Radiation of hybrid mode from a circular corrugated (CC) waveguide are closely related with the coupling of hybrid mode with Gauss-Laguerre beam at the mouth. By using the mode matching method, we analyze propagation of the multi-Gauss-Laguerre beams equivalent to the HE_{m} mode. We show evidence from the numerical calculations that radiation from two-, three- and multi-Gauss-Laguerre beams gives satisfactory approximation in both near- and far-field regions. The radiation pattern from multi-beam Gauss-quadrature integration with a relative uncertainty of $10^{-9}$ is adopted. In two- and three-beam superpositions, the summation of $(m = 0, 2)$ and $(m = 0, 2, 5)$ are carried out. In the LHD, the transmission lines which consists of the CC-waveguide with $2a = 88.9$ mm for $\omega/2\pi = 84$ and 168 GHz are being prepared. In coupling between HE_{11} and TEM_{om} for $w_{0}/a = 0.6436$, the coefficient $A_m$ calculated. The TEM_{02} mode has the largest value in first negative coupling coefficient group and the TEM_{05} mode has the maximum value in second positive coupling group.

In Fig.1 (a)-(c), radiation patterns for different approximation at $z = 100$, 1 and 0.2 m are shown, respectively. With increasing superposition of higher modes in multi-beam approximation, radiation pattern becomes close to that of Fresnel-Kirchhoff (FK) integral. The main beam of FK integral can be approximated well with two-beam ($\text{TEM}_{00} + \text{TEM}_{02}$) or three-beam ($\text{TEM}_{00} + \text{TEM}_{02} + \text{TEM}_{05}$) superpositions in near- and far-field regions. In the multi-beam by sum of $\text{TEM}_{0m}$ up-to $m = 20$, amplitudes of higher order side-lobes agree well with that of FK integral.

Amplitudes of side-lobe $E_y$ for $z = 100$ m are examined as a function of $m_{\text{max}}$. Near a boundary value $m$ between positive and negative coupling coefficients, a new side-lobe appears. With increasing $m_{\text{max}}$, the amplitude of the side-lobe is saturated. The amplitude of side-lobes in two- and three-beam approximations is nearly equal to that of the multi-beam approximation with $m_{\text{max}} = 2$ and $m_{\text{max}} = 5$. The deviation $\epsilon$ from the FK integral in single-, two-, three- and multi-beam superpositions with $w_0/a = 0.6436$ is calculated as a function of distance $z$, where the error $\epsilon$ is defined by

$$\epsilon = \frac{\int |E \times H^* - E_{\text{FK}} \times H_{\text{FK}}^*| 2\pi dr}{\int E_{\text{FK}} \times H_{\text{FK}}^* 2\pi dr}$$

For only far-field radiation, the single-beam approximation is slightly improved by changing $w_0/a = 0.596$ for 0.6436 as pointed out by Crenn [1]. In both near- and far-field radiations, two-beam and three-beam approximation is good with small $\epsilon$. The multi-beam approximation such as $m_{\text{max}} = 20$ are best.

References