## §3. Density Calibration Method by Microwave Reflectometry Based on Bayesian Estimation

Minami, T., Funaba, H., Narihara, K., Yamada, I., Hayashi, H., Kohmoto, T.

A Bayesian estimation method is proposed to the derivation of the calibration factor using a microwave reflectometry. The Bayesian estimation is a probabilistic approach for estimating an unknown probability density function. The Bayesian theorem is defined as follows[1],

$$P(x|d) = \frac{P(d|x)P(x)}{P(d)} \tag{1}$$

,where $P(x)$ is a prior probability, $P(x|d)$ is a posterior probability and a conditional probability of x given d, $P(d|x)$ is a conditional probability of d given x, $P(d)$ is a normalized constant, and $P(d|x)/P(d)$ is a likelihood. The $P(d)$ is defined as

$$P(d) = \int_{-\infty}^{\infty} P(d|x)P(x)dx. \tag{2}$$

For the purpose of the Thomson density calibration, we assume that the prior probability function of the density calibration factor of $C^T$ being x is a gaussian distribution. The prior probability $P(x)$ is defined as follows.

$$P(x) = \frac{1}{\sqrt{2\pi}w_{prior}} exp\Big\{-\frac{1}{2}(\frac{x - C^T_{prior}}{w_{prior}})^2\Big\} \tag{3}$$

, where $C^T_{prior}$ is a prior mean calibration factor, and $w_{prior}$ is a standard deviation ( error bar ) of the calibration factor. The $C^T_{prior}$ can be derived form $C^T_{posterior}$ ( Eq. 6 ) calculated from the previous reflectometer measurement. The first prior calibration factor (the initial value) can be determined from the gas scattering experiment or the estimation taking the solid angle of the scattering light and the transmission efficiency of the collective optics and the spectrometer into account. It is no problem that the first prior calibration factor includes some error, because the primal error is corrected with the reflectometer measurement.

The parameter d is the measurement of the calibration factor with the reflectometer, and is derived from

$$d = \frac{n_e^{ref}}{S} \tag{4}$$

,where $n_e^{ref}$ is a plasma density measured with the reflectometer, and the S is the amount of the scattered light measured with the Thomson scattering device. We also assume that $P(d|x)$ is a gaussian distribution, and the $P(d|x)$ is defined as follows.

$$P(d|x) = \frac{1}{\sqrt{2\pi}w_d} exp\Big\{-\frac{1}{2}(\frac{x - d}{w_d})^2\Big\} \tag{5}$$

, where $w_d$ is a standard deviation of the 'd' that is derived from the reflectometer measurement.

The modified calibration factor to the real value after the one reflectometer measurement is calculated as the posterior value ,which is derived from the posterior probability function $P(x|d)$ (eq.(1)). The posterior mean calibration factor $C^T_{posterior}$ and that the standard deviation $w_{posterior}$ are derived from the following formulas.

$$C^T_{posterior} = \int_0^{\infty} xP(x|d)dx \tag{6}$$

$$w_{posterior} = \sqrt{\int_0^{\infty} x^2 P(x|d)dx - (C^T_{posterior})^2} \tag{7}$$

The Bayesian estimation provides a sequential analysis for the calibration factor determination taking the accuracy of the reflectometer measurement into account. The simultaneous measurement of the Thomson scattering and the reflectometer is repeated. The posterior value of the $C^T$ and $w$ are used as the prior one of the next step, and the derivation of the posterior probability function is repeated recursively. The calibration factor is modified and come close to the real value with the number of the steps of the simultaneous measurements with the Thomson scattering and the reflectometer. If the calibration factor is changed by an unexpected accident such as the misalignment of the laser path, the calibration factor is immediately modified to the new value.

[1] A lot of references about "Bayesian theorem" are available.
(e.x. Data Analysis: A Bayesian Tutorial D.S. Sivia Oxford University Press)