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journal or publication title	Physics of Plasmas
volume	Vol.13
number	Issue 8
page range	pp.082503-1 - 082503-7
year	2006-08-01
URL	http://hdl.handle.net/10655/3821

doi: 10.1063/1.2234296

Properties of energetic-particle continuum modes destabilized by energetic ions with beam-like velocity distributions

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(Received 16 May 2006; accepted 27 June 2006; published online 7 August 2006)

Properties of energetic-particle continuum modes (EPMs) destabilized by energetic ions in tokamak plasmas were investigated using a hybrid simulation code for magnetohydrodynamics and energetic particles. The energetic ions are assumed to have beam-like velocity distributions for the purpose of clarifying the dependence on energetic ion velocity. It was found that for beam velocities lower than the Alfvén velocity, the unstable modes are EPMs while the toroidal Alfvén eigenmodes are unstable for the beam velocities well above the Alfvén velocity. The EPMs destabilized by the copassing energetic ions and those destabilized by the counterpassing energetic ions differ in primary poloidal harmonics and spatial locations. The frequencies of the EPMs are located close to the shear Alfvén continuous spectrum when they are compared at the spatial peak locations of the primary poloidal harmonic or compared at the spatial tails if the primary poloidal harmonic is $m=1$. The frequencies of the EPMs were carefully compared with the energetic-ion orbital frequencies. It was found that the frequencies of the EPMs are in good agreement with the energetic-ion orbital frequencies with a correction for the toroidal circulation frequency. This demonstrates that the energetic-ion orbital frequency determines the EPM frequency. © 2006 American Institute of Physics.
[DOI: 10.1063/1.2234296]

I. INTRODUCTION

Magnetohydrodynamic (MHD) instabilities excited by energetic particles are important issues of fusion plasmas since they lead to redistribution and loss of energetic particles. Energetic-particle continuum modes (EPMs)¹ are a kind of MHD instabilities driven by energetic particles. It was theoretically predicted that EPMs have frequencies in the range of characteristic energetic-particle frequencies such as transit, bounce, and precession frequencies.¹ The frequency of EPMs is inside the shear Alfvén continuum.¹ It was numerically demonstrated in an MHD and energetic-particles hybrid simulation study with the ballooning representation that the real frequency of high- n EPM scales linearly with the maximum energetic particle velocity,² where n is a toroidal mode number. Theoretical and numerical analyses of high- n EPM linear stability have been done with the ballooning representation.^{3–5} The ballooning representation is not applicable to low- n modes. This makes theoretical analysis of low- n modes more difficult. Low- n EPMs have been investigated by computer simulations.^{6,7} Many experimental phenomena related to EPMs in tokamaks, spherical tokamaks, and helical plasmas have been reported.^{8–12}

The purpose of this work is to investigate properties of EPMs in the linearly growing phase. Recently, it has been reported that energetic particles can induce Alfvén modes, which do not resonate with the shear Alfvén continuum near their peak locations, such as cascade modes,^{13,14} EPM gap modes,⁷ and nonlocal EPMs.¹⁵ These Alfvén modes induced by energetic particles are beyond the scope of the present paper. We focus on the energetic-particle continuum modes that have the Alfvén resonance near their peak locations. We investigate EPMs destabilized by energetic ions in tokamak plasmas using a hybrid simulation code for MHD and ener-

getic particles, MEGA.^{15,16} Specifically, we investigate the dependence of EPM frequency and spatial profile on the energetic ion velocity. The energetic ions are assumed to have beam-like velocity distributions for the purpose of clarifying the dependence on energetic ion velocity. We describe the simulation model and method in Sec. II. Section III is devoted to the simulation results. We found that for beam velocities lower than the Alfvén velocity, the unstable modes are EPMs while toroidal Alfvén eigenmodes are destabilized for beam velocities well above the Alfvén velocity. The EPMs destabilized by the copassing energetic ions and those destabilized by the counterpassing energetic ions differ in primary poloidal harmonics and spatial locations. The frequencies of the EPMs were carefully compared with the energetic-ion orbital frequencies. It is demonstrated that the energetic-ion orbital frequency determines the EPM frequency. A summary is given in Sec. IV.

