§5. High-n Ballooning Modes in Helical Systems Allowing a Large Shafranov Shift

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In currentless helical systems such as heliotron/torsatrons, the global rotational transform $s$ of the vacuum magnetic field increases in the minor radius direction, i.e., the global shear $s \equiv d\ln s/d\ln \psi$ is positive ($2\pi \psi$ is the toroidal flux). Shafranov speculated by using the low-$\beta$ approximation that if the global shear is positive, high-$n$ ballooning modes would not become unstable when the Mercier modes are stable.\(^1\) The characteristics of the vacuum configuration, e.g., the global and local shear and the shape of the flux surfaces, however, can be significantly deformed as $\beta(= \text{the kinetic pressure/the magnetic pressure})$ increases in such a currentless system that allows an inherently large Shafranov shift. We examine the high-$n$ ballooning modes in such helical systems and show the physical mechanism of the high-$n$ ballooning modes occurring in the Mercier stable strong positive global shear region emphasizing how the local shear stabilization effects are weakened. Here is considered the situation where the external poloidal field is applied so that the plasma boundary is almost fixed, and are obtained three dimensional currentless equilibria by using the VMEC code.\(^2\)

In the currentless helical systems, the toroidal restoring force against the Shafranov shift is generated by the interaction of the Pfirsh-Schlüter current with the averaged poloidal field due to the helical coil system on the plasma flux surface. The averaged poloidal field is considerably large near the separatrix where $L$ and $M$ are the polarity and the toroidal pitch of the helical coils. Thus, Pfirsh-Schlüter current localizes away from the plasma boundary to take the toroidal force balance as is shown in Fig.1(C), which significantly affects the distribution of the locally enhanced poloidal field outside of the torus. The local shear is given by

$$\hat{s} = s + \tilde{s}, \quad \tilde{s} = \psi \frac{\partial}{\partial \eta} \left( \frac{g_{\varphi \varphi}}{g_{\theta \theta}} \right)$$

where field line coordinates $(\psi, \eta, \alpha)$ are used relating to the Boozer coordinates $(\psi, \theta, \zeta)$ as $\eta = \theta$ and $\alpha = \zeta - q\theta$. Due to the localization of the Pfirsh-Schlüter current, the turning surface appears where $g_{\varphi \varphi} = 0$ in Fig.1(B). The perpendicular wave number $|k_\perp|$ is given by

$$|k_\perp|^2 = \frac{B^2}{|\nabla \psi|^2} + \frac{|\nabla \psi|^2}{\psi^2} \left( \int \hat{s} d\eta \right)^2$$

$$= \frac{B^2}{|\nabla \psi|^2} + \frac{|\nabla \psi|^2}{\psi^2} \left( s\eta + \psi \frac{g_{\varphi \varphi}}{g_{\theta \theta}} \right)^2$$

which plays a stabilizing role due to the shear Alfvén field line bending. The appearance of the turning surface makes the local shear $\hat{s}$ weak in the strong positive global shear region, which destabilizes the high-$n$ ballooning modes.

Fig.1 (B) Equally spaced $(\psi, \theta)$ meshes on some poloidal cross sections for a helical configuration and (C) corresponding contours of Pfirsch-Schlüter current.

References