§10. Spectrum of Subcritically Excited Interchange Mode Turbulence

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Method of self-sustained turbulence is extended to study the frequency spectrum. We here consider the spectrum of fully developed turbulence of current diffusive interchange mode (CDIM) [1].

We study the slab plasma which is inhomogeneous in the z-direction, and the magnetic field is mainly in the y-direction. In order to analyze interchange mode turbulence, the relevant set of equations has been derived for the electrostatic potential $\phi$, current $j$, and pressure $p$; equation of motion, the Ohm's law and the energy balance equation. The Lagrangian nonlinearity is included in a form of

$$\sum_{k \gg k_1} \int \frac{|k_{\perp}|^2}{2(\omega + \omega_2 + \mu_2(k_{\perp})|k_{\perp}| + |k_{\perp}|^2)^2} \frac{d\omega_2}{2}$$

(1)

Other coefficients are derived accordingly. By use of $(\mu_N, \lambda_N, \chi_N)$, the basic equation is rewritten as

$$k_{\parallel} \mu_{\parallel} = \frac{k_{\perp}^2}{-i\omega + \mu_N k_{\perp}^2} \phi(k_{\perp}, \omega) - \frac{k_{\perp}^2 G_0}{-i\omega + \lambda_N k_{\perp}^2} \phi(k_{\perp}, \omega)$$

$$+ (-i\omega + \mu_N k_{\perp}^2) k_{\perp} \phi(k_{\perp}, \omega) = 0$$

(2)

where $G_0 = (d\sigma_2/dz) dp_2/dz$ is a driving parameter.

The mode near $\omega = 0$ are of interests. We set $\omega = 0$ in Eqs. (1) and (2) and find the fluctuation spectra in k-space. From Eq. (2) with $\omega = 0$, special solutions for $(\mu_N, \lambda_N, \chi_N)$ are found; Namely, for given $G_0$,

$$\mu_N(k_{\perp}, 0), \lambda_N(k_{\perp}, 0), \chi_N(k_{\perp}, 0) \propto k_{\perp}^{-2}.$$  

(3)

The spectral function is derived as

$$E(k) = \int d\omega E(k, \omega) = 2P_{\phi} G_0 k_{\perp}^{-3}$$

(4)

where $P_{\phi} = \mu_N(k_0) / \chi_N(k_0) = 1$. For the interchange mode, the turbulent transport coefficient is obtained as

$$\chi_T = 2P_{\phi} G_0^{0.2} k_{\perp}^{-2}.$$

The frequency spectrum is examined, where the averaged frequency is chosen to be zero. The decorrelation time $\tau_{cor}$ is order-estimated as

$$\tau_{cor} = \frac{1}{\omega_{cor}} = \frac{(\mu_N(k_0) k_{\perp}^2)}{1}.$$  

For the frequency range of $\omega_{cor} < \omega$, the spectrum may be given by Lorentzian, which is called as Cauchy's distribution in the statistical physics of Brownian motion [2]. This condition corresponds to the "long time approximation", where the coherence of each mode is lost. We here adopt this ansatz and seek for the solution of the form $\phi(\omega) \propto \frac{1}{\omega^2 + \gamma^2}$ for $\omega = 0$, and

$\phi(\omega)$ is the Fourier spectrum of $\phi(t)$. Since $\mu_{\perp}^2 >> (\omega + \omega_2)$ in the denominator of Eq.(1), the relation

$$\mu_N(k, \omega) \propto \int \frac{d\omega_2}{\omega_2^2 + \gamma^2} \frac{d\omega_2}{\mu_N(k, \omega + \omega_2)}$$

(5)

is obtained. Contribution near $\omega_2 = -\omega$ is important. We finally obtain a special form of $\mu_N(k, \omega)$ and $E(k, \omega)$

$$\mu_N(k, \omega) = \left(\frac{\gamma^2}{\omega^2 + \gamma^2}\right)^2 \mu_N(k, \omega)$$

(6)

$$E(k, \omega) = \left(\frac{\gamma^2}{\omega^2 + \gamma^2}\right)^2 \gamma E(k)$$

(7)

Both spectra have power law dependencies on $\omega$ in asymptotic forms as $\mu_N(k, \omega), E(k, \omega) \propto \omega^{-2}, \omega^{-4}$. Distributions with long tail are possible as a solution of CDIM (current-diffusive interchange mode) turbulence. The spiky burst in temporal evolution can be predicted.

The consideration of scale invariance shows that the system is invariant under the transformation $x \rightarrow \lambda x$ and $t \rightarrow \lambda^2 t$ with $(\phi, j, p) \rightarrow (\lambda^2 \phi, \lambda^2 j, \lambda^2 p)$ and $G_0 \rightarrow \lambda^2 G_0$. The turbulent transport coefficient, is transformed as $\mu \rightarrow \lambda^2 \mu$. We find that $\phi^2 G_0^{1/2}$, $k G_0^{1/2}$ and $\omega G_0^{1/2}$ are scale-invariant. That is, $\phi^2$ satisfies the relations $\phi^2 \sim G_0^1 H(k G_0^{1/2})$ or $\phi^2 \sim G_0^1 F(\omega G_0^{1/2})$ where $H$ and $F$ are arbitrary functions. If the energy spectrum is given in the power law in the $k$-space, then it has a form as $E(k) = G_0^{5/2 - v/2 k^{-2}}$. Equation (4) satisfies this constraint. If the $\omega$-spectrum is given by the power law, one has $\phi^2(\omega) = G_0^{5/2 + v/2 \omega^{-2}}$. The above equation (7) satisfies this constraint when the condition $\gamma = \gamma_{cor}$ is put.

References
