Confinement Improvement in H-Mode-Like Plasmas
in Helical Systems

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Confinement Improvement in H-Mode-Like Plasmas in Helical Systems


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Abstract

The reduction of the anomalous transport due to the inhomogeneous radial electric field is theoretically studied for toroidal helical plasmas. The self-sustained interchange-mode turbulence is analysed for the system with magnetic shear and magnetic hill. For the system with magnetic well like conventional stellarators, the ballooning mode turbulence is studied. Influence of the radial electric field inhomogeneity on the transport coefficients and fluctuations are quantitatively shown. Unified theory of the transport coefficients in the L-mode and H-mode-like plasmas are presented.

Keywords: Toroidal Helical System, H-mode, Anomalous Transport, Radial Electric Field, Self-Sustained Turbulence
The finding of the H-mode in tokamaks (Wagner et al 1982, ASDEX Team 1989) has not only shortened the path to the ignition plasmas but also motivated the confinement physics studies. The modelling on the H-mode has been developed focusing on the role of radial electric field structure (Itoh et al 1988, 1989a), which either gives rise to the bifurcation in the confinement states (Itoh et al 1988, 1989a, Shaing et al 1989a) or suppresses micro-instabilities thus reducing the anomalous transport (Itoh et al 1989b, Shaing et al 1989b, Bigrali et al 1989, Itoh et al 1990). The important role of radial electric field has then been confirmed experimentally (Groebner et al 1990, Ida et al 1990).

For toroidal helical systems (stellarators), the influence of the radial electric field has long been recognized at least from the view point of the absolute trapping of particles and neoclassical transport. It is an urgent task to study the effect on the anomalous transport in this configuration, which has been known to be the dominant loss mechanism in experiments (Grieger et al 1986, Itoh et al 1989c, Lyon 1990).

The inhomogeneous radial electric field affects the micro-instabilities and associated anomalous transport. Since the poloidal gyroradius can be of the order of the gradient length of \( E_r \) in the Wendelstein VII-A (WVII-A) stellarator (Grieger et al 1986), the suppression could be considerable. We have studied the coupling between the temperature gradient and electric field (Itoh et al 1992a) and found that the link between the anomalous transport and them has the property to cause the bifurcation (Itoh et al 1992b). This mutual interaction allows a multiple
solution of the gradient for the given heat flux in the whole plasma column. The transition to the improved confinement state was thus concluded to be slower in comparison with the L-H transition in tokamaks. This prediction was confirmed by the recent finding of the H-mode like discharges in WVII-AS stellarator (Erckmann et al 1983).

Compared to the progress in the transition physics, the theories on the transport coefficients remained premature. The quantitative explanation for the L-mode and H-mode has long been remained to be mystery. Recently, theory for the self-sustained turbulence has been progressed. By solving the interchange-mode turbulence by this novel method, the better understanding of the L-mode transport in torsatron/Heliotron was given (Itoh et al 1982c). The analysis has been extended to treat the ballooning-mode turbulence in the system with magnetic well (Itoh et al 1993a, 1993b, Yagi et al 1993). The influence of the inhomogeneous radial electric field was then analysed for the tokamaks (Itoh et al 1993c). We here apply this formalism to toroidal helical plasmas and present the unified formula of the anomalous transport coefficient for the L and H-like mode in helical systems.

We first study torsatron/Heliotron plasma with the cylindrical coordinates \((r, \theta, z)\). The reduced set of equations (Strauss 1980) is employed. The basic equations consist of the equation of motion, \(n_1 \nabla_1 (d \phi^2 \phi)/dt - \mu \nabla^2_1 \phi = B^2 V_b J + (\Omega'/r) \partial \phi / \partial \theta\), generalized Ohm’s law, \(E + \nabla B = \sigma \nabla \phi A\), and the energy balance equation.
\[ \frac{dp}{dt} = -xv_{i}^{2}p. \] Here \( m_{i} \) is the ion mass, \( n_{i} \) is the ion density, \( \Phi \) is the static potential, \( B \) is the main magnetic field, \( \Omega' \) is the magnetic curvature, \( p \) is the plasma pressure, \( J \) is the current and \( \sigma \) is the classical conductivity. The \( \text{E} \times \text{B} \) nonlinear interactions are renormalized in a form of \( \chi, \mu \) and \( \lambda \). The detailed derivation was reported in Itoh 1993d. The time derivative is defined as \( \partial / \partial t + [\Phi, J] / B \), where \( [ , ] \) denotes the Poisson bracket. The doppler shift of frequency is offset for the homogeneous \( \text{E} \times \text{B} \) rotation. Only the contribution of \( E_{r} \) to \( d / dt \) is retained.

