A theoretical study for parallel electric field in nonlinear magnetosonic waves in three-component plasmas

journal or publication title	Physics of Plasmas
volume	23
page range	072115
year	2016-07-15
URL	http://hdl.handle.net/10655/00012665

doi: 10.1063/1.4958312

Physics of Plasmas

A theoretical study for parallel electric field in nonlinear magnetosonic waves in three-component plasmas

Cite as: Phys. Plasmas **23**, 072115 (2016); https://doi.org/10.1063/1.4958312
Submitted: 21 April 2016 • Accepted: 27 June 2016 • Published Online: 15 July 2016

Mieko Toida

PHYSICS OF PLASMAS 23, 072115 (2016)

A theoretical study for parallel electric field in nonlinear magnetosonic waves in three-component plasmas

Mieko Toida[a)]
National Institute for Fusion Science, National Institutes of Natural Sciences, Toki 509-5292, Japan

(Received 21 April 2016; accepted 27 June 2016; published online 15 July 2016)

The electric field parallel to the magnetic field in nonlinear magnetosonic waves in three component plasmas (two-ion-species plasma and electron-positron-ion plasma) is theoretically studied based on a three-fluid model. In a two-ion-species plasma, a magnetosonic mode has two branches, high-frequency mode and low-frequency mode. The parallel electric field $E_\parallel$ and its integral along the magnetic field, $F = -\int E_\parallel ds$, in the two modes propagating quasiperpendicular to the magnetic field are derived as functions of the wave amplitude ϵ and the density ratio and cyclotron frequency ratio of the two ion species. The theory shows that the magnitude of F in the high-frequency-mode pulse is much greater than that in the low-frequency-mode pulse. Theoretical expressions for $E_\parallel$ and F in nonlinear magnetosonic pulses in an electron-positron-ion plasma are also obtained under the assumption that the wave amplitudes are in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$, where m_e/m_i is the electron to ion mass ratio. *Published by AIP Publishing.*
[http://dx.doi.org/10.1063/1.4958312]

I. INTRODUCTION

Theory and particle simulations have revealed that nonlinear magnetosonic waves can strongly accelerate particles with various nonstochastic mechanisms.[1] The electric field parallel to the magnetic field, $E_\parallel$, in a magnetosonic shock wave plays crucial roles in some of the mechanisms.[2] For example, the parallel electric field can cause trapping and acceleration of electrons in a shock wave propagating obliquely to the magnetic field.[3] In a plasma containing electrons, positrons, and ions, the parallel electric field can strongly accelerate positrons.[4]

In the particle simulations on the trapping and acceleration of electrons by a nonlinear magnetosonic wave, it was observed that $E_\parallel$ can be strong.[3] The values of the integral of $E_\parallel$ along $\boldsymbol{B}$, $F = -\int E_\parallel ds$, were also observed to be quite large; we call F the parallel pseudopotential because $E_\parallel$ contains both longitudinal and transverse components. This cannot be explained by the magnetohydrodynamics; the values of $E_\parallel$ and F have been generally believed to be quite small in a collisionless plasma because $E_\parallel$ is exactly zero in the ideal magnetohydrodynamics.[5]

Motivated by the observation of the strong $E_\parallel$, the theory for $E_\parallel$ and F in the nonlinear magnetosonic wave in a single-ion-species plasma has been developed.[6] For small-amplitude pulses, $E_\parallel$ and F were derived based on a two-fluid model. In a warm plasma, F is given as $eF \sim \epsilon\Gamma_e T_e$, where ϵ is the wave amplitude, Γ_e is the specific heat ratio of electrons, and T_e is the electron temperature. However, in a cold plasma such that $\epsilon^2 m_i v_A^2 > \epsilon\Gamma_e T_e$, where v_A is the Alfvén speed, it is given by

$$eF \simeq \frac{1}{2}\epsilon^2 m_i v_A^2 \left(1 - \frac{m_i}{m_e}\cos^2\theta\right)^{-1}, \quad (1)$$

where m_i is the ion mass, m_e is the electron mass, and θ is the propagation angle of the nonlinear wave. These theories were verified by the electromagnetic particle simulations. Further, for large-amplitude shock waves with $\epsilon \sim O(1)$, the phenomenological relation that can explain simulation results for both warm and cold plasmas was presented as $eF \sim \epsilon(m_i v_A^2 + \Gamma_e T_e)$. These results indicate that F can be large when the external magnetic field $\boldsymbol{B}_0$ is strong. In Ref. 7, the parallel electric field in nonlinear magnetosonic wave in an electron-positron-ion plasma was theoretically analyzed. It was shown that eF in a small-amplitude pulse with $\epsilon \ll 1$ is proportional to $\epsilon^2 m_i v_A^2$ in a cold plasma and it decreases with increasing positron density.

Although the theory for $E_\parallel$ and F has been extended, the parallel electric field in a nonlinear magnetosonic wave in a plasma containing multiple species ions has not been analyzed. Astrophysical and fusion plasmas usually contain multiple species ions. The presence of multiple species ions can significantly influence the properties of magnetosonic waves (for instance, Refs. 8–16). In a two-ion-species plasma, there are two magnetosonic modes, which we call high-frequency mode and low-frequency mode. The frequency of the low-frequency mode goes to zero as the wavenumber k approaches zero. The high-frequency mode has a finite cut-off frequency of the order of the ion cyclotron frequency. Nonlinear behavior of the low- and high-frequency modes can be described by Korteweg-de Vries equation (KdV equation), although the linear dispersion curves of these modes are quite different in the long-wavelength region.[13] Nonlinear coupling between the high- and low-frequency modes can occur.[14] In fact, the numerical simulation showed that high-frequency-mode pulses are generated from a low-frequency mode pulse when its amplitude exceeds a critical value, which depends on the density ratio and cyclotron ratio of the two ion species.[16]

In this paper, we develop a theory for $E_\parallel$ and F in nonlinear magnetosonic pulses propagating quasiperpendicular to the magnetic field in a two-ion-species plasma. We derive

a)Electronic mail: toida.mieko@nifs.ac.jp

the expression for $E_\parallel$ and F in solitary pulses of the high- and low-frequency modes. We also theoretically analyze $E_\parallel$ and F in a nonlinear pulse in an electron-positron-ion plasma (e-p-i plasma) assuming that the amplitude is in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$; the theoretical expressions for $E_\parallel$ and F for such amplitudes were not given in Ref. 7 where $\epsilon \ll 1$ was assumed.

In Sec. II, we overview the properties of linear and nonlinear magnetosonic waves in the three component plasmas (two-ion-species plasma and e-p-i plasma). In Sec. III, we analyze $E_\parallel$ and F in the low-frequency-mode pulse and the high-frequency-mode pulse in a two-ion-species plasma. For the low-frequency mode, there are two pulses with different characteristic lengths when the propagation angle θ is greater than the critical angle θ_c;[17] the value of θ_c is, for example, 71° in a hydrogen-helium (H–He) plasma with the density ratio $n_{\rm He}/n_{\rm H} = 0.1$. We derive the theoretical expressions for F in the two low-frequency-mode pulses, which are given as functions of ϵ, θ, and the density ratio and cyclotron frequency ratio of two ion species. It is found that the values of F in the two low-frequency-mode pulses are much smaller than those in the pulses in the single-ion-species plasma given by Eq. (1). For the high-frequency-mode pulse, we show that the magnitude of F is the same order as Eq. (1). Thus, F in the high-frequency-mode pulse is much greater than F in the low-frequency-mode pulses. In Sec. IV, we analyze nonlinear magnetosonic waves in an e-p-i plasma. We derive a KdV equation assuming that $(m_e/m_i) < \epsilon < 1$. We then obtain F in the nonlinear pulse with its amplitude in this range. It is shown that F in the pulse with ϵ in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$ can be written as the same form as that in the range of $\epsilon \ll 1$. Section V gives a summary of our work.

II. OVERVIEW OF MAGNETOSONIC WAVES IN THREE COMPONENT PLASMAS

We consider magnetosonic waves propagating in the x direction in an external magnetic field $\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta)$ in a three component plasma. We use the following three-fluid equations:

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j \boldsymbol{v}_j) = 0, \tag{2}$$

$$m_j\left(\frac{\partial}{\partial t} + (\boldsymbol{v}_j\cdot\nabla)\right)\boldsymbol{v}_j = q_j\boldsymbol{E} + \frac{q_j}{c}\boldsymbol{v}_j\times\boldsymbol{B}, \tag{3}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E}, \tag{4}$$

$$\nabla\times\boldsymbol{B} = (4\pi/c)\sum_j q_j n_j \boldsymbol{v}_j, \tag{5}$$

where the plasma is assumed to be cold and the displacement current is neglected. For a two-ion-species plasma with the ion species a and b, the subscript j refers to ion species ($j = a$ or b) or electrons ($j = $ e). For an e-p-i plasma, j refers to electrons, positrons, or ions (j = e, p, or i). From Equations (2) to (5), we obtain the linear dispersion relation as

$$c^2k^2\left[c^2k^2\sin^2\theta + \omega_{\rm p}^2(1+\cos^2\theta)\right]\sum_j \frac{\omega_{pj}^2\omega^2}{\omega^2-\Omega_j^2}$$
$$+\left(c^2k^2\sin^2\theta+\omega_{\rm p}^2\right)\left(\sum_j\frac{\omega_{pj}^2}{\omega-\Omega_j}\right)\left(\sum_j\frac{\omega_{pj}^2}{\omega+\Omega_j}\right)\omega^2$$
$$+c^4k^4\omega_{\rm p}^2\cos^2\theta = 0, \tag{6}$$

where

$$\omega_{\rm p}^2 = \sum_j \omega_{\rm pj}^2 \tag{7}$$

and ω_{pj} and Ω_j are plasma and cyclotron frequencies of the particle species j, respectively.

Figure 1(a) shows the dispersion curve of the magnetosonic wave with the propagation angle $\theta = 87°$ in a single-ion-species plasma. As $k \to \infty$, ω approaches ω_r defined as

$$\omega_r = |\Omega_{\rm e}|(m_e/m_i + \cos^2\theta)^{1/2}, \tag{8}$$

which is of the order of $\sqrt{|\Omega_{\rm e}|\Omega_i}$ when $\cos\theta \lesssim (m_e/m_i)^{1/2}$.