II. SIMULATION MODEL

The hybrid simulation model for MHD and energetic particles^{16–18} is employed in the MEGA code. The plasma is divided into the bulk plasma and the energetic ions. The bulk plasma is described by the nonlinear full MHD equations. The electromagnetic field is given by the MHD description. This approximation is reasonable under the condition that the energetic ion density is much less than the bulk plasma density. The MHD equations with the energetic ion effects are

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}), \quad (1)$$

$$\rho \frac{\partial}{\partial t} \mathbf{v} = -\rho \boldsymbol{\omega} \times \mathbf{v} - \rho \nabla \left(\frac{v^2}{2} \right) - \nabla p + (\mathbf{j} - \mathbf{j}_h) \times \mathbf{B} - \nu \rho \nabla \times \boldsymbol{\omega} + \frac{4}{3} \nu \rho \nabla (\nabla \cdot \mathbf{v}), \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (3)$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\mathbf{v}) - (\gamma - 1)p \nabla \cdot \mathbf{v} + (\gamma - 1) \times \left[\nu \rho \omega^2 + \frac{4}{3} \nu \rho (\nabla \cdot \mathbf{v})^2 + \eta \mathbf{j} \cdot (\mathbf{j} - \mathbf{j}_{eq}) \right], \quad (4)$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta(\mathbf{j} - \mathbf{j}_{eq}), \quad (5)$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}, \quad (6)$$

$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}, \quad (7)$$

where μ_0 is the vacuum magnetic permeability, γ is the adiabatic constant, and $\mathbf{j}_{eq}$ is the equilibrium current density. Here, $\mathbf{j}_h$ is the energetic ion current density without $\mathbf{E} \times \mathbf{B}$ drift. The effect of the energetic ions on the MHD fluid is taken into account in the MHD momentum equation [Eq. (2)] through the energetic ion current. The MHD equations are solved using a finite-difference scheme of fourth-order accuracy in space and time.

The drift-kinetic description is employed for the energetic ions. The guiding-center velocity $\mathbf{u}$ is given by

$$\mathbf{u} = \mathbf{v}_{\parallel}^* + \mathbf{v}_E + \mathbf{v}_B, \quad (8)$$

$$\mathbf{v}_{\parallel}^* = \frac{v_{\parallel}}{B^*} [\mathbf{B} + \rho_{\parallel} B \nabla \times \mathbf{b}], \quad (9)$$

$$\mathbf{v}_E = \frac{1}{B^*} [\mathbf{E} \times \mathbf{b}], \quad (10)$$

$$\mathbf{v}_B = \frac{1}{Z_h e B^*} [-\mu \nabla B \times \mathbf{b}], \quad (11)$$

$$\rho_{\parallel} = \frac{m_h v_{\parallel}}{Z_h e B}, \quad (12)$$

$$\mathbf{b} = \mathbf{B}/B, \quad (13)$$

$$B^* = B(1 + \rho_{\parallel} \mathbf{b} \cdot \nabla \times \mathbf{b}), \quad (14)$$

$$m_h v_{\parallel} \frac{dv_{\parallel}}{dt} = \mathbf{v}_{\parallel}^* \cdot [Z_h e \mathbf{E} - \mu \nabla B], \quad (15)$$

where $v_{\parallel}$ is the velocity parallel to the magnetic field, μ is the magnetic moment, which is an adiabatic invariant, and m_h and $Z_h e$ are energetic ion mass and electric charge. The energetic-ion current density without $\mathbf{E} \times \mathbf{B}$ drift in Eq. (2) is

$$\mathbf{j}_h = \int (\mathbf{v}_{\parallel}^* + \mathbf{v}_B) Z_h e f d^3 v - \nabla \times \int \mu \mathbf{b} f d^3 v, \quad (16)$$

where the second term on the right-hand side is the magnetization current. The $\mathbf{E} \times \mathbf{B}$ drift $\mathbf{v}_E$ disappears in $\mathbf{j}_h$ due to the quasineutrality.¹⁶