We solve the mode structure in the vicinity of the mode rational surface, \( r_{s} \). The parallel wave number is expressed as \( k = k_{0} q x / q R \) (\( x = r - r_{s} \), \( q \) is the safety factor). Fourier transformation is used as \( \tilde{\mathcal{F}}(r, \theta, z) = \mathcal{F}(r) \exp(\text{i} \theta \cdot \text{i} z / R)(k) \exp(\text{-i} k x) \). Eliminating \( \mathcal{F} \) and \( \tilde{J} \) from basic equation, we have the eigenmode equation for \( \mathcal{F} \). The marginal stability condition is obtained by setting \( \tau = 0 \), as

\[
\frac{d}{dk} \left( \frac{1}{\lambda k^{2}} \frac{d}{dk} \right) \left( \frac{2 k^{2} + \omega_{B}}{\lambda k^{2}} \frac{d}{dk} \right) \mathcal{F} + \frac{D_{0}}{s^{2}} \mathcal{F} = 0. \tag{1}
\]

We use the normalization \( r / a \rightarrow \tilde{r}, \ t / \tau_{Ap} \rightarrow \tilde{t}, \ \chi \tau_{Ap} / a^{2} \rightarrow \tilde{\chi}, \ \mu \tau_{Ap} / a^{2} \rightarrow \tilde{\mu}, \ \lambda \tau_{Ap} / \mu_{0} a^{4} \rightarrow \tilde{\lambda}, \ \chi \tau_{Ap} / \mu_{0} m_{i} n_{i} / B_{p} \rightarrow \tilde{\chi}, \ \tau \tau_{Ap} \rightarrow \tilde{\tau}, \ \text{and notation} \ \tau \ \text{is the growth rate,} \ s = r (dq / dr) / q, \ \Omega' \beta' / 2 \varepsilon, \ B_{p} = Br / qR, \ \varepsilon = r / R, \ a \ \text{and} \ R \ \text{for the major and minor radii,} \ \beta = 2 \mu_{0} p / B^{2}, \ \text{and} \ \beta' = d \beta / d \tilde{r}. \ \text{The} \ 4
parameter $\omega_{E1}$ denotes the effect of the radial electric field shear,

$$\omega_{E1} = \tau_{Ap}(mdE_r/d\hat{r})(rB)^{-1}. \quad (2)$$

If we neglect $\omega_{E1}$, Eq. (1) reduces to the transport-driven interchange mode equation for the L-mode plasma (Itoh et al 1992c).

Equation (1) determines the relation between the anomalous transport coefficients ($\tilde{\eta}, \tilde{\lambda}, \tilde{\mu}$) and the plasma inhomogeneity ($\delta', E_r'$) (Itoh et al 1992c, 1993a, 1993b, 1993d, Yagi et al 1993). We here treat the effects of $\omega_{E1}$ by the perturbation method as in Itoh et al 1993c. The linear term of $\omega_{E1}$ is kept and Eq. (1) is rewritten as

$$\frac{d}{dy} \frac{1}{F} \frac{d}{dy} \tilde{F} + H \left[ \frac{F^3}{b} \right] \tilde{F} + Ly = 0 \quad (3-1)$$

and

$$L\tilde{F} = \tilde{\omega}_{E1} \frac{b}{\nu D_0} \left[ \frac{d}{dy} \frac{1}{F} \frac{d^2}{dy^2} \tilde{F} - H \left( \frac{d}{dy} \frac{F^2}{dy} + \frac{F^2}{dy^2} \right) \right]. \quad (3-2)$$

We use notations $H = D_0^{3/2} \tilde{\eta}^{-3/2} \tilde{\mu}^{-1/2}$, $F = b + y^2$, $b = \kappa_0^2/k_0^2$, $y = k/k_0$, $k_0^4 = D_0/\tilde{\eta}\tilde{\mu}$, and $\tilde{\omega}_{E1} = \tau_{Ap}(dE_r/dr)(rB)^{-1}$. The approximate relation $\tilde{i} = \tilde{\mu}$ is used in simplifying the expression of the operator $L$. (The validity of the approximation of $\mu = \tilde{x}$ is shown in Itoh et al 1993e.)
For the strongly localized mode, \( y^2 < 1 \), this eigenvalue equation is approximated in a form of the Weber type equation by neglecting the \( dp/dy \) term (Itoh et al 1993a, 1993b, 1993d, Yagi et al 1993) as

\[
d^2\psi/dy^2 + H((1-b^2)-3by^2)\psi + L\psi = 0. \tag{4}
\]

