§17. Numerical Simulation of 3D Image Reconstruction of LHD Plasma with IR Imaging Video Bolometers


The arrangement of software for image reconstruction is in progress in order to realize the 3D tomography of radiation from LHD. Using the observation ports 6.5-U, 6.5-L, 6-T and 10-O, the system of four imaging bolometers was carefully improved in sightline positioning design [1] and installed on LHD for the 17th cycle of experiments. Assuming the toroidal periodicity and symmetry of the plasma, one half period of toroidal angle 18° was taken as the region of interest.

Numerical simulations were carried out on the improved layout of the four-camera system. With the geometry of the layout and on the basis of the physics of radiation detection, the projection matrix \( H \) that relates the unknown 3D plasma image \( f \) to its projection \( g \) in camera system was theoretically calculated with the aid of the CAD software. The 3D radiation distributions obtained with the EMC3-EIRENE code of impurity behavior were used as phantoms. By omitting the non-radiation voxels from image reconstruction, the size of the matrix \( H \) was decreased to 2,528x13,161 so that the singular value decomposition could be accomplished with a personal computer.

Figs. 1 and 2 show the result of applying the Tikhonov (unit matrix) regularization to the underdetermined linear equation \( Hf = g \). Changing the value of the regularization parameter \( \gamma \) lead to the changes of the reconstructed image \( \hat{f} \) and its projection \( \hat{Hf} \) as in Fig. 1. Uniform random numbers, whose upper limit of interval was 10% of the mean of the true projection values, were added to produce the data vector \( g \), and the Lagrange function to be minimized was defined as \( \mathcal{A}(f) = \gamma \| \hat{f} \|^2 + \| Hf - g \|^2 / M \) with \( M = 2,528 \). While the mean of the squared residuals \( \epsilon^2 \) monotonically decreased with \( \gamma \), the generalized cross validation GCV was minimized and well reflected the minimum of the reconstruction error \( \delta^2 \).

The value of \( \epsilon = 2.17 \times 10^{-6} \) at the minimum of GCV was much smaller than the root mean square \( 5.08 \times 10^{-5} \) of the added random numbers. If we had taken the Morozov condition, we would have got a smoothed reconstruction similar to (C) in Fig. 1. This result shows the validity of computation and reconfirms the usefulness of GCV [2, 3] for the tomography in which the angular loss of projection data is very large. The condition number of the matrix \( H \) was evaluated as \( 7.71 \times 10^{34} \) using the software IDL.

For a better regularization according to the non-negativity constraint, additional study has been made to build fast iterative algorithms.

Finally, we obtained the first result of image reconstruction from experimental data of LHD.

![Fig. 1. Result of a numerical simulation: (left) a 3D phantom \( f_c \) and its projection \( Hf_c \) which are displayed in one poloidal section and in the 6.5-L port camera, respectively; (right) reconstructed images \( \hat{f} \) and their projections \( \hat{Hf} \) displayed in the same section and camera for \( \gamma \) values of (A) \( 4.0 \times 10^{-16} \), (B) \( 1.9 \times 10^{-13} \) (min. GCV), and (C) \( 2.7 \times 10^{-9} \).](image)

![Fig. 2. Changes of GCV, \( \epsilon^2 \), and \( \delta^2 \) with the regularization parameter \( \gamma \); while \( \epsilon^2 \) changed monotonically with \( \gamma \), both GCV and \( \delta^2 \) were minimized for the same discrete value of \( \gamma = 1.9 \times 10^{-15} \) in this case.](image)

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