Figure 1(b) shows the magnetosonic wave in a two-ion-species plasma, where the ion species are hydrogen (H) and helium (He) with the density ratio $n_{\rm He}/n_{\rm H} = 0.1$. In a two-ion-species plasma, the magnetosonic wave is split into two modes, high-frequency mode and low-frequency mode. The cut-off frequency of the high-frequency mode ω_{+0} is given by

$$\omega_{+0} = \left(\frac{\omega_{\rm pa}^2}{\Omega_a^2}+\frac{\omega_{\rm pb}^2}{\Omega_b^2}\right)\frac{\Omega_a\Omega_b|\Omega_{\rm e}|}{\omega_{\rm pe}^2}. \tag{9}$$

The frequency ω_{-r} is the resonance frequency of the perpendicular low-frequency mode,

$$\omega_{-\rm r} = \left(\frac{\omega_{\rm pa}^2\Omega_b^2+\omega_{\rm pb}^2\Omega_a^2}{\omega_{\rm pa}^2+\omega_{\rm pb}^2}\right)^{1/2}. \tag{10}$$

The dispersion curves for both the high- and low-frequency modes have a large curvature near the wavenumber k_c defined as

$$k_c = \omega_{-r}/v_{\rm A}. \tag{11}$$

It has been pointed out that the normalized frequency difference,

$$\Delta_\omega = (\omega_{+0}-\omega_{-r})/\omega_{+0}, \tag{12}$$

is an important parameter in nonlinear development of the two modes.[16,17] The value of Δ_ω depends on the density ratio and cyclotron frequency ratio of the two species ions. It increases with increasing Ω_a/Ω_b, where $\Omega_a > \Omega_b$ is assumed. For a fixed Ω_a/Ω_b, Δ_ω becomes maximum when the ion charge densities are equal, $n_a q_a = n_b q_b$.

Figure 1(c) displays the case for an e-p-i plasma, where the positron to electron density ratio is $n_{p0}/n_{e0} = 0.1$. Although the structure of the dispersion curves for an e-p-i plasma is similar to that for a two-ion-species plasma, there are significant quantitative differences between them. The resonance frequency of the lower frequency mode in an e-p-i plasma is estimated as

$$\omega_r \simeq \frac{|\Omega_e|}{\omega_p}\left[\omega_{pi}^2 + \left(\omega_{pe}^2 + \omega_{pp}^2\right)\cos^2\theta\right]^{1/2}, \quad (13)$$

which is the same order as ω_r in the single-ion-species plasma. The cut-off frequency of the higher frequency mode is

$$\omega_{h0} \simeq \frac{\left(\omega_{pi}^2\Omega_e^2 + \omega_{pep}^2\Omega_i^2\right)}{\Omega_i\left(\omega_{pep}^2 + \omega_{pi}^2\right)}, \quad (14)$$

where ω_{pep} is defined as

$$\omega_{pep}^2 = \omega_{pe}^2 + \omega_{pp}^2. \quad (15)$$

We consider the regions for which the dispersion relation can be approximated by the following form:

$$\omega = v_p k(1 - d^2k^2/2). \quad (16)$$

These regions are enclosed by the gray dotted lines in Fig. 1. The nonlinear behavior for these regions can be described by the KdV equation,

$$\frac{\partial B_{z1}}{\partial \tau} + \frac{3v_A}{2B_0}\alpha B_{z1}\frac{\partial B_{z1}}{\partial \xi} + \frac{v_A d^2}{2}\frac{\partial^3 B_{z1}}{\partial \xi^3} = 0, \quad (17)$$

where B_{z1} is the perturbation of B_z, $B_{z1} = B_z - B_{z0}$, and ξ and τ are stretched coordinates,

$$\xi = \epsilon^{1/2}(x - v_p t), \quad \tau = \epsilon^{3/2}t, \quad (18)$$

with $\epsilon \sim |B_{z1}/B_0|$. The solitary wave solution of Eq. (17) is

$$B_{z1}/B_0 = b_n \mathrm{sech}^2[(x - Mv_A t)/D], \quad (19)$$

where the soliton width D and the Mach number M are given as

$$D = 2b_n^{-1/2}d, \quad (20)$$

$$M = 1 + \alpha b_n/2. \quad (21)$$

In the following, we discuss the parallel electric field $E_\parallel$ in the solitary pulses, given by Eq. (19), in the three component plasmas.

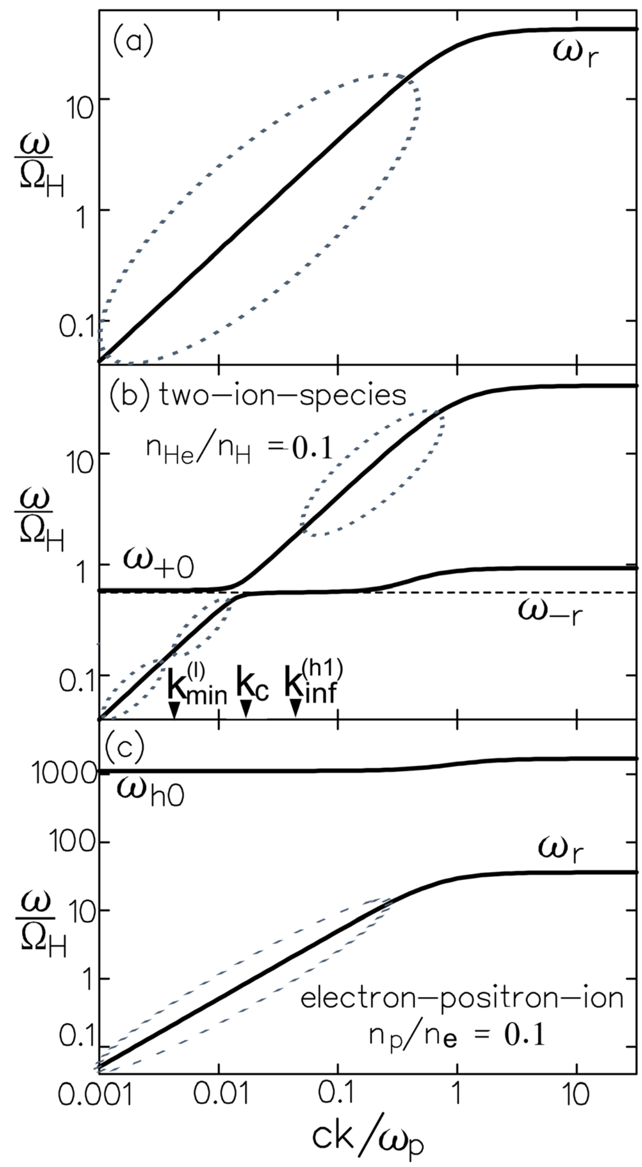

FIG. 1. Dispersion curves for magnetosonic waves in a single-ion-species plasma (a), two-ion-species plasma (b), and electron-positron-ion plasma (c). The propagation angles of the magnetosonic waves are $\theta = 87°$ for all the cases. For a two-ion-species plasma, the ion species are hydrogen (H) and helium (He), with the density ratio $n_{He}/n_H = 0.1$. For an electron-positron-ion plasma, the density ratio of positrons to electrons is $n_p/n_e = 0.1$. The dispersion relations for the regions enclosed by the gray dotted lines can be approximated by Eq. (16) and the nonlinear behavior for these regions can be described by the KdV equation.

III. TWO-ION-SPECIES PLASMA

We present properties of nonlinear pulses of the low- and high-frequency modes propagating quasiperpendicular to the magnetic field in a two-ion-species plasma. We then derive the theoretical expressions for $E_\parallel$ and F in the nonlinear pulses.

A. Nonlinear pulses of low-frequency mode

For the low-frequency mode, there are two nonlinear pulses with different characteristic lengths. We here show the characteristic quantities of the two pulses.[17]

The linear dispersion relation of the low-frequency mode is written as $\omega/k = v_A(1 + \mu k^2)$ in the long wavelength limit, where v_A is the Alfvén speed and μ is given by

$$\mu = -\frac{v_A^4}{2c^2}\left[\sum_j \frac{\omega_{pj}^2}{\Omega_j^4} - \frac{v_A^2}{c^2\sin^2\theta}\left(\sum_j \frac{\omega_{pj}^2}{\Omega_j^3}\right)^2\right]. \quad (22)$$

We define the angle θ at which μ becomes zero as θ_{cl}. The value of θ_{cl} is, for example, 71° in a H–He plasma with $n_{He}/n_H = 0.1$. For $\theta > \theta_{cl}$, μ is negative and ω is written as Eq. (16) with

$$v_{\rm p} = v_{\rm A}, \quad d^2 = d_{l1}^2 = -2\mu. \tag{23}$$

This is valid for the range of wavenumbers,

$$k \ll k_{\rm min}^{(1)}, \tag{24}$$

where $k_{\rm min}^{(1)}$ is

$$k_{\rm min}^{(1)} = k_c(\cos\theta/\cos\theta_{\rm cl}), \tag{25}$$

which is smaller than k_c. Assuming that $\Delta_\omega \gg m_e/m_i$ and $\cos^2\theta \ll 1$, where Δ_ω was given by Eq. (12), we can express d_{l1} and $\theta_{\rm cl}$ as

$$d_{l1} = (2\Delta_\omega - r\cos^2\theta)^{1/2}/(k_c\sin\theta), \tag{26}$$

$$\cos^2\theta_{\rm cl} = 2\Delta_\omega/r. \tag{27}$$

Here, r is defined as

$$r = \frac{(\Omega_a^2 + \Omega_b^2)}{\Omega_a^2\Omega_b^2}\omega_{-{\rm r}}^2 - 1. \tag{28}$$

When Ω_a and Ω_b are of the same order of magnitude, r is of the order of unity. The nonlinear behavior for the region given by Eq. (24) is described by the KdV equation (17) with the coefficient α,

$$\alpha = \sin\theta. \tag{29}$$

For $\theta > \theta_{\rm cl}$ and the range of wavenumbers,

$$k_{\rm min}^{(1)} \ll k \ll k_{\rm c}, \tag{30}$$

the dispersion relation is also written as Eq. (16), and the KdV equation is obtained. However, d, and α are different from those for $k \ll k_{\rm min}^{(1)}$. For $k_{\rm min}^{(1)} \ll k \ll k_{\rm c}$, they are

$$d = d_{l2} = (2\Delta_\omega)^{1/2}/(k_{\rm c}\sin\theta), \quad \alpha = 1. \tag{31}$$

Thus, for the low-frequency mode, we have two solitary pulses given by Eq. (19) with the characteristic lengths $d = d_{l1}$ and $d = d_{l2}$. The pulse with d_{l1} is for the wavenumber region $k \ll k_{\rm min}^{(1)}$, whereas the pulse with d_{l2} is for $k_{\rm min}^{(1)} \ll k \ll k_c$. We call the former the longer-wavelength pulse and the latter the shorter-wavelength pulse. The amplitudes of the pulses for which the KdV equations are valid were shown in Ref. 17.

B. $E_\parallel$ in low-frequency-mode pulses

We derive the parallel electric field $E_\parallel$ and the parallel pseudopotential F in the low-frequency-mode pulses, which are the longer-wavelength pulse and the shorter-wavelength pulse.