It is important to start simulations from MHD equilibria consistent with energetic-ion distributions. When the energetic-ion pressure is isotropic in the velocity space, the energetic-ion contribution in Eq. (2) is just a scalar pressure gradient in the same form as the bulk pressure gradient.¹⁶ Then, the equilibrium can be obtained from the Grad-Shafranov equation neglecting the energetic-ion orbit width. However, since the energetic-ion pressure is anisotropic in the velocity space and the energetic-ion orbit width is not negligibly small in the initial conditions investigated in the present paper, we solved an extended Grad-Shafranov equation developed in Ref. 19 in the cylindrical coordinates (R, φ, z) , where R is the major radius coordinate, φ is the toroidal angle coordinate, and z is the vertical coordinate. The equilibrium energetic-ion distribution f_0 is represented as a function of toroidal canonical momentum P_{φ} , total velocity v , and magnetic moment μ ,

$$f_0 = f_0(P_{\varphi}, v, \mu). \quad (17)$$

Since f_0 is a function of the invariants in the axisymmetric plasma without electric field, it is an equilibrium distribution. A detailed description of the extended Grad-Shafranov equation is given in Ref. 15.

In this work, we assume initial energetic-ion distributions in the form

$$f_0(P_{\varphi}, v, \mu) = \sum_{l=0}^L a_l \left(\frac{P_{\varphi_{\max}} - P_{\varphi}}{P_{\varphi_{\max}} - P_{\varphi_{\min}}} \right)^l \times \exp \left[- \left(\frac{v - v_0}{\Delta v} \right)^2 \right] \delta(\mu). \quad (18)$$

Using the least-squares method, a_l is chosen so that

$$P_{h\parallel} = \int m_h v_{\parallel}^2 f_0(P_{\varphi}, v, \mu) d^3 v \quad (19)$$

is close to the specified energetic-ion parallel pressure profiles. We have chosen $L=5$ for numerical convergence. The parameter v_0 is the beam velocity and $\Delta v = 0.1 v_A$. Here, v_A is the Alfvén velocity. The magnetic moments of energetic ions are assumed to be 0 for the purpose of clarifying the relation between EPM frequency and energetic-ion orbital frequency.

The δf method²⁰⁻²² is used for the energetic ions. The marker particles are initially loaded uniformly in the phase space. With the uniform loading employed in this work, the number of energetic ions that each marker particle represents is in proportion to the initial distribution function, namely the particle density in the phase space. We introduce a normalization factor α that is chosen so that

$$\int P_{h\parallel} dV = \alpha \sum_{i=1}^N m_h v_{\parallel}^2 f_0(P_{\varphi_i}, v_i, \mu_i) \quad (20)$$

is initially satisfied, where N is the total number of marker particles used.

The time evolution of each particle's weight is given by

$$\frac{d}{dt} w_i = -\alpha \left[\frac{dP_{\varphi}}{dt} \frac{\partial f_0}{\partial P_{\varphi}} + \frac{dv}{dt} \frac{\partial f_0}{\partial v} \right]_{\mathbf{x}=\mathbf{x}_i, \mathbf{v}=\mathbf{v}_i}, \quad (21)$$

with the initial condition $w_i=0$. The energetic-ion current $\mathbf{j}_h$ in Eqs. (2) and (16) is calculated using the particle weight,

$$\mathbf{j}_h = \mathbf{j}_{h0} + \sum_{i=1}^N w_i Z_h e (\mathbf{v}_{\parallel i}^* + \mathbf{v}_{Bi}) S(\mathbf{x} - \mathbf{x}_i) - \nabla \times \left[\mathbf{b} \sum_{i=1}^N w_i \mu_i S(\mathbf{x} - \mathbf{x}_i) \right], \quad (22)$$

where $S(\mathbf{x} - \mathbf{x}_i)$ is the shape factor of the marker particle and $\mathbf{j}_{h0}$ is the energetic-ion current density in the equilibrium. A benchmark test of the code is reported in Ref. 15.

