§4. Vlasov and Drift Kinetic Simulation Method Based on the Symplectic Integrator

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Vlasov and drift kinetic simulation methods based on the symplectic integrators are benchmarked for test problems on the linear and nonlinear Landau dampings and the Kelvin-Helmholtz (K-H) instability. The explicit symplectic integrator for the separable Hamiltonian straightforwardly leads to generalization of the splitting scheme for the Vlasov-Poisson system. The $N$th-order version improves the total energy conservation decreasing the error as $\propto \Delta t^N$ where $\Delta t$ denotes the time step size. An Eulerian drift kinetic simulation scheme derived from the implicit symplectic integrator for the non-separable Hamiltonian exactly satisfies the conservation of the energy and the enstrophy in the K-H instability, and results in successful application to the plasma echo. More detailed results are reported in Ref. [1].

Higher-order splitting scheme for the Vlasov-Poisson system

The higher-order time-integration scheme for the Vlasov equation is obtained by applying the explicit symplectic integrator to the coordinate transformations in the time-splitting scheme as follows [2];

$$
\begin{align*}
  f_{i-1}(q, p) &= f_{i-1}(q - c_i \Delta t \frac{\partial T}{\partial q}|_{f=f_{i-1}}, p) \\
  f_i(q, p) &= f_{i-1}(q, p + d_i \Delta t \frac{\partial V}{\partial q}|_{f=f_{i-1}})
\end{align*}
$$

for $i = 1, \ldots, k$. For an even number of $N$, Eq. (1) can also be given by successive operations of the second-order splitting scheme, in accordance to the $N$th-order symplectic integrator by Yoshida [3].

Conservation of the total energy definitely depends on the accuracy of the time-integration scheme. The scheme is benchmarked for a test problem on the nonlinear Landau damping. Dependence of the error in the energy conservation, which is defined by $|E(t) - E(0)|_{\text{max}}/E(0)$, on the time step size is summarized in Fig. 1, where the solid, dashed, dotted and dot-dashed lines represent powers of $\Delta t^\beta$ with $\beta = 1, 2, 4,$ and 6, respectively.

Nondissipative time-integration method for the drift kinetic system

The nondissipative simulation method for the drift kinetic equation is implemented by means of the implicit midpoint rule;

$$
\hat{f}^{n+1} = f^n - \Delta t \{ \hat{f}, \hat{H} \} \quad \text{with} \quad \hat{f} = \frac{f^{n+1} + f^n}{2}.
$$

It is remarked that the implicit scheme here is employed for improvement of the conservation property of the $L^2$ norm. The scheme is benchmarked for test problems of the Kelvin-Helmholtz (K-H) instability and the plasma echo. Conservation of the total energy and the enstrophy is clearly shown in simulation of the K-H instability [1]. The test simulation for the plasma echo in the drift kinetic system with a slab geometry demonstrates successful reproduction of the phase mixing process with the time-reversibility.

References