1. Longer-wavelength pulse

For the longer-wavelength pulses with characteristic wavelength d_{l1}, we can derive $E_\parallel$ using the conventional reductive perturbation method to obtain the KdV equation.[19–21] We introduce the stretched coordinates given by Eq. (18) and expand quantities as

$$n_j = n_{j0} + \epsilon n_{j1} + \epsilon^2 n_{j2} + \cdots, \tag{32}$$

$$v_{jx} = \epsilon v_{jx1} + \epsilon^2 v_{jx2} + \cdots, \tag{33}$$

$$v_{jy} = \epsilon^{3/2} v_{jy1} + \epsilon^{5/2} v_{jy2} + \cdots, \tag{34}$$

$$v_{jz} = \epsilon v_{jz1} + \epsilon^2 v_{jz2} + \cdots, \tag{35}$$

$$B_z = \sin\theta + \epsilon B_{z1} + \epsilon^2 B_{z2} + \cdots, \tag{36}$$

$$E_y = \epsilon E_{y1} + \epsilon^2 E_{y2} + \cdots, \tag{37}$$

$$B_y = \epsilon^{3/2} B_{y1} + \epsilon^{5/2} B_{y2} + \cdots, \tag{38}$$

$$E_x = \epsilon^{3/2} E_{x1} + \epsilon^{5/2} E_{x2} + \cdots, \tag{39}$$

$$E_z = \epsilon^{3/2} E_{z1} + \epsilon^{5/2} E_{z2} + \cdots. \tag{40}$$

We consider up to the second order terms of these quantities, and we write $E_\parallel$ as

$$E_\parallel = \frac{\boldsymbol{E}\cdot\boldsymbol{B}}{B} = \frac{\boldsymbol{E}_1\cdot\boldsymbol{B}_0}{B_0}\left(1 - \frac{\boldsymbol{B}_1\cdot\boldsymbol{B}_0}{B_0^2}\right) + \frac{\boldsymbol{E}_1\cdot\boldsymbol{B}_1}{B_0} + \frac{\boldsymbol{E}_2\cdot\boldsymbol{B}_0}{B_0}. \tag{41}$$

We obtain the relations between $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{B}_1$ (for the details of calculation, see Appendix A Subsection 1). By virtue of Eqs. (A17) and (A18), we see that the lowest order term of $E_\parallel$ is zero,

$$\frac{\boldsymbol{E}_1\cdot\boldsymbol{B}_0}{B_0} = \epsilon^{3/2}(E_{x1}\cos\theta + E_{z1}\sin\theta) = 0. \tag{42}$$

Further, we see that $\boldsymbol{E}_1\cdot\boldsymbol{B}_1$ is zero,

$$\boldsymbol{E}_1\cdot\boldsymbol{B}_1 = \epsilon^{5/2}(E_{y1}B_{y1} + E_{z1}B_{z1}) = 0. \tag{43}$$

We thus have $E_\parallel = \boldsymbol{E}_2\cdot\boldsymbol{B}_0/B_0$. From Eq. (A26), we obtain $E_\parallel$ of the longer-wavelength pulse as

$$\begin{aligned} E_\parallel &= \epsilon^{5/2}(E_{x2}\cos\theta + E_{z2}\sin\theta), \\ &= \epsilon^{5/2}\frac{v_p^4 B}{c\omega_{\rm p}^2\tan\theta}\left(\sum_j\frac{\omega_{pj}^2}{\Omega_j^3}\right)\frac{\partial^3}{\partial\xi^3}\left(\frac{B_{z1}}{B_0}\right). \end{aligned} \tag{44}$$

Then, we have the parallel pseudopotential, $F = -\int E_\parallel ds = -\int (E_\parallel/\cos\theta)dx$, as

$$F = -\epsilon^2\frac{v_p^4 B_0}{c\omega_{\rm p}^2\sin\theta}\left(\sum_j\frac{\omega_{pj}^2}{\Omega_j^3}\right)\frac{\partial^2}{\partial\xi^2}\left(\frac{B_{z1}}{B_0}\right). \tag{45}$$

Substituting the soliton solution (19) into Eq. (45), we find the peak value of F of the longer-wavelength pulse as

$$F_{Ml1} = \frac{v_p^4 B_0}{2c\omega_{\rm p}^2 d_{l1}^2}\left(\sum_j\frac{\omega_{pj}^2}{\Omega_j^3}\right)b_n^2, \tag{46}$$

where we have approximated that $\sin\theta = 1$. This can be expressed as

$$eF_{Ml1} = \frac{m_e v_A^2}{2(2\Delta_\omega - r\cos^2\theta)} \frac{\omega_{+0}\left(\omega_{pa}^2\Omega_b^3 + \omega_{pb}^2\Omega_a^3\right)}{\Omega_a^2\Omega_b^2\left(\omega_{pa}^2 + \omega_{pb}^2\right)} b_n^2, \quad (47)$$

where ω_{+0}, Δ_ω, and r are defined by Eqs. (9), (12), and (28), respectively. We thus obtain the theoretical expression for F as a function of the amplitude b_n, the propagation angle θ, and the density ratio and cyclotron frequency ratio of the two species ions.

2. Shorter-wavelength pulse

For the shorter-wavelength pulses with the characteristic length d_{l2}, we expand v_{jz}, B_y, and E_z as

$$v_{jz} = \cos\theta(\epsilon v_{jz1} + \epsilon v_{jz2} + \cdots), \quad (48)$$

$$B_y = \cos\theta(\epsilon^{3/2}B_{y1} + \epsilon^{5/2}B_{y2} + \cdots), \quad (49)$$

$$E_z = \cos\theta(\epsilon^{3/2}E_{z1} + \epsilon^{5/2}E_{z2} + \cdots). \quad (50)$$

The expansion of other quantities is the same as those in Eqs. (32)–(40). We also assume that

$$\cos^2\theta \ll \epsilon \ll 2\Delta_\omega. \quad (51)$$

Equations (48)–(51) enable us to focus on the region $k_{\rm min}^{(1)} \ll k \ll k_c$ and obtain the KdV equation with the characteristic length d_{l2}.[17]

We derive $E_\parallel$ in the shorter-wavelength pulse (see Appendix A Subsection 2). From Eq. (A30), we see that $\boldsymbol{E}_1 \cdot \boldsymbol{B}_0 = \epsilon^{3/2}(E_{x1}\cos\theta + E_{z1}\cos\theta\sin\theta) = 0$ and $\boldsymbol{E}_1 \cdot \boldsymbol{B}_1 = E_{y1}B_{y1} + E_{Z1}B_{z1} = 0$. By virtue of Eq. (A32), we find that

$$E_\parallel = \epsilon^{5/2}(E_{x2}\cos\theta + E_{z2}\cos\theta\sin\theta)$$
$$= \epsilon^{5/2}\frac{v_p^4 B\cos\theta\sin\theta}{c\omega_p^2}\left(\sum_j \frac{\omega_{pj}^2}{\Omega_j^3}\right)\frac{\partial^3}{\partial\xi^3}\left(\frac{B_{z1}}{B_0}\right). \quad (52)$$

The parallel pseudopotential is

$$F = -\epsilon^2 \frac{v_p^4 B_0 \sin\theta}{c\omega_p^2}\left(\sum_j \frac{\omega_{pj}^2}{\Omega_j^3}\right)\frac{\partial^2}{\partial\xi^2}\left(\frac{B_{z1}}{B_0}\right). \quad (53)$$

Substituting the soliton solution (19) with $d = d_{l2}$ into Eq. (53), we can write the peak value of F of the shorter-wavelength pulse as

$$eF_{Ml2} = \frac{e v_p^4 B_0}{2c\omega_p^2 d_{l2}^2}\left(\sum_j \frac{\omega_{pj}^2}{\Omega_j^3}\right) b_n^2$$
$$= \frac{m_e v_A^2}{4\Delta_\omega}\frac{\omega_{+0}\left(\omega_{pa}^2\Omega_b^3 + \omega_{pb}^2\Omega_a^3\right)}{\Omega_a^2\Omega_b^2\left(\omega_{pa}^2 + \omega_{pb}^2\right)} b_n^2. \quad (54)$$

3. Magnitude of F

By virtue of Eqs. (47) and (54), we can estimate the magnitude of F in the longer-wavelength pulse and the shorter-wavelength pulse as

$$eF_{Ml1} \sim m_e v_A^2 b_n^2/(4\Delta_\omega - 2\cos^2\theta) \quad (55)$$

and

$$eF_{Ml2} \sim m_e v_A^2 b_n^2/(4\Delta_\omega), \quad (56)$$

where we have assumed that Ω_a and Ω_b are the same order of magnitude. For quasiperpendicular pulses with $\cos^2\theta \ll \Delta_\omega$, both eF_{Ml1} and eF_{Ml2} are of the order of $m_e v_A^2 b_n^2/(4\Delta_\omega)$. The value of $4\Delta_\omega$ of the H–He plasma with $n_{\rm He}/n_{\rm H} = 0.1$ is 0.12, which is much greater than $m_e/m_i \sim 10^{-3}$. In such a two-ion-species plasma with $\Delta_\omega \gg m_e/m_i$, eF_{Ml1} and eF_{Ml2} are much smaller than eF_M in the single-ion-species plasma, which is of the order of $m_i v_A^2 b_n^2/2$ as shown by Eq. (1).

As the plasma approaches the single-ion-species plasma, Δ_ω goes to zero. Then, eF_{Ml1} and eF_{Ml2} increase with decreasing Δ_ω. For the plasma with $\Delta_\omega \le m_e/m_i$, Eqs. (55) and (56) are not valid, and we have to consider the terms of the order of m_e/m_i, which are neglected in these equations.

C. Nonlinear pulses of high-frequency mode

We here outline the properties of the high-frequency mode, in which the parameter η defined by

$$\eta = (\omega_{pa}^2 + \omega_{pb}^2)^{1/2}/\omega_{\rm pe}, \quad (57)$$

plays an important role.

