§12. Equation Determining the Radial Electric Field in the Presence of the Fast Ions

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Since fast ions due to NBI have a large gyroradius and drift velocity, their particle flux across the flux surface has significant effects on the radial electric field, which is determined by the ambipolar condition. To evaluate these effects, Monte-Carlo simulation is usually used where the orbits of the fast ions and the Coulomb collisions with the background thermal species are calculated. Here, the equation determining the radial electric field in the presence of the fast ions is proposed. The main assumptions for the background thermal species are 1) the standard small gyroradius ordering, 2) the transport ordering, 3) the drift ordering, 4) no inductive electric field, and 5) the fast time variation of the radial electric field in comparison to the background density and pressure.

\[ \frac{\partial \ln \Phi(\psi)}{\partial t} = O(\nu_{ee}) \sim O(\nu_{ii}) \]

Finally, we treat effects of the fast ions on the thermal species as the external forces. Since the behavior of the fast ions is calculated by the Monte-Carlo simulation, the effects of the fast ions enter the final equation as the radial flux and the Coulomb interaction (friction) with the background thermal species in terms of the external force. Note that the inductive electric field is neglected. As a result of it, we have the following equation in the Boozer coordinate system \((\psi, \theta, \zeta)\):

\[ \epsilon_0 (1 + \epsilon_{rr}) \langle |\nabla \psi|^2 \rangle \frac{\partial}{\partial t} \left( \frac{d\Phi(\psi)}{d\psi} \right) = \epsilon_f \Gamma_f^\psi + \sum_a \epsilon_a \Gamma_a^\psi + \sum_a R_{af} \]

where the summation is taken only for the thermal species, and \(\Gamma_f^\psi\) and \(\Gamma_a^\psi\) are the radial fluxes of the fast ions and thermal species, respectively:

\[ \Gamma_f^\psi = \langle n_f \vec{u}_f \cdot \nabla \psi \rangle \]

\[ \Gamma_a^\psi = \frac{J + \epsilon I}{\epsilon_a \langle B^2 \rangle} \left( \vec{B}_T \cdot \nabla \cdot \vec{\Pi}_{a1} \right) - \frac{J}{\epsilon_a \langle B^2 \rangle} \left( \vec{B} \cdot \nabla \cdot \vec{\Pi}_{a1} \right) \sim \frac{J + \epsilon I}{\epsilon_a \langle B^2 \rangle} \left( \vec{B}_T \cdot \nabla \cdot \vec{\Pi}_{a1} \right) \]

Both are considered as the \(1/\nu\) ripple diffusion. Note that \(\Gamma_f^\psi\) includes the fast ion particle loss. The residual term \(\sum_a R_{af}\) is the Coulomb frictional flux of the fast ions with thermal species:

\[ R_{af} = \left( \vec{B} \times \nabla \psi \cdot \vec{F}_{af} \right) \frac{\langle B^2 \rangle}{B^2} - \frac{1}{\langle B^2 \rangle} \left( \left( \frac{\partial G}{\partial \theta} - I \frac{\partial G}{\partial \zeta} \right) \vec{B} \cdot \vec{F}_{af} \right) \]

Other quantities are defined as follows:

\[ \epsilon_{rr} = \left( \frac{\langle |\nabla \psi|^2 \rangle}{v_A^2} \right) + \frac{g_2 \left( \frac{\partial G}{\partial \theta} - I \frac{\partial G}{\partial \zeta} \right)}{\langle v_A^2 \rangle} \]

\[ v_A^2 = \frac{B^2}{\sum_a m_a n_a \mu_0} \]

The functions \(G\) and \(g_2\) are given as the solutions of the following equations:

\[ \vec{B} \cdot \nabla G = \frac{1}{\sqrt{g}} \left( \left( \frac{B^2}{B^2} \right) - 1 \right) \]

\[ \vec{B} \cdot \nabla \left( g_2 \frac{B^2}{B^2} \right) = \vec{B} \times \nabla \psi \cdot \nabla \left( \frac{1}{B^2} \right) \]

Both are determined only by the magnetic field configuration.

The 1st residual term comes from the gyrofriction with the fast ions. The 2nd residual term is usually small because the function \(G\) has only oscillatory components. If both residual terms are neglected, then the equation determining the temporal development of the radial electric field reduces to the usual form except for the geometric effect \(\epsilon_{rr}\).