The simulation domain is $R_c - a \leq R \leq R_c + a$, $0 \leq \varphi \leq 2\pi$, and $-a \leq z \leq a$, where a is the minor radius and R_c is the major radius of the simulation domain. The aspect ratio is $R_c/a = 3.2$. The shape of the outermost magnetic surface in the simulation is circular. For the purpose of data analysis, magnetic flux coordinate systems (r, φ, ϑ) , where r is the radial coordinate and ϑ is the poloidal angle, were constructed for the MHD equilibria investigated in the present paper. The number of marker particles used is 5.2×10^5 . The number of grid points is $101 \times 16 \times 101$ for the cylindrical coordinates (R, φ, z) . In the simulation runs reported in the present paper, the viscosity and resistivity are $\nu = 10^{-6} v_A R_c$ and $\eta = 10^{-6} \mu_0 v_A R_c$, respectively.

III. SIMULATION RESULTS

A. Dependence on beam velocity

In this subsection, the safety factor profile is $q = 1.2 + 1.8(r/a)^2$ and the plasma density is uniform. The poloidal magnetic field is negative ($B_{\vartheta} < 0$) while the toroidal magnetic field is positive ($B_{\varphi} > 0$). The energetic-ion beta profile is a Gaussian profile $\beta_h(r) = \beta_{h0} \exp[-(r/0.4a)^2]$ with $\beta_{h0} = 5 \times 10^{-3}$. Here, β_h is a ratio of the energetic-ion pressure and the magnetic pressure, $\beta_h \equiv 2\mu_0 P_{h\parallel}/3B^2$, and the factor 3 appears because the perpendicular pressure is assumed to be zero. The safety factor profile, the energetic-ion beta profile, and the bulk plasma beta profile ($\beta_{\text{bulk}} \equiv 2\mu_0 p/B^2$) are shown in Fig. 1. We focus on unstable modes with toroidal mode number $n=1$. An Alfvén continuum gap is located at $r/a = 0.41$ and $q = 1.5$ for $n=1$. The energetic-ion orbit width is characterized by a parameter $v_A/\Omega_h = 8 \times 10^{-2}a$, where Ω_h is the energetic-ion cyclotron frequency and v_A is the Alfvén velocity.

The unstable modes with the toroidal mode number $n=1$ were investigated for different beam velocities of $v_0 = -1.4v_A, -1.2v_A, -v_A, -0.8v_A, -0.6v_A, 0.8v_A, v_A, 1.2v_A, 1.4v_A$, and $1.6v_A$. Since the plasma current has the same di-

FIG. 1. Spatial profiles of the energetic-ion beta value β_h , the bulk beta value β_{bulk} , and the safety factor q .

rection as the magnetic field in the equilibria considered in the present paper, the negative sign of the beam velocity corresponds to counterpassing ions, and the positive sign corresponds to copassing ions. The radial velocity profile of the unstable mode for the counterpassing energetic ions with $v_0 = -0.6v_A$ is shown in Fig. 2. The phase is chosen so as to maximize the peak value of the cosine part of the primary harmonic. The primary poloidal harmonic is $m=1$ and the other harmonics are negligibly small. For the copassing energetic ions with $v_0 = v_A$, the primary poloidal harmonic of the unstable mode is $m=2$. The radial velocity profile of the unstable mode for $v_0 = v_A$ is shown in Fig. 3. Thus, the primary poloidal harmonics of the unstable modes are different between the copassing and counterpassing energetic ions.

Frequencies of the unstable modes were analyzed with a form $\exp[i\omega t - im\vartheta - in\varphi]$. We found that the frequencies (ω) of all the unstable modes are positive, namely, all of the modes rotate poloidally in the ion diamagnetic drift direction as expected for modes destabilized by ions. The frequencies of the unstable modes are shown in Fig. 4 with the Alfvén continuous spectra with toroidal mode number $n=1$. The frequencies are normalized by the Alfvén frequency $\omega_A = v_A/R_0$, where R_0 is the plasma major radius with the Shafranov shift. In the figure, the frequencies are plotted at the spatial peak locations of the $m=2$ harmonic for the co-

FIG. 2. Spatial profiles of (a) the cosine part and (b) the sine part of radial velocity v_r harmonics of the unstable mode for $v_0 = -0.6v_A$ with toroidal mode number $n=1$.

