§16. Iterative Methods of Tikhonov-Phillips Image Reconstruction of Large Size with a Nonlinear Constraint

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Study has been made progressively on numerical methods of fast solving the inverse problems of large size that one meets for scientific imaging. The efficient iterative algorithms of the Tikhonov-Phillips and maximum entropy (ME) regularizations are ready for use with respect to ill-conditioned linear equations [1]. The outcome of this fiscal year was to find a useful method of applying a nonlinear constraint to the solutions.

As well known, when the objective image never takes negative values for example, the nonlinear constraint of non-negativity can be introduced using the logarithmic function in the ME method and the skimmer function in the Hopfield neural network. Meanwhile, an advantage of the iterative solvers is the possibility of adopting a relating nonlinear measure in the updating procedures as in ART and SIRT.

In the electron microscopic tomography for bio-imaging, the image background has a lightness of the vitreous ice and makes the non-negativity constraint almost meaningless. Rather, the histogram of the reconstructed image separates to two peaks, which are located around the grades of the background and the virus particle. Thus, an extension was studied. In the iteration of the fast least-squares solvers, which are based on rewriting the problem of minimizing the Lagrange function $\Lambda(\boldsymbol{x})=\|A\boldsymbol{x}-\boldsymbol{b}\|^2+\lambda^2\|C\boldsymbol{x}\|^2$ as

$$\min_{\boldsymbol{x}} \left\| \begin{bmatrix} A \\ \lambda C \end{bmatrix} \boldsymbol{x} - \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{0} \end{bmatrix} \right\|^2 \qquad (1)$$

[2, 3], the restart by replacing as

$$x_j=x_j \; (x_j \geqq tol), \quad x_j=\mu \; (x_j<tol) \qquad (2)$$

was adopted. The value of μ was set to the background level, which could be estimated practically by taking the mean of the pixel values in the linear solution.

Results of simulations on the 2D electron tomography are shown in Figs. 1 and 2. In presupposing the multi-slice imaging of the virus ε15 with rotation angle limits of ±70°, a phantom $\boldsymbol{x}_o$ with 128x1024 pixels was numerically produced with the grade values 1 and 0.1 of the particle and background. For 1024 parallel beams in each of 70 equal-spaced angular directions, the projection $A\boldsymbol{x}_o$ was calculated and additively corrupted with zero-mean Gaussian random numbers, for which $\|\Delta\boldsymbol{b}\|$ was as large as 20% of $\|A\boldsymbol{x}_o\|$.

Reconstruction was studied for various values of *tol*. Figs. 1 and 2 show the results of applying the CGLS as a minimization tool in the scheme of Eq. (1) with 300 restarts of 10 CGLS iterations. With the Tikhonov regularization ($C=I$), the nonlinear constraint on the lower bound μ lead to a remarkable improvement of reconstruction: that is, the noise was greatly decreased, and the sidelobes of missing wedge type around particles disappeared. With the Phillips regularization (the Laplacian matrix C), the reconstruction was improved with more effective noise elimination.

Another problem related to the lightness of the background, that is, the appearance of large artifacts at the right and left edges of the reconstruction region [1], was cleared up by contriving a practical method.

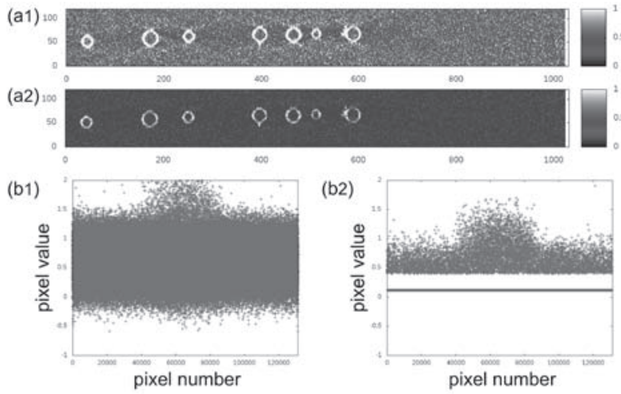

Fig. 1. Nonlinear effect in the Tikhonov regularization: (a1), (b1) the reconstructed image and its pixel values with no constraint; (a2), (b2) those with *tol*=0.4 (λ=5.0).

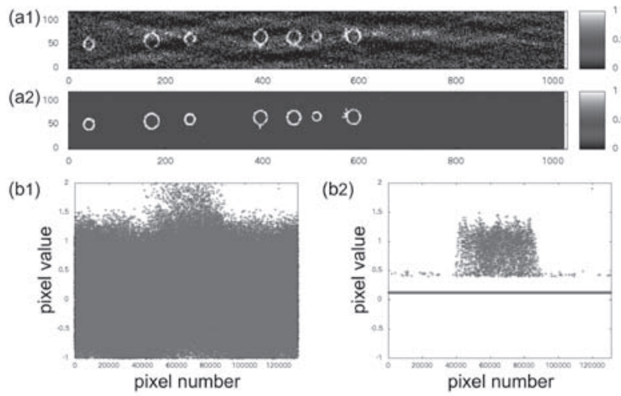

Fig. 2. Nonlinear effect in the Phillips regularization: (a1), (b1) the reconstructed image and its pixel values with no constraint; (a2), (b2) those with *tol*=0.4 (λ=5.0).

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1) Iwama, N. et al.: Ann. Rep. NIFS (2012-2013) 472.
2) Hansen, P.C.: *Discrete Inverse Problems*, SIAM (2010).
3) Morikuni, K. and Hayami, K.: SIAM J. Matrix Anal. Appl. **34** (2013) 1.