§13. Physics of Collapses — Probabilistic Occurrence of ELMs and Crashes —

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A new aspect of the transitions in plasmas, i.e., probabilistic nature of the occurrence, is analyzed [1]. The dynamical model of transition has been studied in many literature [2]. We here adopt two basic equations: One is the evolution of global structures (say, gradient) and the other is the dynamics of the loss rate that produces the hysteresis of the flux-gradient relation. We take two representative variables, i.e., the pressure gradient α and the loss rate γ . The loss rate is directly related to the turbulence level. The model equation takes the form

$$\frac{\partial}{\partial t} \alpha = S - \gamma \alpha$$

$$\zeta \frac{\partial}{\partial t} \gamma = \alpha - I - a(\gamma - I) - b(\gamma - I)^3$$

The notation is as follows: S is the energy influx into the layer, ζ denotes the possible difference of dynamical time between α and γ ; the cubic equation $a(\gamma - I) + b(\gamma - I)^3$ describes the shape of the hysteresis in the gradient-flux relation.

If all the coefficients (S, a, b, ζ) are constant in time, the stable stationary solutions or limit cycle is obtained. However, the statistical variance is as important a quantity as the statistical average, for the confined plasmas which is far from thermal equilibrium. The nonlinear simulation has shown a large temporal variation around the average [3]. Based on these considerations, we consider that parameters (S, a, b) are statistical variables and have fluctuation parts in time. We put

$$S = S_0 + \epsilon_f \text{ and } a = a_0 + \epsilon_a$$

and consider ϵ_f and ϵ_a as statistical variables, i.e.,

$$\langle \epsilon_f \rangle = 0, \langle \epsilon_a \rangle = 0, \langle \epsilon_f^2 \rangle \neq 0 \text{ and } \langle \epsilon_a^2 \rangle \neq 0.$$

Temporal evolution is studied. Example is shown for the statistical variance in the influx S, ϵ_f . The Gaussian statistics is assumed, and the amplitude of deviation $\overline{\epsilon_f} = \sqrt{\langle S(t) - S_0 \rangle^2} = 0.05$ is applied. There appear irregular ELMs. We evaluate the averaged frequency of oscillations, $\langle f \rangle$, and the standard deviation $\sigma, \sigma = \sqrt{\langle T^2 \rangle - \langle T \rangle^2} / \langle T \rangle$, where T is the observed interval of oscillations. In Fig.2, $\langle f \rangle$ and σ are plotted for $\overline{\epsilon_f} = 0.05$ as a function of S_0 . We observe the probabilistic excitation and that the transition occurs well below the critical value. As S_0 becomes small, the standard deviation σ becomes large and we find, in this case, an approximate relation $\sigma \propto \langle f \rangle^{-0.74}$.

The probabilistic nature is also caused by the other statistical property of turbulence. The same statement is made for the variation in the hysteresis curve.

Fig.1 Average frequency and standard deviation of the bursts vs mean value of source S_0 . In the absence of noise (dashed-dotted line), sharp discontinuity appear at $S_0 = S_I$. ($\overline{\epsilon_a} = 0.05$). Statistical distribution of the critical gradient is observed.

References

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