FIG. 3. Spatial profiles of (a) the cosine part and (b) the sine part of radial velocity v_r harmonics of the unstable mode for $v_0 = v_A$ with toroidal mode number $n = 1$.

passing energetic ions. For $v_0 = 0.8v_A$, no unstable mode was found. In Fig. 4, we see that the frequencies of the unstable modes for $v_0 = v_A$ and $1.2v_A$ are located close to the Alfvén continuum. For the counterpassing energetic ions, the frequencies are plotted at the locations where the $m = 1$ harmonic is 10% of the peak value at the plasma center. We see in Fig. 4 that the frequencies of the unstable modes for $v_0 = -0.6v_A$ and $-0.8v_A$ are located on the Alfvén continuum. Thus, the unstable modes for the low velocities $-0.8v_A \leq v_0 \leq 1.2v_A$ have the shear Alfvén resonance near their peak locations. This indicates that the unstable modes for the low velocities are EPs. It is interesting to note that the excited mode frequencies are close to the spectrum gap in order to minimize the continuum damping,^{23,24} as suggested in Refs. 4 and 5. On the other hand, the frequencies of the unstable modes for beam velocities well above the Alfvén velocity are in the shear Alfvén continuous spectrum gap. The unstable modes for the high velocities are toroidal Alfvén eigenmodes (TAE).^{25,26} The radial velocity profile of the unstable mode

FIG. 4. Frequencies and peak locations of the unstable modes destabilized by copassing energetic ions with different beam velocities are represented by triangles. For counterpassing energetic ions, since the primary poloidal harmonic $m = 1$ peaks at the plasma center, frequencies and tail locations of the $m = 1$ harmonic are represented by circles. Also shown are the safety factor (q) profile and the shear Alfvén continuous spectra with toroidal mode number $n = 1$.

FIG. 5. Spatial profiles of (a) the cosine part and (b) the sine part of radial velocity v_r harmonics of the unstable mode for $v_0 = -1.2v_A$ with toroidal mode number $n = 1$.

for $v_0 = -1.2v_A$ is shown in Fig. 5. For the copassing energetic ions, the unstable modes for $v_0 = 1.4v_A$ and $1.6v_A$ are TAEs. However, as is shown in Fig. 6, the phase of the $m = 2$ harmonic is opposite to the $m = 1$ harmonic. This makes a contrast to the TAE destabilized by the counterpassing energetic ions shown in Fig. 5.

Let us consider the reason why the primary harmonics are different between the EPs destabilized by the copassing and the counterpassing energetic ions. The orbital frequency of an energetic ion is given by

$$\omega_E = \left(n - \frac{l}{q} \right) \frac{v_0}{R_0}, \quad (23)$$

where l is an arbitrary integer. The orbital frequency ω_E is the characteristic frequency of the perturbed energetic-ion

FIG. 6. Spatial profiles of (a) the cosine part and (b) the sine part of radial velocity v_r harmonics of the unstable mode for $v_0 = 1.6v_A$ with toroidal mode number $n = 1$.

distribution with poloidal mode number l and toroidal mode number n . In Eq. (23), the minus sign of l/q arises from a fact that the poloidal magnetic field is negative in the theta direction in the equilibria investigated. The resonance condition between the mode and the energetic ions is $\omega = \omega_E$. For $n=1$, $q \sim 1.5$, and $|v_0| \sim v_A$, we can find that both sets ($l=1$ and $v_0 \sim v_A$) and ($l=2$ and $v_0 \sim -v_A$) give $\omega_E \sim \omega \sim 0.3\omega_A > 0$. The primary resonance between the energetic ions and the MHD fluid takes place with $l=m \pm 1$, because the energetic ions interact with the MHD fluid through the curvature drift and the grad-B drift, which are in proportion to $\cos \vartheta$ or $\sin \vartheta$.²⁷ Thus, the copassing energetic ions $v_0 \sim v_A$ with $l=1$ resonate primarily with the $m=2$ poloidal harmonic of the MHD modes while the counterpassing ions $v_0 \sim -v_A$ with $l=2$ resonate primarily with the $m=1$ poloidal harmonic. This explains well the primary poloidal harmonics of the EPs destabilized by the copassing and the counterpassing energetic ions, respectively. As is shown in Figs. 5 and 6, the phase relation of the $m=1$ and 2 poloidal harmonics of the TAE destabilized by the copassing energetic ions is different from that by the counterpassing energetic ions. This difference also might be related to the difference in the resonance condition between the copassing and the counterpassing energetic ions.

