§15. Coherent Structure of Zonal Flow and Nonlinear Saturation

Itoh, K., Toda, S., Sanuki, H.
Hallatschek, K. (IPP, Germany)
Itoh, S.-I., Yagi, M. (RIAM, Kyushu Univ.)

The growth of the zonal flow in the presence of the drift-wave turbulence has been discussed on the basis of space and time scale separation. We also consider the space and time scale separation between zonal flow and ambient turbulence, and the evolution of the zonal flow and ambient turbulence are governed by

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial r} V_Z \right) = \frac{\partial^2}{\partial r^2} \frac{c^2}{B^2} \int d^2 k \frac{k_\theta k_r}{(1 + k_\perp^2 \rho_s^2)^2} \bar{N}_k - \gamma_{damp} \frac{\partial}{\partial r} V_Z$$

and

$$\frac{\partial}{\partial t} N_k + \frac{\partial \omega_k}{\partial \mathbf{k}} \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial \omega_k}{\partial \mathbf{x}} \cdot \frac{\partial N_k}{\partial \mathbf{k}} = 0 ,$$

where V_Z is the zonal flow velocity, $\bar{N}_k$ is the slow modulation of drift-wave action N_k , which is induced by V_Z , and γ_{damp} denotes the damping rate of zonal flow by other processes. The zonal flow has a slow dependence as expressed by $\exp(iKr - i\Omega t)$.

Calculating up to the third order nonlinear terms, we have the equation for the zonal flow vorticity as

$$\begin{aligned} \frac{\partial}{\partial t} U + D_{rr} \left(\frac{\partial^2}{\partial r^2} U + K_0^{-2} \frac{\partial^4}{\partial r^4} U \right) - D_3 \frac{\partial^2}{\partial r^2} U^3 \\ - \mu_{||} (1 + 2q^2) \frac{\partial^2}{\partial r^2} U = 0 \end{aligned}$$

where

$$D_{rr} = -\frac{c^2}{B^2} \int d^2 k \frac{R(K, \Omega) k_\theta^2 k_r}{(1 + k_\perp^2 \rho_s^2)^2} \frac{\partial N_k}{\partial k_r} ,$$

$$R(K, \Omega) = (\Delta \omega_k)^{-1} D_3 / D_{rr} \simeq R(K, \Omega)^2 k_\theta^2 \left(\partial^2 / \partial k_r^2 \right) ,$$

$\gamma_{damp} = \mu_{||} (1 + 2q^2) K^2$, where $\mu_{||}$ is the turbulent shear viscosity for the flow along the field line and q is the safety factor. With an appropriate normalization, $x = r/L$, $\tau = t/t_Z$ and $u = U/U_0$, where $L^{-2} = K_0^2 (1 - \mu)$, $t_Z = D_{rr}^{-1} K_0^{-2} (1 - \mu)^{-2}$ and $U_0^2 = D_{rr} D_3^{-1} (1 - \mu)$, one has

$$\frac{\partial}{\partial \tau} u + \frac{\partial^2}{\partial x^2} u - \frac{\partial^2}{\partial x^2} u^3 + \frac{\partial^4}{\partial x^4} u = 0 \quad (1)$$

We investigate the case in which the flow is generated from the state with a small noise level where no net flow exists, $\int dx u = 0$. Conservation of the total momentum holds for the periodic boundary condition and the flow evolves to satisfy the condition $\int dx u = 0$. The stationary solution of eq. (1) in the domain $0 < x < d$, for the periodic boundary condition, is given by an elliptic integral as

$$\int \left(1 - 2u^2 + u^4 - \kappa^2 \right)^{-1/2} du = \pm \frac{x}{\sqrt{2}} , \quad (2)$$

where κ is an integral constant satisfying $0 \leq \kappa < 1$.

The integral constant κ is given by the periodicity

$$\int_{-u_c}^{u_c} \left(1 - 2u^2 + u^4 - \kappa^2 \right)^{-1/2} du = d/2\sqrt{2}n \quad (u_c = \sqrt{1 - \kappa^2} ,$$

and $n = 1, 2, 3, \dots$).

The temporal evolution of eq. (1) is solved numerically. It is shown that the growth is dominated by the component which has the largest linear growth rate. That is, the integer n is given by the integer which is closest to $d/n = 4\sqrt{2}\pi$. Figure 1 illustrates the stable stationary state. The peak value of $u(x)$ is given as $u_c \simeq 0.95$. Compared to a simple sinusoidal function (eigenfunction of the linear operator), the result in Fig. 1 is closer to a piecewise constant function.

Fig.1 Stationary state of the normalized solution $u(x)$ for the case of $d = 8\pi$. Radial length x and vorticity u are normalized values.

Reference

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