The dispersion relation of the quasiperpendicular high-frequency mode can be approximated by Eq. (16) with

$$v_{\rm p} = v_{\rm h}, \quad d = d_{\rm h}, \quad (58)$$

where $v_{\rm h}$ and $d_{\rm h}$ are defined as

$$v_{\rm h} = \eta|\Omega_{\rm e}|c/\omega_{\rm pe}, \quad (59)$$

$$d_{\rm h} = \frac{c}{\omega_{\rm pe}}\left(1 - \frac{\cos^2\theta}{\eta^2}\right)^{1/2}. \quad (60)$$

This is valid for the range of the wavenumbers[18]

$$ck_{\rm inf}^{(\rm h1)}/\omega_{\rm pe} \ll ck/\omega_{\rm pe} \ll 1, \quad (61)$$

where $k_{\rm inf}^{(\rm h1)}$ is defined as

$$k_{\rm inf}^{(\rm h1)} = \left(\frac{\bar{\mu}_h}{3|d_{\rm h}|}\right)^{1/4}, \quad (62)$$

with

$$\overline{\mu_{\rm h}} = \frac{\omega_{\rm pe}^2\eta^2}{c^2}\left(1 - 2\sum_i \frac{\omega_{pi}^2\Omega_i}{\omega_{\rm pe}^2|\Omega_{\rm e}|\eta^4} + \sum_i \frac{\omega_{pi}^2\Omega_i^2}{\omega_{\rm pe}^2\Omega_{\rm e}^2\eta^6}\right). \quad (63)$$

Because $\overline{\mu_{\rm h}}$ can be rewritten as

$$\overline{\mu_{\rm h}} = 2\Delta_\omega \frac{\omega_{\rm pe}^2\eta^2}{c^2}\frac{\omega_{-r}^4}{\Omega_a^2\Omega_b^2}, \quad (64)$$

we can estimate the magnitude of $k_{\rm inf}^{(\rm h1)}$ as

$$ck_{\rm inf}^{\rm (h1)}/\omega_{\rm pe} \sim \eta^{1/2}\Delta_\omega. \tag{65}$$

The nonlinear behavior of the high-frequency mode for the wavenumber region (61) can be described by the KdV equation (17) with coefficient α given by[18]

$$\alpha = \frac{\left(\omega_{\rm pa}^2\Omega_a + \omega_{\rm pb}^2\Omega_b\right)}{\omega_{\rm pe}^2|\Omega_{\rm e}|\eta^4}. \tag{66}$$

We write this α as α_h, which is also expressed with Δ_ω as

$$\alpha_h = 1 + \frac{2\omega_{-r}^2}{\Omega_a\Omega_b}\Delta_\omega. \tag{67}$$

D. $E_\parallel$ in high-frequency-mode pulse

We derive $E_\parallel$ in the high-frequency-mode pulse. We expand E_x, v_{jy}, and v_{jz} as

$$E_x = \eta^{-1}(\epsilon^{3/2}E_{x1} + \epsilon^{5/2}E_{x2} + \cdots), \tag{68}$$

$$v_{jy} = \eta^{-1}(\epsilon^{3/2}v_{jy1} + \epsilon^{5/2}v_{jy2} + \cdots), \tag{69}$$

$$v_{jz} = \eta^{-1}(\epsilon^{2}v_{jz1} + \epsilon^{3}v_{jz2} + \cdots). \tag{70}$$

We assume that

$$\epsilon \gg \eta. \tag{71}$$

Equations (68)–(71) enable us to obtain the KdV equation for the high-frequency mode for the region given by Eq. (61). We can also derive $E_\parallel$ in the nonlinear pulse for this region (see Appendix A Subsection 3).

From Eqs. (A42), we see that $\boldsymbol{E}_1 \cdot \boldsymbol{B}_0 = E_{x1}\cos\theta/\eta + E_{z1}\sin\theta = 0$ and $\boldsymbol{E}_1 \cdot \boldsymbol{B}_1 = E_{y1}B_{y1} + E_{z1}B_{z1} = 0$. By virtue of Eq. (A43), we find $E_\parallel$ as

$$\begin{aligned} E_\parallel &= \epsilon^{5/2}(E_{x2}\cos\theta/\eta + E_{z2}\sin\theta) \\ &= \epsilon^{5/2}\frac{B_0\Omega_{\rm e}c^3}{\omega_{\rm pe}^4}\cos\theta\sin\theta\frac{\partial^3}{\partial\xi^3}\left(\frac{B_{z1}}{B_0}\right). \end{aligned} \tag{72}$$

The parallel pseudopotential is

$$F = \epsilon^2\frac{B_0\Omega_{\rm e}c^3}{\omega_{\rm pe}^4}\sin\theta\frac{\partial^2}{\partial\xi^2}\left(\frac{B_{z1}}{B_0}\right). \tag{73}$$

Substituting the soliton solution into Eq. (73), we have the maximum value of F of the high-frequency-mode pulse as

$$eF_{\rm Mh} = \frac{m_e v_{\rm A}^2}{2\alpha_h\eta^2 d_h^2}\frac{c^2}{\omega_{\rm pe}^2}\sin\theta b_n^2. \tag{74}$$

Using Eq. (60), we rewrite Eq. (74) as

$$eF_{\rm Mh} = \frac{\rho_0 v_{\rm A}^2}{2\alpha_h n_{e0}}\left(1 - \frac{\cos^2\theta}{\eta^2}\right)^{-1} b_n^2, \tag{75}$$

where we have estimated that $\sin\theta \simeq 1$ as in Ref. 6.

Comparing Eqs. (1) and (75), we find that $eF_{\rm Mh}$ is slightly smaller than $eF_{\rm M}$ in the nonlinear pulse in the single-ion-species plasma because α_h is slightly greater than 1 as shown by Eq. (67). In the limit of $\Delta_\omega \to 0$, α_h becomes 1 and Eq. (75) reduces to Eq. (1).

We now compare the magnitude of F in the high-frequency-mode pulse, $F_{\rm Mh}$, and that in the low-frequency-mode pulse, $F_{\rm Ml1}$ or F_{Ml2}. From Eqs. (55), (56), and (75), we can estimate the ratio of $F_{\rm Mh}$ to $F_{{\rm Ml}j}$, where $j = 1$ or 2, as

$$\frac{F_{\rm Mh}}{F_{\rm lj}} \sim \frac{m_i}{m_e}\Delta_\omega\left(1 - \frac{\cos^2\theta}{\eta^2}\right)^{-1}, \tag{76}$$

where we have used $\cos\theta \ll 1$. Equation (76) shows that the magnitude of F of the high-frequency-mode pulse is much greater than that of the low-frequency-mode pulse when $\Delta_\omega \gg m_e/m_i$.

We discuss the reason why F in the high-frequency-mode pulse is much greater than F in the low-frequency-mode pulses. As shown by Eqs. (26), (31), and (60), there is a large difference between the characteristic widths of the high-frequency-mode pulse and of the low-frequency-mode pulses. For quasiperpendicular pulses such that $\cos\theta < \eta$, the characteristic width of the high-frequency-mode pulse, which can be estimated as $d \sim c/\omega_{\rm pe}$, is much smaller than that of the low-frequency-mode pulses, $d \sim (c/\omega_{\rm pi})/\Delta_\omega$. From Eqs. (46), (54), and (75), we find that if $\Omega_a \sim \Omega_b$, both F in the high-frequency-mode pulse and F in the low-frequency-mode pulses can be estimated as

$$eF \sim m_i v_{\rm A}^2 (c/\omega_{\rm pe})^2 b_n^2 d^{-2}, \tag{77}$$

indicating that F is proportional to d^{-2}. We also find that the strength of $E_\parallel$ is proportional to d^{-3}. Therefore, the huge difference between F in the high-frequency-mode pulse and F in the low-frequency-mode pulses can be caused by the large difference between the characteristic widths of these pulses.

IV. ELECTRON-POSITRON-ION PLASMA

A. $E_\parallel$ and F in pulses with $\epsilon \ll 1$

In this subsection, we present the theory for $E_\parallel$ and F in nonlinear pulses in an e-p-i plasma that was derived in Ref. 7 with the conventional reductive perturbation method in which $\epsilon \ll 1$ was assumed.

In the limit of $\omega \to 0$, the dispersion relation of the magnetosonic wave in an e-p-i plasma can be written as the same form as that in a two-ion-species plasma; $\omega/k = v_{\rm A}(1 + \mu k^2)$ with μ given by Eq. (22), where j refers to electrons, positrons, or ions (j = e, p, or i). Nonlinear behavior is described by the KdV equation, and the parallel electric field and the parallel pseudopotential in the nonlinear pulse can be written as

$$E_\parallel = \epsilon^{5/2}\frac{4\pi v_{\rm A}^4}{B_0^2\tan\theta}\left(\sum_j\frac{n_{j0}m_j^2}{q_j}\right)\left(\frac{c}{\omega_{\rm p}}\right)^2\frac{\partial^3}{\partial\xi^3}\left(\frac{B_{z1}}{B_0}\right), \tag{78}$$

$$F_B = -\epsilon^2 \frac{4\pi v_A^4}{B_0^2 \sin\theta}\left(\sum_j \frac{n_{j0} m_j^2}{q_j}\right)\left(\frac{c}{\omega_p}\right)^2 \frac{\partial^2}{\partial \xi^2}\left(\frac{B_{z1}}{B_0}\right), \quad (79)$$

in the cold plasma approximation. These equations are obtained from Eqs. (47) and (48) in Ref. 7 in the limit of $v_A/c \to 0$.

If $n_{i0} \gg (m_e/m_i)n_{e0}$, F_B can be approximated as

$$eF_B = -\epsilon^2 \frac{m_i v_A^2}{Z \sin\theta}\left(\frac{c}{\omega_p}\right)^2 \frac{\partial^2}{\partial \xi^2}\left(\frac{B_{z1}}{B_0}\right), \quad (80)$$

where $Z = q_i/e$. Substituting the soliton solution into this, one has the maximum value of F_B as

$$eF_{BM} \simeq \frac{m_i v_A^2}{4Z \sin\theta} \frac{(c/\omega_p)^2}{|\mu|} b_n^2. \quad (81)$$

For quasiperpendicular waves with $\theta > \theta_c$, where θ_c is the angle at which $\mu = 0$, we can approximate μ as

$$\mu \sim -\frac{v_A^2}{2\Omega_i^2} \frac{(n_{e0} + n_{p0})m_e}{\rho}. \quad (82)$$

Substituting this into Eq. (81), we can express the dependence of F on the positron-to-electron density ratio, n_{p0}/n_{e0}, as

$$F \propto \frac{(1 - n_{p0}/n_{e0})}{(1 + n_{p0}/n_{e0})\left[1 + n_{p0}/n_{e0} + (1 - n_{p0}/n_{e0})m_e/m_i\right]}, \quad (83)$$

where we have used $\rho \simeq n_{i0} m_i = (n_{e0} - n_{p0})m_i/Z$. It indicates that F decreases with increasing n_{p0}/n_{e0} and F increases with increasing n_{i0}/n_{e0}.

These equations were derived under the assumption that $\epsilon \ll 1$.

B. Nonlinear pulses with ϵ in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$

In this subsection, we discuss pulses with amplitudes in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$. We first derive KdV equation for this range. We then obtain the theoretical expressions for $E_\parallel$ and F in nonlinear pulses with such amplitudes, which were not shown in Ref. 7.