B. Comparison between EPM frequency and energetic-ion orbital frequency

It was theoretically predicted that EPs have frequencies in the range of characteristic energetic-particle frequencies such as transit, bounce, and precession frequencies.¹ The purpose of this subsection is to compare carefully the EPM frequency with the energetic-ion orbital frequency. We focus on EPs destabilized by copassing energetic ions. In the previous subsection, the peaks of the EPs destabilized by the copassing ions are located in the region $q > 1.5$. In this subsection, we chose a safety factor profile $q = 1.35 + 1.65(r/a)^2$ in order to make the drive of the energetic ions more powerful with the region $q > 1.5$ closer to the plasma center. For the same purpose, the energetic-ion beta value at the plasma center is $\beta_{h0} = 1\%$ instead of 0.5% in the previous subsection. For toroidal mode number $n=1$, a shear Alfvén continuous spectrum gap is located at $r/a = 0.30$ and $q = 1.5$.

The radial velocity profile of the unstable mode for $v_0 = v_A$ is shown in Fig. 7. The primary poloidal harmonic is $m=2$. The change in sign of the sine part of the $m=2$ harmonic across the peak location of the cosine part at $r/a = 0.34$ indicates the Alfvén resonance. Frequencies of the unstable modes are plotted at the peak locations in Fig. 8 with the shear Alfvén continuous spectra. It is clear that the frequencies of the unstable modes for the beam velocities $v_0 \leq 1.2v_A$ are located close to the Alfvén continuous spectrum in the figure. These unstable modes are EPs. On the other hand, the frequencies for the beam velocity $v_0 = 1.4v_A$ are in the shear Alfvén continuous spectrum gap. This unstable mode is a TAE. We investigated the spatial profiles of the perturbed energetic-ion pressure. The spatial profile of the perturbed energetic-ion pressure for $v = v_A$ is shown in Fig. 9. The primary poloidal harmonic is $m=1$. This is con-

FIG. 7. Spatial profiles of (a) the cosine part and (b) the sine part of radial velocity v_r harmonics of the unstable mode for $v_0 = v_A$ with toroidal mode number $n=1$.

sistent with the resonance condition discussed in the previous subsection, which predicts the poloidal mode number of the energetic-ion orbital frequency is $l=1$. The $m=1$ harmonic of the perturbed energetic-ion pressure peaks at $r/a = 0.19$ and $q = 1.41$.

In Fig. 8, we see that the modes destabilized by faster energetic ions have higher frequency. We compare the energetic-ion orbital frequencies with the frequencies of the unstable modes. We know $l=1$ from the perturbed energetic-ion pressure profile shown in Fig. 9 and the resonance condition discussed in the previous subsection. We found a correction is needed for the toroidal circulation frequency nv_0/R_0 in Eq. (23). The correction comes from the fact that the center of the beam ion orbits shifts from the plasma center by σR_0 , where $\sigma \equiv qv_0m_h/Z_h e B_0 R_0$, where B_0 is the magnetic field intensity at the plasma center. Thus, the corrected energetic-ion orbital frequency is given by

FIG. 8. Frequencies and peak locations of the unstable modes destabilized by copassing energetic ions with different beam velocities are represented by triangles. Also shown are the safety factor (q) profile and the shear Alfvén continuous spectra with toroidal mode number $n=1$. The safety factor profile $q = 1.35 + 1.65(r/a)^2$ is different from Fig. 4.

