§4. Application of the CAS3D Code

—Ballooning Modes in Tokamaks—

Nakajima, N., Ichiguchi, K., Okamoto, M., Nührenberg, C. (Max Planck Inst.)
Nührenberg, J. (Max Planck Inst.)

The Finite-Element-Fourier (FE-Fourier) code package, CAS3D (Code for the Analysis of the MHD Stability of 3-D Equilibria), \(^1\) which is based on a formulation of ideal MHD energy principle in the Boozer coordinates and developed by Nührenberg, C., provides the computational tool that is necessary to study the global MHD stability of 3-D toroidal plasmas. Within the framework of linear ideal MHD the energy integral for the plasma potential energy \(W_p\) connected with the displacement \(\vec{\xi}\) can be given as

\[
W_p = \frac{1}{2} \int d\tau \left[ |\vec{C}|^2 - A(\vec{C} \cdot \nabla s)^2 + \gamma P(\nabla \cdot \vec{\xi})^2 \right]
\]

where \(s\) is the flux-surface label, and the destabilizing term \(A\) and the stabilizing term \(|\vec{C}|^2\) are expressed by

\[
A = \frac{2}{|\nabla s|^2} (\vec{J} \times \nabla s) \cdot (\vec{B} \cdot \nabla) \nabla s,
\]

\[
\vec{C} = \nabla \times (\vec{\xi} \times \vec{B}) + \frac{\vec{J} \times \nabla s}{|\nabla s|^2} \vec{\xi} \cdot \nabla s
\]

Since the fluid compressional contribution \(\nabla \cdot \vec{\xi}\) has the stabilizing effects, the incompressible condition \(\nabla \cdot \vec{\xi} = 0\) is used. Thus, only 2 scalar components, i.e., \(\xi^s\) and \(\eta^s\) of \(\vec{\xi}\) are solved. To treat perturbations with a high toroidal mode number, the phase transformation is done as follows:

\[
\begin{align*}
\xi^s &= X^s \cos[\theta + \phi] + X^s \sin[\theta + \phi], \\
\eta^s &= Y^s \sin[\theta + \phi] + Y^s \cos[\theta + \phi]
\end{align*}
\]

where \(M\) and \(N\) are the poloidal and toroidal mode number, which is considered as the target mode number of the perturbation we treat. From this phase transformation, perturbations with a high toroidal mode number \(N \gg 1\) are treated by not requiring much memory and CPU time.

Before applying the CAS3D code to heliotron/torsatron plasma so as to investigate low to high-\(n\) interchange and ballooning modes, it is applied to the ballooning calculation of a tokamak plasma as a benchmark test. The ballooning mode structure with \((M, N) = (7, -6)\) is expressed in Fig.1. And the ballooning mode structure with \((M, N) = (128, -96)\) is shown in Fig.2. Comparing these two figures, we can see the advantage of the CAS3D.

![Fig.1 Ballooning mode structure of a tokamak plasma. The curve labeled 1 corresponds to the mode with \((M, N) = (7, -6)\).](image1)

![Fig.2 Ballooning mode structure of the same tokamak plasma. The curve labeled 1 corresponds to the mode with \((M, N) = (128, -96)\).](image2)

References