1. Linear dispersion relation

Before deriving KdV equation, we show the linear dispersion relation. With the assumption that $(m_e/m_i)^{1/2} < \epsilon < 1$, we focus on the frequency region $\Omega_i < \omega < \sqrt{|\Omega_e|\Omega_i}$, which will be explained below. For this region, the linear dispersion relation can be written as Eq. (16), where v_p and d are given by

$$v_p = \frac{c\Omega_i}{\omega_{pi}} \sin\theta, \quad (84)$$

$$d^2 = d_{epi}^2 = c^2 \frac{\omega_{pep}^2}{\omega_{pi}^4} \frac{\Omega_i^2}{\Omega_e^2}\left(1 - \frac{\omega_{pi}^2}{\omega_{pep}^2} \frac{\Omega_e^2}{\Omega_i^2} \frac{\cos^2\theta}{\sin^2\theta}\right). \quad (85)$$

From this, we can expect that the KdV equation with the characteristic length d_{epi} is obtained for $\Omega_i < \omega < \sqrt{|\Omega_e|\Omega_i}$. Equation (20) shows that the width of the solitary pulse is $D \sim d_{epi}\epsilon^{-1/2}$. When $\theta \gg \theta_c$, where θ_c is the critical angle at which d becomes zero, d is of the order of c/ω_{pe}. Then, we can estimate the wavenumber and frequency of the nonlinear pulse as $k \sim 1/D \sim \epsilon^{1/2}(\omega_{pe}/c)$ and $\omega \sim v_p k \sim \epsilon^{1/2}(m_i/m_e)^{1/2}\Omega_i$. This indicates that the assumption $(m_e/m_i)^{1/2} < \epsilon < 1$ corresponds to $\Omega_i < \omega < \sqrt{\Omega_i|\Omega_e|}$.

2. Derivation of KdV equation

We now derive the KdV equation using the assumption that $\eta < \epsilon < 1$, where η is defined as

$$\eta = (m_e/m_i)^{1/2}. \quad (86)$$

We also assume that

$$\cos\theta \sim O(\eta), \quad \sin\theta \simeq 1, \quad n_{i0} \gg (m_e/m_i)n_{e0}. \quad (87)$$

We expand E_x, v_{jy}, and v_{jz} as Eqs. (68)–(70); the expansion of the other quantities are the same as in Eqs. (32)–(40).

From the lowest order terms of the normalized equations, we obtain the lowest order quantities of density and velocities as

$$n_{j1} = n_{j0} B_{z1}/\sin\theta, \quad (88)$$

where j = e, i, or p,

$$v_{ex1} = v_{px1} = \frac{B_{z1}}{\sin\theta}, \quad v_{ey1} = -v_{py1} = \frac{\eta}{S q_i n_{i0}} \frac{\partial B_{z1}}{\partial \xi},$$
$$v_{ez1} = -v_{pz1} = \frac{\eta\cos\theta}{S^2 q_i n_{i0} e(n_{e0} + n_{p0})} \frac{\partial^2 B_{z1}}{\partial \xi^2},$$
$$v_{ix1} = \frac{\eta^2 \sin\theta}{R_i S q_i n_{i0}} \frac{\partial B_{z1}}{\partial \xi}, \quad v_{iy1} = v_{iz1} = 0, \quad (89)$$

where S and R_i are given by

$$S = \frac{\omega_{pe}^2 d v_p}{|\Omega_e| c^2}, \quad R_i = \frac{\Omega_i d}{v_p}. \quad (90)$$

These velocities are different from the velocities obtained under the assumption that $\epsilon \ll 1$. The lowest order terms of electric and magnetic fields are

$$E_{x1} = -\frac{\eta \sin\theta}{S q_i n_{i0}} \frac{\partial B_{z1}}{\partial \xi}, \quad E_{y1} = B_{z1},$$
$$E_{z1} = -B_{y1} = \frac{\cos\theta}{S q_i n_{i0}} \frac{\partial B_{z1}}{\partial \xi}. \quad (91)$$

After some manipulations (for the details of calculations, see Appendix B), we then obtain the KdV equation (17) for the frequency region $\Omega_i \ll \omega \ll \sqrt{\Omega_i \Omega_e}$ with v_p and d given by Eqs. (84) and (85), respectively, and the coefficient α,

$$\alpha = 1/\sin\theta. \quad (92)$$

3. $E_\parallel$ and F in nonlinear pulses

We derive $E_\parallel$ in the nonlinear pulses with amplitudes $(m_e/m_i)^{1/2} < \epsilon < 1$. Equation (91) leads to $\boldsymbol{E}_1 \cdot \boldsymbol{B}_0 = E_{x1}\cos\theta/\eta + E_{z1}\sin\theta = 0$ and $\boldsymbol{E}_1 \cdot \boldsymbol{B}_1 = E_{y1}B_{y1} + E_{z1}B_{z1} = 0$. Taking E_{x2} and E_{z2} into account, we obtain $E_\parallel$, from Eq. (B15), as

$$E_\parallel = \epsilon^{5/2}(E_{x2}\cos\theta/\eta + E_{z2}\sin\theta), \\ = \epsilon^{5/2}\frac{c^3\Omega_i \sin\theta\cos\theta}{\omega_{pi}^2\omega_{\rm pep}^2}B\frac{\partial^3}{\partial\xi^3}\left(\frac{B_{z1}}{B_0}\right). \tag{93}$$

We then have the parallel pseudopotential,

$$F = -\epsilon^2\frac{c^3\Omega_i\sin\theta}{\omega_{pi}^2\omega_{\rm pep}^2}B\frac{\partial^2}{\partial\xi^2}\left(\frac{B_{z1}}{B_0}\right), \tag{94}$$

which is rewritten as

$$eF = -\epsilon^2\frac{m_i v_{\rm A}^2}{Zn_{i0}}\left(\frac{c}{\omega_{\rm pep}}\right)^2\frac{\partial^2}{\partial\xi^2}\left(\frac{B_{z1}}{B_0}\right), \tag{95}$$

where we have used the approximation (87).

Substituting the soliton solution into this, we obtain the maximum value of F as

$$eF_{\rm M} = \frac{m_i v_{\rm A}^2}{2Zn_{i0}}\frac{(c/\omega_{\rm pep})^2}{d_{epi}^2}b_n^2. \tag{96}$$

This is written as

$$eF_{\rm M} = \frac{m_i v_{\rm A}^2}{2}\frac{n_{i0}Z}{(n_{e0}+n_{p0})^2}\left(1 - \frac{n_{i0}m_i}{(n_{e0}+n_{p0})m_e}\cos^2\theta\right)^{-1}b_n^2, \tag{97}$$

indicating that $eF_{\rm M}$ increases with increasing n_{i0}/n_{e0} and decreasing n_{p0}/n_{e0}. Equations (80) and (81) for $\epsilon \ll 1$, which were derived without the assumption that $\sin\theta \simeq 1$ and $m_e/m_i \simeq 0$, become Eqs. (95) and (96), respectively, in the limit of $\sin\theta = 1$ and $m_e/m_i = 0$.

V. SUMMARY

We theoretically studied the parallel electric field in nonlinear magnetosonic waves in three-component plasmas (two-ion-species plasma and e-p-i plasma) based on a three-fluid model. We derived the expressions for $E_\parallel$ and its integral along the magnetic field, $F = -\int E_\parallel ds$, in solitary pulses propagating quasiperpendicular to the magnetic field.

In a two-ion-species plasma, the magnetosonic wave has two branches: high-frequency mode and low-frequency mode. We derived $E_\parallel$ and F in the nonlinear pulses of the two modes as functions of the wave amplitude ϵ, the propagation angle θ, and the density ratio and ion cyclotron frequency ratio of the two species ions.

For the low-frequency mode, there are two pulses: longer-wavelength pulse and shorter-wavelength pulse. The theory showed that the magnitude of F in the two low-frequency-mode pulses can be estimated as $eF \sim \epsilon^2 m_e v_{\rm A}^2/\Delta_\omega$, where Δ_ω is the normalized frequency difference. For the high-frequency mode, we showed that F in the nonlinear pulse can be estimated as $eF \sim \epsilon^2 m_i v_{\rm A}^2$, which is the same order as F in a pulse in a single-ion-species plasma. Thus, it was found that F in high-frequency-mode pulse is much greater than F in the low-frequency-mode pulses when $\Delta_\omega \gg m_e/m_i$.

For an e-p-i plasma, we considered nonlinear pulses with amplitudes in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$, which were not discussed in Ref. 7. The KdV equation and $E_\parallel$ and F in the nonlinear pulses were obtained. It was shown that F in the pulse with ϵ in the range of $(m_e/m_i)^{1/2} < \epsilon < 1$ can be written as the same form as that in the range of $\epsilon \ll 1$. The magnitude of F increases as the ion density increases.

As for a future work, effects of $E_\parallel$ on particle motions and pulse propagation in a two-ion-species plasma should be analyzed. The strong $E_\parallel$ in the high-frequency-mode pulse may cause the damping of the pulse through the Landau resonance. This may be important, as well as the damping due to heavy ion acceleration through the transverse electric field in the pulse.[22]

ACKNOWLEDGMENTS

This work was conducted as a part of the collaboration program under Grant No. NIFS16KNSS077 of the National Institute for Fusion Science and was supported in part by a Grant-in-Aid for Scientific Research (C), Grant No. 15K05367, from the Japan Society for the Promotion of Science.

APPENDIX A: DERIVATIONS OF $E_\parallel$ FOR A TWO-ION-SPECIES PLASMA

We normalize the length, velocity, and time to λ, $v_{\rm p}$, and λ/v_p, respectively, where λ is a characteristic length. The number density, mass, charge, magnetic field, and electric field are normalized to $n_{\rm e0}$, m_e, e, B_0, and v_pB_0/c, respectively. The normalized basic equations are as follows:

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\boldsymbol{v}_j) = 0, \tag{A1}$$

$$\left(\frac{\partial}{\partial t} + (\boldsymbol{v}_j\cdot\nabla)\right)\boldsymbol{v}_j = R_j(\boldsymbol{E} + \boldsymbol{v}_j\times\boldsymbol{B}), \tag{A2}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times\boldsymbol{E}, \tag{A3}$$

$$\frac{\partial B_z}{\partial x} = -S\sum_j q_j n_j v_{jy}, \tag{A4}$$

$$\sum q_j n_j = 0, \tag{A5}$$

where R_j and S are

$$R_j = \frac{\Omega_j\lambda}{v_p}, \quad S = \frac{\omega_{\rm pe}^2\lambda v_p}{|\Omega_{\rm e}|c^2}. \tag{A6}$$

We note that R_j and S depend on λ. This indicates that the expansion of quantities for the high-frequency mode is different from that for the low-frequency mode because the high- and low-frequency modes have different λs.

1. Low-frequency mode: Longer-wavelength pulse

Introducing stretched coordinates given by Eq. (18) and applying expansion given by Eqs. (32)–(40) to the above normalized equations (A1)–(A5), we have the following set of equations.