The stability boundary is obtained by the perturbative method. Let \( \{u_j\} \) be the \( j \)-th eigenfunction of the unperturbed equation \( (\omega_{bi} = 0) \). The fundamental eigenmode \( u_0 \) and the first harmonics \( u_1 \) are expressed as \( u_0 = \xi^{1/4} \pi^{-1/2} \exp(-\xi y^2/2) \) and \( u_1 = \sqrt{2/\pi} \xi^{3/4} y \exp(-\xi y^2/2) \), respectively. The odd and even parity modes are mixed by the operator \( L \) when \( \omega_{bi} \neq 0 \). We write the eigenfunction as

\[
\psi(y) = u_0 + \rho u_1 \ldots, \tag{5}
\]

where the coefficient \( \rho \) indicates the coupling between the even and odd modes. The eigenvalue is approximately given as

\[
\frac{\hat{H}^2}{\xi^2} = 1 - \frac{\langle 0|L|1\rangle <1|L|0\rangle}{\xi^2} \tag{6}
\]

where

\[
\hat{H} = H(1-b^2), \tag{7-1}
\]

and
\[ \xi^2 = 3bH. \quad (7-2) \]

The coefficient \( \rho \) is given as

\[ \rho = \langle 0 \mid L \mid 0 \rangle (\hat{H} - 3\xi)^{-1}. \quad (8) \]

Substituting the eigenfunctions, the integrals \( \langle 0 \mid L \mid 1 \rangle \) and \( \langle 1 \mid L \mid 0 \rangle \) are computed. We have

\[ \langle 0 \mid L \mid 1 \rangle \approx \frac{2b}{D_0} \hat{\omega}_{E1} \left[ \frac{3\xi b}{4} - \frac{3}{8} + H(b^2 + \frac{b}{\xi} + \frac{3}{4\xi^2}) \right] \quad (9-1) \]

\[ \langle 1 \mid L \mid 0 \rangle \approx \frac{2b}{D_0} \hat{\omega}_{E1} \left[ \frac{3\xi b}{4} - \frac{7}{8} + H(b^2 + \frac{b}{\xi} + \frac{3}{4\xi^2}) \right] \quad (9-2) \]

For the unperturbed mode, the least stable mode is specified (Itoh et al 1993d) by the condition \( b \approx 0.34 \) and \( H = H_0 = 1.26 \). Substituting these numbers into Eq. (9), we have \( \langle 0 \mid L \mid 1 \rangle \langle 1 \mid L \mid 0 \rangle \approx -0.58 \hat{\omega}_{E1}^2 / D_0 \). Equation (6) gives the relation

\[ \frac{D_0^{3/2} \lambda}{s^2 \xi^2} = H_0 \left[ 1 + G \hat{\omega}_{E1}^2 \right] \quad (10) \]

and

\[ G = 0.52 D_0^{-1}. \quad (11) \]
From Eq. (10), the anomalous transport coefficient in the presence of the inhomogeneous radial electric field is obtained as

$$\tilde{\chi} = \frac{\tilde{\chi}_L}{1 + G\hat{\omega}_{EI}^2}.$$  \hspace{1cm} (12-1)

and

$$\tilde{\chi}_L = \frac{0.8D_0^{3/2}\lambda}{\tilde{\chi}S^2}.$$ \hspace{1cm} (12-2)

The form of $\tilde{\chi}_L$ is the anomalous transport obtained for the L-mode plasma. The coefficient $\lambda/\tilde{\chi}$ is given by $\delta^2/a^2$ where $\delta$ is the collisionless skin depth (Itoh 1983e). Equation (12) quantifies the effect of $E_r'$ on the thermal conductivity, unifying the L- and H-mode plasmas. It is emphasized that the coefficients are given explicitly in terms of the equilibrium quantities. This is because the self-sustained turbulence is solved by our theoretical formalism. The suppression of the transport is prominent when $\hat{\omega}_{EI} \sim \sqrt{D_0}$.

The theory also predicts the change in the fluctuations. The relations $k_\theta^2 = b k_0^2$ and $k_0^4 = D_0 \chi^2$ (note that the relation $\chi = \mu$ is used) show that poloidal mode number becomes larger as $\chi$ is reduced. Using Eq. (12), we have

$$k_\theta^2 \propto s^2 \delta^{-2} D_0^{-1/2} \left[ 1 + G\hat{\omega}_{EI}^2 \right].$$ \hspace{1cm} (13)
Fluctuation level is also reduced. The renormalized diffusivity has a relation \( \chi \sim (rB_e^2/(\chi_{k, \perp}^2)) \) (Itoh 1993d), which gives the estimate, \( \Phi/B \sim \chi \). Using Eq. (12), we have

\[
e\Phi/T \sim (x_L eB/T) \left[ 1 + G_{\omega_E}^2 \right]^{-1}. \tag{14}
\]

The fluctuation level is reduced and the radial and poloidal correlation lengths become shorter as \( \chi \) is subject to considerable reduction.

Figure 1 illustrates the dependences on \( D_0 \) (the drive of the pressure gradient) and \( \omega_{E1} \) (influence of the \( E_r \)) of the thermal conductivity. The influences of the radial electric field inhomogeneity on the correlation length and the amplitude of fluctuations are shown in Fig. 2. Reduction of the anomalous transport in the range of \( \omega_{E1} > \sqrt{D_0} \) is shown.

