§27. The Role of Nonlinear Dynamics in Self-organization

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We have studied the role of nonlinear dynamics in the formation of global structures in plasma systems.

First, we studied a model of hole formation in Vlasov systems. Several steady state solutions have been proposed and they are presented in Schamel's review article[1]. However, they seem very artificial and complicated. Saeki and Genma proposed the simplest and physically interesting solution [2]: The essential point is to introduce particles with negative energy, namely trapped particles. The water-bag model is now written for the distribution function, which depends on phase variables only through the energy $H$.

$$f(H) = \begin{cases} \frac{m n_0}{E_0 2\sqrt{2}} & (-e\phi_0 \leq H \leq E_0) \\ 0 & \text{(otherwise)} \end{cases}$$

The Saggeev potential for the potential defined by $\frac{d^2\phi}{dx^2} = -\frac{dV}{d\phi}$ is analytically given and implies that there is a maximum of $\phi$, which corresponds to an electron hole.

Dawson's one-dimensional plasma model[3] can be transformed into the system of hard spheres in a one-dimensional chain, each of which is under the harmonic potential around an equidistant equilibrium site. Namely if we denote by $x_i(t)$ and $v_i(t)$ the displacement from the equilibrium position and the velocity of the $i$-th sheet, then they form a harmonic oscillator $\ddot{x}_i = v_i$ and $\dot{v}_i = -\omega^2 x_i$ as long as they do not collide. When a neighboring pair of sheets meet, namely, $x_i - x_{i+1} = a$, where $a$ is the distance between the neighboring equilibrium sites, they exchange the momenta. Namely if one denotes the velocities before and after the collision by $v_i^< \text{ and } v_i^>$ respectively, we have $v_i^> = v_{i+1}^>$ and $v_{i+1}^> = v_i^<$. This collision process is usually introduced as an initial condition for the motion after the collision. However, we can now describe this hard sphere collision as a term in an autonomous differential equation[4]. The idea is to introduce a "half"-delta function $\delta_-(x)$, which is infinite only when $x = 0^-$. Then we have

$$\delta_-(x_i - x_{i+1} - a) = \frac{\delta_-(t - t_\circ)}{v_i^> - v_{i+1}^>}$$

Namely the delta function of time is concentrated just before the collision at time $t_\circ$. Thus the exchange of the velocity is expressed as

$$\dot{v}_i = -(v_i - v_{i+1})^2 \delta_-(x_i - x_{i+1} - a)$$

Indeed if we integrate both sides from $t = t_\circ + 0^-$ to $t = t_\circ + 0^+$, we have $v_i^> = v_{i+1}^>$. Although Eq.(3) is singular in that it includes a delta function, it is autonomous and allows the orbital stability analysis, which is now in progress.

References