The continuity equation is

$$\epsilon^{3/2}\left(-\frac{\partial n_{j1}}{\partial \xi}+n_{j0}\frac{\partial v_{jx1}}{\partial \xi}\right) + \epsilon^{5/2}\left(-\frac{\partial n_{j2}}{\partial \xi}+\frac{\partial n_{j1}}{\partial \tau}+\frac{\partial}{\partial \xi}(n_{j1}v_{jx1})+n_{j0}\frac{\partial v_{jx2}}{\partial \xi}\right)+\cdots=0. \tag{A7}$$

The x, y, and z components of the equations of motion for ions and electrons are

$$\epsilon^{3/2}\left(\frac{\partial v_{jx1}}{\partial \xi}+R_j(E_{x1}+v_{jy1}\sin\theta)\right)+\epsilon^{5/2}\left[\frac{\partial v_{jx2}}{\partial \xi}-\frac{\partial v_{jx1}}{\partial \tau} -v_{jx1}\frac{\partial v_{jx1}}{\partial \xi}+R_j(E_{x2}+v_{jy2}\sin\theta+v_{jy1}B_{z1}-v_{jz1}B_{y1})\right]+\cdots=0, \tag{A8}$$

$$\epsilon R_j(E_{y1}+v_{jz1}\cos\theta-v_{jx1}\sin\theta) +\epsilon^2\left(\frac{\partial v_{jy1}}{\partial \xi}+R_j(E_{y2}+v_{jz2}\cos\theta-v_{jx2}\sin\theta-v_{jx1}B_{z1})\right)+\cdots=0, \tag{A9}$$

$$\epsilon^{3/2}\left(\frac{\partial v_{jz1}}{\partial \xi}+R_j(E_{z1}-v_{jy1}\cos\theta)\right)+\epsilon^{5/2}\left[\frac{\partial v_{jz2}}{\partial \xi}-\frac{\partial v_{jz1}}{\partial \tau} -v_{jx1}\frac{\partial v_{jz1}}{\partial \xi}+R_j(E_{z2}-v_{jy2}\cos\theta+v_{jx1}B_{y1})\right]+\cdots=0. \tag{A10}$$

The y and z components of Faraday's law are

$$\epsilon^{3/2}\left(-\frac{\partial B_{z1}}{\partial \xi}+\frac{\partial E_{y1}}{\partial \xi}\right)+\epsilon^{5/2}\left(-\frac{\partial B_{z2}}{\partial \xi}+\frac{\partial B_{z1}}{\partial \tau}+\frac{\partial E_{y2}}{\partial \xi}\right)+\cdots=0, \tag{A11}$$

$$\epsilon^{2}\left(-\frac{\partial B_{y1}}{\partial \xi}-\frac{\partial E_{z1}}{\partial \xi}\right)+\epsilon^{3}\left(-\frac{\partial B_{y2}}{\partial \xi}+\frac{\partial B_{y1}}{\partial \tau}-\frac{\partial E_{z2}}{\partial \xi}\right)+\cdots=0. \tag{A12}$$

The x, y, and z components of Ampere's law are

$$\epsilon\sum_j(n_{j0}q_jv_{jx1})+\epsilon^2\sum_j q_j(n_{j1}v_{jx1}+n_{j0}v_{jx2})\cdots=0, \tag{A13}$$

$$\epsilon^{3/2}\left(\frac{\partial B_{z1}}{\partial \xi}+S\sum_j q_jn_{j0}v_{jy1}\right) +\epsilon^{5/2}\left(\frac{\partial B_{z2}}{\partial \xi}+S\sum_j q_j(n_{j1}v_{jy1}+n_{j0}v_{jy2})\right)\cdots=0, \tag{A14}$$

$$\epsilon\textstyle\sum_j(n_{j0}q_jv_{jz1})+\epsilon^2\left(\frac{\partial B_{y1}}{\partial \xi}-S\sum_j q_j(n_{j1}v_{jz1}+n_{j0}v_{jz2})\right) +\epsilon^{5/2}\left(\frac{\partial B_{y2}}{\partial \xi}-S\sum_j q_j(n_{j0}v_{jz2}+n_{j1}v_{jz1})\right)\cdots=0. \tag{A15}$$

The charge neutrality is

$$\epsilon\sum_j q_jn_{j1}+\epsilon^2\sum_j q_jn_{j2}+\cdots=0. \tag{A16}$$

From the lowest-order terms in Eqs. (A7), (A11), and (A12), we obtain the following relations among the lowest-order quantities:

$$n_{j1}=n_{j0}v_{jx1},\quad E_{y1}=B_{z1},\quad E_{z1}=-B_{y1}. \tag{A17}$$

We multiply the $O(\epsilon^{3/2})$ terms in Eqs. (A8) and (A10) by $q_jn_{j0}\cos\theta$ and $q_jn_{j0}\sin\theta$, respectively, and sum up them. Further, we take its summation over the particle species j. Then, by virtue of the $O(\epsilon)$ terms in Eqs. (A13) and (A15), we find

$$E_{x1}\cos\theta+E_{z1}\sin\theta=0. \tag{A18}$$

With the aid of Eqs. (A17) and (A18), we have, from the lowest order terms in Eqs. (A8)–(A10),

$$v_{jx1}=B_{z1}\sin\theta,\quad v_{jz1}=-B_{z1}\cos\theta,\quad v_{jy1}=-\frac{1}{R_j}\frac{\partial B_{z1}}{\partial \xi}-\frac{B_{y1}}{\cos\theta}. \tag{A19}$$

We now consider the second order terms of the normalized equations. From Eqs. (A13) and (A15), we have

$$\sum_j n_{j0}q_jv_{jx2}=0, \tag{A20}$$

$$\frac{\partial B_{y1}}{\partial \xi}=S\sum_j n_{j0}q_jv_{jz2}, \tag{A21}$$

where we have used Eqs. (A17) and (A19). Multiplying the $O(\epsilon^2)$ term in Eq. (A9) by $n_{j0}q_j/R_j$ and taking the summation over j, we have

$$\sum_j\frac{n_{j0}q_j}{R_j}\frac{\partial v_{jy1}}{\partial \xi}=-\sum_j n_{j0}q_jv_{jz2}\cos\theta=-\frac{\cos\theta}{S}\frac{\partial B_{y1}}{\partial \xi}, \tag{A22}$$

where we have used Eqs. (A20) and (A21). Substituting v_{jy1} given by Eq. (A19) into Eq. (A22), we obtain B_{y1} as

$$B_{y1}=-\frac{v_p^3\cos\theta}{c^2\lambda\left(v_p^2/v_A^2-\cos^2\theta\right)}\sum_j\frac{\omega_{pj}^2}{\Omega_j^3}\frac{\partial B_{z1}}{\partial \xi}. \tag{A23}$$

In order to obtain $E_{x2}\cos\theta+E_{z2}\sin\theta$, we multiply the $O(\epsilon^{5/2})$ terms in Eqs. (A8) and (A10) by $n_{j0}q_j\cos\theta$ and $n_{j0}q_j\sin\theta$, respectively, and sum up them. Taking its summation over j, we have

$$\sum_j q_j n_{j0} \frac{\partial v_{jz2}}{\partial \xi} \sin\theta + \sum_j q_j n_{j0} R_j (E_{x2}\cos\theta + E_{z2}\sin\theta) + \sum_j q_j n_{j0} R_j \left[(v_{jy1}B_{z1} - v_{jz1}B_{y1})\cos\theta + v_{jx1}B_{y1}\sin\theta \right] = 0, \tag{A24}$$

where we have used $v_{jx1}\cos\theta + v_{jz1}\sin\theta = 0$, which is obtained from Eq. (A19). We also find that the third term of the left hand side of Eq. (A24) becomes zero, with the aid of Eq. (A19), as follows:

$$\begin{aligned}
&\sum_j q_j n_{j0} \left[R_j (v_{jy1}B_{z1} - v_{jz1}B_{y1})\cos\theta + R_j (v_{jx1}B_{y1})\sin\theta \right] \\
&= \sum_j q_j n_{j0} \left[\left(\frac{\partial B_{z1}}{\partial \xi} - R_j \frac{B_{y1}}{\cos\theta} \right) B_{z1}\cos\theta + R_j B_{z1} B_{y1}\cos^2\theta + R_j (B_{z1}B_{y1})\sin^2\theta \right] \\
&= \sum_j q_j n_{j0} \frac{\partial B_{z1}}{\partial \xi} B_{z1}\cos\theta = 0.
\end{aligned} \tag{A25}$$

We then have

$$\sum_j q_j n_{j0} R_j (E_{x2}\cos\theta + E_{z2}\sin\theta) = -\frac{\sin\theta}{S} \frac{\partial^2 B_{y1}}{\partial \xi^2}. \tag{A26}$$

Substituting Eq. (A23) into this and using the unnormalized quantities, we obtain Eq. (44).

2. Low-frequency-mode: Shorter-wavelength pulse

For the shorter-wavelength pulse, we expand v_{jz}, B_y, and E_z as Eqs. (48)–(50). Then, the equations of motions are written as

$$\epsilon^{3/2}\left(\frac{\partial v_{jx1}}{\partial \xi} + R_j \left(E_{x1} + v_{jy1}\sin\theta \right) \right) + \epsilon^{5/2}\left[\frac{\partial v_{jx2}}{\partial \xi} - \frac{\partial v_{jx1}}{\partial \tau} - v_{jx1}\frac{\partial v_{jx1}}{\partial \xi} + R_j \left(E_{x2} + v_{jy2}\sin\theta + v_{jy1}B_{z1} \right) \right] - \epsilon^{5/2}\cos^2\theta v_{jz1}B_{y1} + \cdots = 0, \tag{A27}$$

$$\epsilon R_j \left(E_{y1} - v_{jx1}\sin\theta \right) + \epsilon^2 \left(\frac{\partial v_{jy1}}{\partial \xi} + R_j \left(E_{y2} - v_{jx2}\sin\theta - v_{jx1}B_{z1} \right) \right) + \epsilon\cos^2\theta v_{jz1} + \epsilon^2\cos^2\theta v_{jz2} + \cdots = 0, \tag{A28}$$

$$\epsilon^{3/2}\cos\theta\left(\frac{\partial v_{jz1}}{\partial \xi} + R_j (E_{z1} - v_{jy1}) \right) + \epsilon^{5/2}\cos\theta\left[\frac{\partial v_{jz2}}{\partial \xi} - \frac{\partial v_{jz1}}{\partial \tau} - v_{jx1}\frac{\partial v_{jz1}}{\partial \xi} + R_j (E_{z2} - v_{jy2} + v_{jx1}B_{y1}) \right] + \cdots = 0, \tag{A29}$$

where we have assumed that $\cos^2\theta \ll \epsilon$. The other equations are the same as in Appendix A Subsection 1; Eqs. (A7), and (A11)–(A16) are also used for the shorter-wavelength pulse.

