initial rotation frequency increases, since the rotation remains at saturation. It is found that the different dependence of the saturation level on the poloidal rotation results from the dependence of the electromagnetic slowing down torque on the frequency of the poloidal rotation. As the initial rotation frequency increases, the slowing down torque decreases for the *plasma mode* and it increases for the *wall mode*. Therefore, for the *plasma mode*, the rotation tends to survive in the nonlinear phase, and the saturation level of the *plasma mode* is suppressed by the poloidal rotation. Partially because the dependence of the electromagnetic slowing down torque on the rotation frequency can be explained by the linear property of the RWMs, and partially because the linear property of the

modes does not depend on the direction of the rotation under the situation in which the effect of the Doppler shift is more dominant than the centrifugal force and the Coriolis force, the nonlinear behaviors presented in this work might be seen in the cases with the toroidal rotation.

In the present work, poloidal rotation is used instead of toroidal rotation, in order to examine the nonlinear behaviors of RWMs under the situation in which the Doppler shift is predominant. Although there are many types of reduced MHD equations including the parallel (toroidal) flow,¹⁸⁻²¹ most of them cannot correctly treat the toroidal Doppler shift. In order to treat the toroidal Doppler shift correctly, the time evolution of $v_{\parallel} \tilde{\mathbf{b}}$ should be included in the vorticity equation, where $v_{\parallel}$ is parallel velocity, $\tilde{\mathbf{b}} = \tilde{\mathbf{B}}/|\tilde{\mathbf{B}}|$, and $\tilde{\mathbf{B}}$ is the perturbed magnetic field. However, such an effect is neglected except for Izzo's model.¹⁸ In Izzo's model, the toroidal velocity v_{ζ} is assumed to be $v_{\zeta} \sim \epsilon v_p$, which is not relevant to studying RWMs under the toroidal rotation with a few percent of Alfvén velocity, where v_p is the poloidal velocity and ϵ is the inverse aspect ratio. In order to correctly include the toroidal Doppler shift, a new set of the reduced MHD equations must be derived by assuming large toroidal velocity, $v_{\zeta} \sim v_p/\epsilon$. Deriving a new set of the reduced MHD equations and investigating nonlinear behaviors of RWMs with both toroidal and poloidal rotations are planned for future work.

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