A new aspect of the transitions in plasmas, i.e., probabilistic nature of the occurrence, is analyzed [1]. The dynamical model of transition has been studied in many literature [2]. We here adopt two basic equations: One is the evolution of global structures (say, gradient) and the other is the dynamics of the loss rate that produces the hysteresis of the flux-gradient relation. We take two representative variables, i.e., the pressure gradient $\alpha$ and the loss rate $\gamma$. The loss rate is directly related to the turbulence level. The model equation takes the form

$$\frac{\partial}{\partial t} \alpha = S - \gamma \alpha$$

$$\frac{\partial}{\partial t} \gamma = \alpha - 1 - a(\gamma - 1) - b(\gamma - 1)^3$$

The notation is as follows: $S$ is the energy influx into the layer, $\zeta$ denotes the possible difference of dynamical time between $\alpha$ and $\gamma$; the cubic equation $a(\gamma - 1) + b(\gamma - 1)^3$ describes the shape of the hysteresis in the gradient-flux relation.

If all the coefficients $(S, a, b, \zeta)$ are constant in time, the stable stationary solutions or limit cycle is obtained. However, the statistical variance is as important a quantity as the statistical average, for the confined plasmas which is far from thermal equilibrium. The nonlinear simulation has shown a large temporal variation around the average [3]. Based on these considerations, we consider that parameters $(S, a, b)$ are statistical variables and have fluctuation parts in time. We put $S = S_0 + \varepsilon_f$ and $a = a_0 + \varepsilon_a$

and consider $\varepsilon_f$ and $\varepsilon_a$ as statistical variables, i.e.,

$$(\varepsilon_f) = 0, (\varepsilon_a) = 0, (\varepsilon_f^2) \neq 0 \text{ and } (\varepsilon_a^2) \neq 0.$$ 

Temporal evolution is studied. Example is shown for the statistical variance in the influx $S$, $\varepsilon_f$. The Gaussian statistics is assumed, and the amplitude of deviation $\varepsilon_f = \sqrt{(S(t) - S_0)^2} = 0.05$ is applied. There appear irregular ELMs. We evaluate the averaged frequency of oscillations, $\langle f \rangle$, and the standard deviation $\sigma$, $\sigma = \sqrt{\langle T^2 \rangle - \langle T \rangle^2} / \langle T \rangle$, where $T$ is the observed interval of oscillations. In Fig.2, $\langle f \rangle$ and $\sigma$ are plotted for $\varepsilon_f = 0.05$ as a function of $S_0$. We observe the probabilistic excitation and that the transition occurs well below the critical value. As $S_0$ becomes small, the standard deviation $\sigma$ becomes large and we find, in this case, an approximate relation $\sigma \propto \langle f \rangle^{-0.74}$.

The probabilistic nature is also caused by the other statistical property of turbulence. The same statement is made for the variation in the hysteresis curve.

References