From the lowest order terms of the equations, we have the following relations among the lowest order quantities:

$$\begin{aligned}
&n_{j1} = n_{j0} v_{jx1}, \quad v_{ex1} = v_{ix1} = B_{z1}/\sin\theta, \\
&v_{jy1} = -\frac{1}{R_j \sin\theta}\left(1 - \sum_j \frac{n_{j0} m_j}{\rho_0} \right) \frac{\partial B_{z1}}{\partial \xi}, \quad v_{jz1} = -\frac{B_{z1}}{\sin^2\theta}, \\
&E_{x1} = -\frac{1}{\rho_0 \sin\theta} \sum_j \frac{n_{j0} m_j}{R_j} \frac{\partial B_{z1}}{\partial \xi}, \quad E_{y1} = B_{z1}, \quad E_{z1} = -B_{y1} = -\frac{E_{x1}}{\sin\theta}.
\end{aligned} \tag{A30}$$

From the $O(\epsilon^{5/2})$ terms in Eqs. (A27) and (A29), we obtain

$$\cos\theta\sin\theta \sum_j q_j n_{j0} \frac{\partial v_{jz2}}{\partial \xi} + \sum_j q_j n_{j0} R_j (E_{x2}\cos\theta + E_{z2}\cos\theta\sin\theta) = 0, \tag{A31}$$

where we have used Eqs. (A20) and (A30). By virtue of the $O(\epsilon^2)$ term in Eq. (A15), Eq. (A31) can be written as

$$E_{x2}\cos\theta + E_{z2}\cos\theta\sin\theta = \frac{1}{\sum_j q_j n_{j0} R_j} \frac{\cos\theta\sin\theta}{S\rho_0} \sum_j \frac{n_{j0} m_j}{R_j} \frac{\partial^3 B_{z1}}{\partial \xi^3}. \tag{A32}$$

Using the unnormalized quantities, we can write this as Eq. (52).

3. High-frequency mode

For the high-frequency mode, we expand E_x, v_{jy}, and v_{jz} as Eqs. (68)–(70). Then, the x, y, and z components of equation of motion for electrons are written as

$$R_e\eta^{-1}\epsilon^{3/2}(E_{x1}+v_{ey1}\sin\theta)+R_e\eta^{-1}\epsilon^{5/2}(E_{x2}+v_{ey2}\sin\theta+v_{ey1}B_{z1})+\cdots=0, \tag{A33}$$

$$R_e\epsilon\left(E_{y1}-v_{ex1}\sin\theta\right)\;+\;\eta^{-1}\epsilon^2\left(\frac{\partial v_{ey1}}{\partial\xi}+R_e\eta\left(E_{y2}+\frac{\cos\theta}{\eta}v_{ez1}-v_{ex2}\sin\theta-v_{ex1}B_{z1}\right)\right)+\cdots=0, \tag{A34}$$

$$R_e\epsilon^{3/2}\left(E_{z1}-\frac{\cos\theta}{\eta}v_{ey1}\right)+\epsilon^{5/2}\left[\eta^{-1}\frac{\partial v_{ez1}}{\partial\xi}+R_e\left(E_{z2}-v_{ey2}\frac{\cos\theta}{\eta}+v_{ex1}B_{y1}\right)\right]=0. \tag{A35}$$

Here, R_e is of the order of η^{-1}. The x, y, and z components of equation of motion for ions are

$$\epsilon^{3/2}\left(\frac{\partial v_{ix1}}{\partial\xi}+R_i\eta^{-1}\left(E_{x1}+v_{iy1}\sin\theta\right)\right)+\epsilon^{5/2}\left[\frac{\partial v_{ix2}}{\partial\xi}-\frac{\partial v_{jx1}}{\partial\tau}-v_{ix1}\frac{\partial v_{ix1}}{\partial\xi}+R_i\eta^{-1}\left(E_{x2}+v_{iy2}\sin\theta+v_{iy1}B_{z1}\right)\right]+\cdots=0, \tag{A36}$$

$$\eta^{-1}\epsilon^2\frac{\partial v_{iy1}}{\partial\xi}+\eta^{-1}\epsilon^3\left(\frac{\partial v_{iy2}}{\partial\xi}-\frac{\partial v_{iy1}}{\partial\tau}-v_{ix1}\frac{\partial v_{iy1}}{\partial\xi}\right)+\epsilon R_i\left(E_{y1}-v_{jx1}\sin\theta\right)+\cdots=0, \tag{A37}$$

$$\begin{aligned}&\eta^{-1}\epsilon^{5/2}\left(\frac{\partial v_{iz1}}{\partial\xi}\right)+\eta^{-1}\epsilon^{7/2}\left(\frac{\partial v_{iz2}}{\partial\xi}-\frac{\partial v_{iz1}}{\partial\tau}-v_{ix1}\frac{\partial v_{iz1}}{\partial\xi}\right)\\&\quad+\epsilon^{3/2}R_i\left(E_{z1}-v_{iy1}\frac{\cos\theta}{\eta}\right)+\epsilon^{5/2}R_i\left(E_{z2}+v_{ix1}B_{y1}-v_{iy2}\frac{\cos\theta}{\eta}\right)+\cdots=0.\end{aligned} \tag{A38}$$

Here, R_i is of the order of η. The x, y, and z components of Ampere's law are

$$\epsilon\sum_j(n_{j0}q_jv_{jx1})+\epsilon^2\sum_j q_j(n_{j1}v_{jx1}+n_{j0}v_{jx2})\cdots=0, \tag{A39}$$

$$\epsilon^{3/2}\left(\frac{\partial B_{z1}}{\partial\xi}+S\eta^{-1}\sum_j q_jn_{j0}v_{jy1}\right)+\epsilon^{5/2}\left(\frac{\partial B_{z2}}{\partial\xi}+S\eta^{-1}\sum_j q_j(n_{j1}v_{jy1}+n_{j0}v_{jy2})\right)\cdots=0, \tag{A40}$$

$$\epsilon^2\left(\frac{\partial B_{y1}}{\partial\xi}-S\eta^{-1}\sum_j q_j(n_{j0}v_{jz1})\right)+\epsilon^{5/2}\left(\frac{\partial B_{y2}}{\partial\xi}-S\eta^{-1}\sum_j q_j(n_{j0}v_{jz2}+n_{j1}v_{jz1})\right)\cdots=0, \tag{A41}$$

where $S\eta^{-1}$ is of the order of unity. The continuity equation and the Faraday's law are written as the same as those for the low-frequency mode.

We assume that $\epsilon\ll\eta$. Then, from the lowest order terms, we can express the lowest order quantities in terms of B_{z1} as

$$\begin{gathered}n_{e1}=n_{e0}v_{ex1},\quad v_{ex1}=B_{z1}/\sin\theta,\\ v_{ey1}=\frac{1}{S\eta^{-1}}\frac{\partial B_{z1}}{\partial\xi},\quad v_{ez1}=\frac{\cos\theta}{S^2\eta^{-1}}\frac{\partial^2B_{z1}}{\partial\xi^2},\\ n_{i1}=n_{i0}v_{ix1},\quad v_{ix1}=R_iB_{z1}/S,\quad v_{iy1}=v_{iz1}=0,\\ E_{x1}=-\frac{\sin\theta}{S\eta^{-1}}\frac{\partial B_{z1}}{\partial\xi},\quad E_{y1}=B_{z1},\\ E_{z1}=-B_{y1}=\frac{\cos\theta}{S}\frac{\partial B_{z1}}{\partial\xi}.\end{gathered} \tag{A42}$$

From the second order terms of Eqs. (A33) and (A35), we have

$$E_{\parallel}=E_{x2}\frac{\cos\theta}{\eta}+E_{z2}\sin\theta=-\frac{\sin\theta}{R_e\eta}\frac{\partial v_{ez1}}{\partial\xi}=-\frac{\cos\theta\sin\theta}{R_eS^2}\frac{\partial^3B_{z1}}{\partial\xi^3}, \tag{A43}$$

which leads to Eq. (72) with unnormalized quantities.

APPENDIX B: DERIVATION OF NONLINEAR EQUATION AND $E_\parallel$ FOR AN E-P-I PLASMA

For nonlinear pulses with amplitudes in the range of $\eta < \epsilon < 1$ in an e-p-i plasma, we have E_x, v_{jy}, and v_{jz} as Eqs. (68)–(70). When $n_{i0} \gg (m_e/m_i)n_{e0}$, the coefficients of the normalized equations can be estimated as

$$R_e \sim R_p \sim \eta^{-1}, \quad R_i \sim \eta. \tag{B1}$$

Therefore, the equations of motion for electrons and positrons can be written as Eqs. (A33)–(A35); for positrons, the subscript e in these equations is replaced by the subscript p. The equations of motion for ions are given by Eqs. (A36)–(A38). The continuity equation is given by Eq. (A7). The Faraday's and Ampere's laws are written as Eqs. (A11) and (A12) and (A39)–(A41), respectively.

From the lowest order terms, we obtain Eqs. (88)–(91). From the $O(\eta^{-1}\epsilon^3)$ term in Eq. (A37) and the $O(\eta^{-1}\epsilon^{7/2})$ term in Eq. (A38), we have

$$v_{iy2} = v_{iz2} = 0. \tag{B2}$$

We multiply the $O(\epsilon^{5/2})$ terms in (A33) for electrons and positrons by $-en_{e0}$ and en_{p0}, respectively, and sum up them. Then, we have

$$-q_i n_{i0} E_{x2} - e(n_{e0}v_{ey2} - n_{p0}v_{py2})\sin\theta - \frac{1}{2S\eta^{-1}}\frac{\partial B_{z1}^2}{\partial \xi} = 0, \tag{B3}$$

where we have used $en_{e0} = en_{p0} + q_i n_{i0}$. We then multiply the $O(\eta^{-1}\epsilon^2)$ terms in (A34) for electrons and positrons by $-en_{e0}/R_e\eta$ and $en_{p0}/R_p\eta$, respectively, and sum up them. We obtain

$$\begin{aligned}&-e(n_{e0} - n_{p0})E_{y2} - \frac{e}{\eta}\left(\frac{n_{e0}}{R_e}\frac{\partial v_{ey1}}{\partial \xi} - \frac{n_{p0}}{R_p}\frac{\partial v_{py1}}{\partial \xi}\right) - \frac{e\cos\theta}{\eta}(n_{e0}v_{ez1} - n_{p0}v_{pz1})\\ &+e(n_{e0}v_{ex2} - n_{p0}v_{px2})\sin\theta + e(n_{e0}v_{ex1} - n_{p0}v_{px1})B_{z1} = 0.\end{aligned} \tag{B4}$$

From the $O(\epsilon^{5/2})$ terms in Eqs. (A11) and (A40), we have

$$\begin{aligned}\frac{\partial E_{y2}}{\partial \xi} &= \frac{\partial B_{z2}}{\partial \xi} - \frac{\partial B_{z1}}{\partial \tau}\\ &= S\eta^{-1}e(n_{e1}v_{ey1} - n_{p1}v_{py1}) + S\eta^{-1}e(n_{e0}v_{ey2} - n_{p0}v_{py2}) - \frac{\partial B_{z1}}{\partial \tau}.\end{aligned} \tag{B5}$$