The case for the system with magnetic well, such as tokamaks and conventional stellarators, was studied in a separate article (Itoh et al. 1993c). The result was given as

\[
\hat{\chi} = \frac{x_L}{1 + G_w \omega_{ES}^2} \tag{15}
\]

where \( \omega_{ES} = \tau_{Ap}(dE_r/d\phi)(s\Omega B)^{-1} \), \( x_L = 0.6\alpha^{1.5} \chi/\chi \), \( \alpha = q^2 \mu' / \varepsilon \) and \( G_w \sim 1/2\alpha^2 \) in the small shear limit. The influence of the curvature of the radial electric field was also obtained. The reduction of \( \chi \) is realized when \( \omega_{ES} \) becomes of the order of \( \alpha \).
In summary, we have developed the theory of the anomalous transport in toroidal helical plasmas under the influence of the pressure gradient and inhomogeneous radial electric field. The marginal stability condition of the nonlinear interchange mode was solved. The explicit relation of the transport coefficients and the plasma inhomogeneity \( \{ p', E_r' \} \) was obtained keeping the geometrical factors such as \( q, R/a \) and magnetic shear. This theory explains the plasma confinement in \( H \)-mode as well as in \( L \)-mode simultaneously.

This theory also predicts that the fluctuation level could be reduced strongly when the inhomogeneity of the radial electric field establishes. At the same time, the correlation length becomes shorter. This theory could be tested by experiments not only on the macroscopic quantity such as the thermal conductivity (Eqs. (12), (15)), but also on the microscopic fluctuations (Eqs. (13), (14)). Since the \( H \)-mode-like improvement was observed in experiments (Erckmann et al 1983, Toi et al 1993), the comparison would be an urgent task.

Compared to the results predicted for tokamaks (Itoh et al 1993c), we find that the difference in transport coefficients appears only in the geometrical factors. This was concluded for \( L \)-mode plasmas in Itoh et al 1993d, and is now extended to the improved confinement. The generic features of the anomalous transport, i.e., the driving by the pressure gradient, the role of the collisionless skin depth and the effect of the radial electric field inhomogeneity, are unchanged. The similar
statement is found in the report on experiment (Erckmann et al 1993). This theory quantifies, from the first principle, the differences in transport coefficients in various toroidal plasmas, which are caused by the difference in the geometry.

This theory confirms the previous qualitative arguments on the $E_r$ stabilization Itoh et al 1989b, 1990, 1991, Shaing et al 1989b, Bigrali et al 1989, Sugama et al 1991). The main progresses are that the coefficients of $\omega_{BI}^2$ in $\chi$ is explicitly given in terms of the plasma parameters, by solving the self-sustained turbulence, that formula for the L- and H-mode confinement are obtained in a unified manner, and that the fluctuation level is simultaneously obtained. These have become possible by the new theoretical framework, i.e., the $E \times B$ nonlinearity is renormalized in a form of diffusion operator and the mean-field approximation is employed (Itoh et al 1993d).

The present result (12) is obtained except for the numerical factor. Nonlinear simulation such as those in Wakatani et al 1992 would give this coefficient and allow us to examine the validity of the ansatz in detail. Here the analytic nonlinear solution was obtained under the very simplified situations: in order to explain the experiments precisely, aso necessary would be the investigation of effects such as the finite ion gyro-radius, diamagnetic drift for kinetic corrections, parallel flow or perpendicular compressibility. These research topics are open for future study.
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References

Figure Caption

Fig. 1  Bird's eye view of the $D_0$ and $\omega_{E_1}$ dependence of the thermal conductivity $\hat{\lambda}$ (a). The normalized value $\hat{\lambda}/\hat{\lambda}_0$ ($\hat{\lambda}_0=\bar{\lambda}/\bar{s}^2$) is plotted.

Fig. 2  Bird's eye view of the $D_0$ and $\omega_{E_1}$ dependences of the the correlation length $\xi_\phi (m/\bar{k}_8)$ (a) and the fluctuation amplitude $\phi$ (b). In the graph, the normalized values $(\xi_\phi /\bar{s})^2$ and $\phi/[B\bar{\xi}_0]$ ($\bar{\xi}_0=\bar{\lambda}/\bar{s}^2$) are plotted.
Fig. 1.
Fig. 2

\[
\left( \frac{\varepsilon_0 f_0^2}{\delta} \right)^2
\]

\[
D_0
\]

\[
\tilde{\omega}_{E1}
\]

\[
\tilde{\phi}
\]

\[
\tilde{X}_0 B
\]

\[
D_0
\]

\[
\tilde{\omega}_{E1}
\]
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