§7. Probabilistic Excitation in Plasma Transitions

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We shall show the probabilistic nature of the transition (e.g., L/H transition), by using a simple (zero-dimensional) dynamical model. This model equations are simplified and modelled from the theories, which involve the hysteresis characteristic. Two different origins of the statistical variance are examined, i.e., a fluctuation in the source flux, which is caused by the turbulence from the upstream, and a noise in the hysteresis characteristic. These are naturally occurred by the turbulent fluctuations in high temperature plasmas. The statistical variance affects the transition nature and introduces a change from the deterministic phenomenon to the probabilistic one.

Here, a layer with a finite width is considered and the averaged value within this layer is treated as a scalar quantity. Through highly simplified, the model form of the hysteresis successfully applied to the investigation of the dynamics of dithering ELMs. The model equation used here with respect to the mean density n and the loss rate γ takes the forms

$$\frac{\partial n}{\partial t} = S - \gamma n \quad (1)$$

and

$$\zeta \frac{\partial \gamma}{\partial t} = n - 1 + a(\gamma - 1) - b(\gamma - 1)^3, \quad (2)$$

where S is the particle flux into the layer, ζ denotes the dynamical time difference between n and γ and the cubic function $a(\gamma - 1) - b(\gamma - 1)^3$ describes the shape of the hysteresis in the flux-density relation. We set $S = S_0 + \epsilon_s$, $a = a_0 + \epsilon_a$ and ϵ_s and ϵ_a as the statistical variables, e.g., $\langle \epsilon_s \rangle = 0$ and $\langle \epsilon_a^2 \rangle \neq 0$. The variance ϵ_s represents that of the fluctuations of the source from the upstream. We study the probabilistic nature by using the statistical variances: ϵ_s and ϵ_a . In confined plasmas, the case for the power law noise is considered to be more realistic. Therefore, we put emphasis on the study about the noise which obeys the power law.

The variation in the hysteresis curve is investigated in the presence of the noise which obeys the power law. The variance of ϵ_a is considered with $\sigma_p = \sqrt{\langle a - \langle a \rangle \rangle^2}$. The parameters are chosen as $a_0 = 0.5$, $b = 1$, $\zeta = 0.01$ and $S = 1.0$. Using these parameters, we examine the temporal evolution of the particle flux and the period of the oscillation obtained from eqs. (1) and (2). We here introduce the noise, the probabilistic function for ϵ_a which obeys the power law, i.e., $P(\epsilon_a) \propto |\epsilon_a|^{-2}$ in a certain region.

We examine the flux values where the transition from the H-mode to L-mode ends and the period of bursts as the other condition, e.g., the critical density. We choose $\sigma_p = 0.05$. The probabilistic distributions of the critical values of flux Γ_c and periods T_c of bursts are obtained, respectively. The deterministic picture (i.e., in the case without fluctuations) predicts the critical value of flux is $\Gamma_{c0} = 2.0674$ and the period is $T_{c0} = 0.878$. The dependence of $\log_e(P(\Gamma_c))$ on

$\log_e |\Gamma_c - \Gamma_{c0}|$ and $\log_e(P(T_c))$ on $\log_e |T_c - T_{c0}|$ are studied, respectively. In the case of $P(\Gamma_c)$, we fit the critical values in the region $4.5 \times 10^{-4} \leq |\Gamma_c - \Gamma_{c0}| \leq 2.5 \times 10^{-3}$ using the function $P(\Gamma_c) = -18 |\Gamma_c - \Gamma_{c0}|^{-2.06}$ with the $R = 0.98$, where R is the correlation coefficient. On the other hand, the fitting is done in the region $6.7 \times 10^{-4} \leq |T_c - T_{c0}| \leq 5.5 \times 10^{-3}$ by the function $P(T_c) = -15 |T_c - T_{c0}|^{-1.91}$ with $R = 0.98$. In the presence of ϵ_a , the distribution functions of $P(\Gamma_c)$ and $P(T_c)$ are found to obey the same power law with the variance in a some range as $P(\Gamma_c) \propto |\Gamma_c - \Gamma_{c0}|^{-2}$ and $P(T_c) \propto |T_c - T_{c0}|^{-2}$, respectively. This result is similar to that in the case of the critical density. We also study noises with other power dependence ($|\epsilon_a|^{-k}$) with $k = 1, 3$. It is confirmed that the probability distributions in terms of the critical parameters have the same power law.

Finally, we compare the effects of the variance ϵ_s in the source and those of the variance ϵ_a in the mean fluctuation level on the critical parameters by use of the power law noise with $k = 2$. Parameters are chosen as $S_0 = 1.0$, $a_0 = 0.5$, $b = 1.0$ and $\zeta = 0.01$. In the first case, noise part ϵ_s is set $P(\epsilon_s) \propto |\epsilon_s|^{-2}$ and the noise in the source, i.e., $\epsilon_s \neq 0$ and $\epsilon_a = 0$. The variance of ϵ_s is $\bar{\epsilon}_s = 0.1$, so that $\bar{\epsilon}_s/S_0 = 0.1$. The probabilistic function of Γ_c has a sharp distribution around the value of Γ_{c0} . In this case, the probabilistic function of T_c has the distribution around T_{c0} to some content. Next, we study the case of only the noise in the mean fluctuation level, i.e., $\epsilon_s = 0$ and $\epsilon_a \neq 0$. The noise part ϵ_a is set $P(\epsilon_a) \propto |\epsilon_a|^{-2}$ and ϵ_s equals to zero. To compare the pervious case: $\epsilon_s \neq 0$, $\epsilon_a = 0$, we set $\bar{\epsilon}_a/a_0 = 0.1$. In this case, all critical parameters have the much wider distributions than the case of $\epsilon_s \neq 0$, $\epsilon_a = 0$. The correlation plot on Γ_c - T_c plane, Fig. 1, has the wide distribution of both Γ_c and T_c axes. The significant difference of the distributions from the origin of the fluctuations is examined. The statistical study on the experiments of the transition will provide a unique information to understand the physics of the transition as well as the nature of the turbulence in plasmas.

Fig. 1 Correlation plot on Γ_c - T_c plane.

Reference

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