We differentiate Eq. (B4) with respect to ξ. This is written, by virtue of Eq. (B5), as

$$\begin{aligned}&-\frac{e}{\eta}\left(\frac{n_{e0}}{R_e}\frac{\partial^2 v_{ey1}}{\partial \xi^2} - \frac{n_{p0}}{R_p}\frac{\partial^2 v_{py1}}{\partial \xi^2}\right) - \frac{e\cos\theta}{\eta}\left(n_{e0}\frac{\partial v_{ez1}}{\partial \xi} - n_{p0}\frac{\partial v_{pz1}}{\partial \xi}\right) + e\sin\theta\frac{\partial}{\partial \xi}(n_{e0}v_{ex2} - n_{p0}v_{px2})\\ &+e\frac{\partial}{\partial \xi}\left[(n_{e0}v_{ex1} - n_{p0}v_{px1})B_{z1}\right] - S\eta^{-1}q_i n_{i0} e(n_{e1}v_{ey1} - n_{p1}v_{py1})\\ &-S\eta^{-1}q_i n_{i0} e(n_{e0}v_{ey2} - n_{p0}v_{py2}) + q_i n_{i0}\frac{\partial B_{z1}}{\partial \tau} = 0.\end{aligned} \tag{B6}$$

With the aid of Eq. (B3), the term including v_{ex2}, v_{px2}, v_{ey2}, and v_{py2} in Eq. (B6) can be written as

$$\begin{aligned}&e\sin\theta\frac{\partial}{\partial \xi}(n_{e0}v_{ex2} - n_{p0}v_{px2}) - S\eta^{-1}q_i n_{i0} e(n_{e0}v_{ey2} - n_{p0}v_{py2})\\ &= e\sin\theta\frac{\partial}{\partial \xi}(n_{e0}v_{ex2} - n_{p0}v_{px2}) + \frac{S\eta^{-1}}{\sin\theta}q_i^2 n_{i0}^2 E_{x2} + \frac{q_i n_{i0}}{2\sin\theta}\frac{\partial B_{z1}^2}{\partial \xi}.\end{aligned} \tag{B7}$$

From the $O(\epsilon^{5/2})$ term in Eq. (A36) and the $O(\epsilon^2)$ term in Eq. (A16), we have

$$R_i\eta^{-1}E_{x2} = \frac{\partial v_{ix1}}{\partial \tau} + v_{ix1}\frac{\partial v_{ix1}}{\partial \xi} - \frac{e}{n_{i0}q_i}\frac{\partial}{\partial \xi}(n_{e0}v_{ex2} - n_{p0}v_{px2} + n_{e1}v_{ex1} - n_{p1}v_{px1}). \tag{B8}$$

This is rewritten as

$$S\eta^{-1}q_i^2 n_{i0}^2 E_{x2} = \frac{Sq_i^2 n_{i0}^2}{R_i}\left(\frac{\partial v_{ix1}}{\partial \tau} + v_{ix1}\frac{\partial v_{ix1}}{\partial \xi}\right) - \frac{Sq_i n_{i0} e}{R_i}\frac{\partial}{\partial \xi}(n_{e0}v_{ex2} - n_{p0}v_{px2} + n_{e1}v_{ex1} - n_{p1}v_{px1}). \tag{B9}$$

Using Eq. (B9), we eliminate E_{x2} in Eq. (B7). Then, we have

$$e\sin\theta\frac{\partial}{\partial\xi}(n_{e0}v_{ex2}-n_{p0}v_{px2})-S\eta^{-1}q_in_{i0}e(n_{e0}v_{ey2}-n_{p0}v_{py2})$$
$$=q_in_{i0}\sin\theta\left(\frac{\partial v_{ix1}}{\partial\tau}+v_{ix1}\frac{\partial v_{ix1}}{\partial\xi}\right)$$
$$-e\sin\theta\frac{\partial}{\partial\xi}(n_{e1}v_{ex1}-n_{p1}v_{px1})+\frac{q_in_{i0}}{2\sin\theta}\frac{\partial B_{z1}^2}{\partial\xi}. \quad \text{(B10)}$$

By virtue of this equation, Eq. (B6) can be written as

$$-\frac{e}{\eta}\left(\frac{n_{e0}}{R_e}\frac{\partial^2 v_{ey1}}{\partial\xi^2}-\frac{n_{p0}}{R_p}\frac{\partial^2 v_{py1}}{\partial\xi^2}\right)-\frac{e\cos\theta}{\eta}\left(n_{e0}\frac{\partial v_{ez1}}{\partial\xi}-n_{p0}\frac{\partial v_{pz1}}{\partial\xi}\right)$$
$$+e\frac{\partial}{\partial\xi}\left[(n_{e0}v_{ex1}-n_{p0}v_{px1})B_{z1}\right]-S\eta^{-1}q_in_{i0}e(n_{e1}v_{ey1}-n_{p1}v_{py1})$$
$$+\frac{q_in_{i0}}{2\sin\theta}\frac{\partial B_{z1}^2}{\partial\xi}+q_in_{i0}\sin\theta\left(\frac{\partial v_{ix1}}{\partial\tau}+v_{ix1}\frac{\partial v_{ix1}}{\partial\xi}\right)$$
$$-e\sin\theta\frac{\partial}{\partial\xi}(n_{e1}v_{ex1}-n_{p1}v_{px1})+q_in_{i0}\frac{\partial B_{z1}}{\partial\tau}=0. \quad \text{(B11)}$$

Substituting Eq. (89) into Eq. (B11), we obtain the KdV equation,

$$2q_in_{i0}\frac{\partial B_{z1}}{\partial\tau}+\frac{3q_in_{i0}}{\sin\theta}B_{z1}\frac{\partial B_{z1}}{\partial\xi}$$
$$+\frac{c^2}{\omega_{\rm pe}^2\lambda^2}\frac{\omega_{\rm pep}^2\Omega_i}{\omega_{\rm pi}^2|\Omega_{\rm e}|}\left(1-\frac{\omega_{pi}^2|\Omega_{\rm e}|^2}{\omega_{\rm pep}^2\Omega_i^2}\frac{\cos^2\theta}{\sin^2\theta}\right)\frac{\partial^3 B_{z1}}{\partial\xi^3}=0. \quad \text{(B12)}$$

With unnormalized quantities, this is rewritten as

$$\frac{\partial B_{z1}}{\partial\tau}+\frac{3v_p}{2\sin\theta B_0}B_{z1}\frac{\partial B_{z1}}{\partial\xi}$$
$$+v_pc^2\frac{\omega_{\rm pep}^2}{\omega_{pi}^4}\frac{\Omega_i^2}{\Omega_{\rm e}^2}\left(1-\frac{\omega_{pi}^2|\Omega_{\rm e}|^2}{\omega_{\rm pep}^2\Omega_i^2}\frac{\cos^2\theta}{\sin^2\theta}\right)\frac{\partial^3 B_{z1}}{\partial\xi^3}=0. \quad \text{(B13)}$$

We now derive the second order term of $E_{\parallel}$. We multiply the $O(\epsilon^{5/2})$ terms in Eqs. (A33) and (A35) by $\cos\theta/\eta$ and $\sin\theta$, respectively, and sum up them. Then, we have

$$E_{x2}\cos\theta\eta^{-1}+E_{z2}\sin\theta=-\frac{1}{R_e}\frac{\partial v_{ez1}}{\partial\xi}\sin\theta. \quad \text{(B14)}$$

Substituting Eq. (89) into this equation, we obtained

$$E_{x2}\cos\theta\eta^{-1}+E_{z2}\sin\theta=-\frac{\sin\theta\cos\theta}{R_eS^2q_in_{i0}e(n_{e0}+n_{p0})}\frac{\partial^3 B_{z1}}{\partial^3\xi}. \quad \text{(B15)}$$

The unnormalized form of this is Eq. (93).

[1]Y. Ohsawa, Phys. Rep. **536**, 147 (2014).
[2]S. Takahashi, H. Kawai, Y. Ohsawa, S. Usami, C. Chiu, and W. Horton, Phys. Plasmas **16**, 112308 (2009).
[3]N. Bessho and Y. Ohsawa, Phys. Plasmas **6**, 3076 (1999); **9**, 979 (2002).
[4]H. Hasegawa, S. Usami, and Y. Ohsawa, Phys. Plasmas **10**, 3455 (2003); H. Hasegawa, K. Kato, and Y. Ohsawa, *ibid.* **12**, 082306 (2005); T. Iwata, S. Takahashi, and Y. Ohsawa, *ibid.* **19**, 022302 (2012).
[5]H. Alfvén and C. G. Fälthammmer, *Cosmical Electrodynamics* (Clarendon, Oxford, 1963).
[6]S. Takahashi and Y. Ohsawa, Phys. Plasmas **14**, 112305 (2007).
[7]S. Takahashi, M. Sato, and Y. Ohsawa, Phys. Plasmas **15**, 082309 (2008).
[8]S. J. Buchsbaum, Phys. Fluids **3**, 418 (1960).
[9]D. G. Swanson, Phys. Rev. Lett. **36**, 316 (1976).
[10]J. Jacquinot, B. D. Mcvey, and J. E. Scharer, Phys. Rev. Lett. **39**, 88 (1977).
[11]A. B. Mikhailovskii and A. I. Smolyakov, JETP **61**, 109 (1985).
[12]U. Motschmann, K. Sauer, T. Roatsch, and J. F. Mckenzie, J. Geophys. Res. **96**, 13841, doi:10.1029/91JA00638 (1991).
[13]M. Toida and Y. Ohsawa, J. Phys. Soc. Jpn. **63**, 573 (1994).
[14]M. Toida, Y. Ohsawa, and T. Jyounouchi, Phys. Plasmas **2**, 3329 (1995).
[15]S. Boldyrev, Phys. Lett. A **204**, 386 (1995); Phys. Plasmas **5**, 1315 (1998).
[16]M. Toida, H. Higashino, and Y. Ohsawa, J. Phys. Soc. Jpn. **76**, 104502 (2007).
[17]M. Toida and Y. Kondo, Phys. Plasmas **18**, 062303 (2011).
[18]S. Irie and Y. Ohsawa, Phys. Plasmas **10**, 1253 (2003).
[19]H. Washimi and T. Taniuti, Phys. Rev. Lett. **17**, 996 (1966).
[20]T. Kakutani, H. Ono, T. Taniuti, and C. C. Wei, J. Phys. Soc. Jpn. **24**, 1159 (1968).
[21]T. Kakutani and H. Ono, J. Phys. Soc. Jpn. **26**, 1305 (1969).
[22]M. Toida and Y. Aota, Phys. Plasmas **20**, 082301 (2013).