§23. Self-sustained Turbulence and H-mode Confinement in Toroidal Plasmas

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The equation for the dressed test mode for the nonlinear ballooning mode turbulence in the presence of the radial electric field has been derived [1] as

$$d^2 p / d\eta^2 + (\alpha \Lambda / K) \{ 1 - (1/2 + \alpha - s) \eta^2 \} p - M \Lambda [1 + 3(s - \alpha)^2 \eta^2] p + L_1 p = 0$$

$$L_1 p \equiv \omega_{E1} [K F^{-1} d^3 p / d\eta^3 - \Lambda (1 + M/K) F^2 dp / d\eta]$$

In this equation, normalization is: $r / a \rightarrow \hat{r}$,
 $t / \tau_{Ap} \rightarrow \hat{t}$, $\chi \tau_{Ap} / a^2 \rightarrow \hat{\chi}$, $\mu \tau_{Ap} / a^2 \rightarrow \hat{\mu}$,
 $\tau_{Ap} / \mu_0 \sigma_c a^2 \rightarrow 1 / \hat{\sigma}$, $\lambda \tau_{Ap} / \mu_0 a^4 \rightarrow \hat{\lambda}$, $\gamma \tau_{Ap} \rightarrow \hat{\gamma}$.
 We employ the notation $\tau_{Ap} \equiv a \sqrt{\mu_0 m_i n_i} / B_p$,
 $\Lambda = \hat{\lambda} n^4 q^4$, $K = \hat{\chi} n^2 q^2$, $M = \hat{\mu} n^2 q^2$,
 $F = 1 + (s\eta - \alpha \sin \eta)^2$, $s = r (dq / dr) / q$,
 $\omega_{E1} = \tau_{Ap} (dE_r / d\hat{r}) (srB)^{-1}$ and $\alpha = -q^2 R \beta'$.
 Other notation is standard.

The eigenfunction is written as

$$p(y) = u_0 + \sum_{n=1}^{\infty} a_n u_n$$

where the function u_n is explicitly given as
 $u_n = H_n(y) \exp(-y^2/4)$ and $H_n(y)$ is the n -th order Hermite function. The odd and even parity modes are mixed by the operator L_I when $\omega_{E1} \neq 0$. The normalized eigenvalue is calculated by

$$\Omega = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n} \langle 0 | L_1 | n \rangle \langle n | L_1 | 0 \rangle.$$

(In the previous work [1], only neighbouring terms were kept.) The turbulent transport coefficient is given as

$$\hat{\chi}_H = \frac{\hat{\chi}_L}{1 + G_1 \omega_{E1}^2}$$

$$G_1 = 8 \alpha (1 + 2\alpha - 2s) \left\{ 2 + \frac{6(s - \alpha)^2}{1 + 2(\alpha - s)} \right\}^{-1}$$

$$\times \left\{ \frac{3}{8} (1 - (s - \alpha)^2) + \frac{2(1 + 6(s - \alpha)^2)}{4\alpha} \right\}^2$$

$$+ \frac{1}{2} \left\{ \frac{1 - 5(s - \alpha)^2}{4} - \frac{2(s - \alpha)^2}{\alpha} \right\}^2 + \frac{3}{8}$$

The contour lines of $\hat{\chi}_H$ are illustrated in Fig.1 on the $\alpha - \omega_{E1}$ plane. The correction appears in the numerical coefficient G_1 .

The cross-field momentum flux can be calculated by the relation

$$P_{\theta,r} = -\mu_H \nabla V_{\theta}.$$

Normalized momentum flux is given as $\hat{P}_{\theta,r} = \hat{\mu}_H \omega_{E1}$, and the normalized velocity gradient is expressed by ω_{E1} . Figure 2 illustrates $\hat{P}_{\theta,r}$ as a function of ω_{E1} . Viscosity is calculated and is close to $\hat{\chi}_H$. It is shown that the radial momentum flux is a decreasing function of the velocity shear in the high rotation shear limit. This indicates that, for a fixed momentum flux, two solutions of the rotation shear are available. This provides a basis for the bifurcation in the radial electric field structure in the H-mode modelling.

Fig.1 Contour plot of the normalized thermal conductivity as a function of (α, ω_{E1}) .

Fig.2 Anomalous cross-field momentum flux $\hat{P}_{\theta,r}$ as a function of the velocity gradient ω_{E1} .

Reference

- [1] Itoh S-I, Itoh K, Fukuyama A and Yagi M
 1994 *Phys. Rev. Lett.* **72** 1200