FIG. 9. Spatial profiles of (a) the cosine part and (b) the sine part of the perturbed energetic-ion pressure δP_h harmonics for $v_0=v_A$ with toroidal mode number $n=1$.

$$\omega_E = \frac{v_0}{R_0} \left(\frac{n}{1+\sigma} - \frac{l}{q} \right). \quad (24)$$

For $q=1.4$ and $v_0=v_A$, the correction term is $\sigma=3 \times 10^{-2}$. This term gives a correction of roughly 10% to the energetic-ion orbital frequency ω_E . In Fig. 10, the frequencies of the unstable modes ω are compared with the energetic-ion orbital frequency ω_E calculated for all v_0 . In the calculation of ω_E , the values of safety factor q are represented at the peak locations of perturbed energetic-ion pressure. We see good agreement between the mode frequency ω and the energetic-ion orbital frequency ω_E . In Fig. 11, we plot the frequencies of the unstable modes and the energetic-ion orbital frequency given by Eq. (24) versus the beam velocity v_0 . The energetic-ion orbital frequency in Fig. 11 is calculated assuming a constant safety factor value $q=1.4$. This is a reasonable assumption since as is shown in Fig. 9, the perturbed energetic-ion pressure peaks near the plasma center where the magnetic shear is weak. We confirm again that the mode

FIG. 10. Comparison between the frequencies of the unstable modes and the energetic-ion orbital frequency $\omega_E=v_0[n/(1+\sigma)-l/q]/R_0$ with $n=1$, $l=1$, $\omega_A=v_A/R_0$, and $\sigma=qv_0m_h/Z_h e B_0 R_0$. The values of safety factor q used for the calculation of ω_E are represented at the peak locations of perturbed energetic-ion pressure.

FIG. 11. Comparison between the energetic-ion orbital frequency (solid line) and the frequencies of the unstable modes (circles). A unique value of safety factor $q=1.4$ is used for the calculation of the energetic-ion orbital frequency $\omega_E=v_0[n/(1+\sigma)-l/q]/R_0$ with $n=1$, $l=1$, $\omega_A=v_A/R_0$, and $\sigma=qv_0m_h/Z_h e B_0 R_0$.

frequency ω is in good agreement with the energetic-ion orbital frequency ω_E . The results shown in Figs. 10 and 11 demonstrate that the frequencies of the unstable modes are determined by the energetic-ion orbital frequency, and the unstable modes for the beam velocities $v_0 \leq 1.2v_A$ are the energetic-particle continuum modes. For $v_0=1.4v_A$, the TAE is unstable instead of the energetic-particle continuum modes, because the TAE resonates with the energetic ions for this beam velocity and is not affected by the continuum damping.

IV. SUMMARY

Properties of energetic-particle continuum modes (EPM) in tokamak plasmas were investigated using the hybrid simulation code for magnetohydrodynamics and energetic particles, MEGA. The energetic ions were assumed to have beam-like velocity distributions for the purpose of clarifying the dependence on energetic-ion velocity. We focused on the unstable modes with toroidal mode number $n=1$. It was found that the unstable modes are EPMs for the beam velocities lower than the Alfvén velocity while the toroidal Alfvén eigenmodes (TAEs) are unstable for the beam velocities well above the Alfvén velocity. For the safety factor profile investigated, the primary poloidal harmonic of the EPMs destabilized by the copassing energetic ions is $m=2$ while they are $m=1$ for the counterpassing energetic ions. This difference was explained by the resonance condition between the energetic beam ions and the EPMs. It is interesting to note that the two primary harmonics of the TAEs destabilized by the copassing energetic ions have opposite signs to each other while the same sign was observed for the TAEs destabilized by the counterpassing energetic ions.

The frequencies of the EPMs were carefully compared with the energetic-ion orbital frequencies. The energetic-ion orbital frequencies were calculated with the safety factor values at the peak locations of the perturbed energetic-ion pressure. It was found that the frequencies of the EPMs are in good agreement with the energetic-ion orbital frequencies with a correction for the toroidal circulation frequency. The good agreement was confirmed again when a constant value

of the safety factor was assumed for the calculation of energetic-ion orbital frequency. These results demonstrate that the energetic-ion orbital frequency determines the EPM frequency.

ACKNOWLEDGMENTS

The author gratefully acknowledges helpful discussions with Professor N. Nakajima, Professor K. Toi, Dr. M. Isobe, and Dr. K. Shinohara on several points in the paper. Numerical computations were performed at the Plasma Simulator (NEC SX-7/160M5) of the National Institute for Fusion Science.

This work was partially supported by a Grant-in-Aid for Scientific Research of the Japan Society for the Promotion of Science (No. 16560728).

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