§16. Consideration of Mode-Content Analysis Using a Millimeter-Wave Beam Position and Profile Monitor


In an ECRH system, it is important to precisely align a propagating millimeter-wave (mmw) beam to a transmission line to avoid mode conversion to the other higher-order modes. We have been developing a real-time beam-position and profile monitor (BPM) to measure the intensity profile of a high power (Megawatt level) mmw propagating even in an evacuated corrugated waveguide without any disturbances. It was improved to obtain higher spatial resolution 1. The BPM consists of a reflector, two-dimensional Peltier-device array and a water-cooled heat sink which are installed in a miter-bend of the transmission line. Test results using a circular electric heater as a simulated heat source is shown in Fig. 1.

![Fig. 1: Variation of each Peltier device voltage is mapped. The white dashed-line circle indicates the heater position attached and the black dashed-line shows a cross section of a waveguide.](image)

Using the signals obtained by the BPM, a method of mode content analysis is considered according to the method proposed in the reference2. For simplicity, linear polarized modes in a circular corrugated waveguide are considered, which are expressed as the following equations:

\[ E \left( r, \theta \right) = \hat{y} \sqrt{2} f_{o} J_{n}(X_{\sigma} \cdot r/a) \cos(n\theta) \]

\[ E \left( r, \theta \right) = \hat{y} \sqrt{2} f_{o} J_{n}(X_{\sigma} \cdot r/a) \sin(n\theta) \]

\[ E \left( r, \theta \right) = \hat{x} \sqrt{2} f_{o} J_{n}(X_{\sigma} \cdot r/a) \cos(n\theta) \]

\[ E \left( r, \theta \right) = \hat{x} \sqrt{2} f_{o} J_{n}(X_{\sigma} \cdot r/a) \sin(n\theta) \]

where \( n, m \) are mode numbers and \( X_{\sigma} \) is the eigen value of the mode \( \sigma \) with \((n, m)\). \( a \) expresses the radius of the waveguide and the normalization constant \( f_{o} \) is

\[ f_{o} = \frac{Z_{0}}{a \sqrt{\pi} J_{n+1}(X_{\sigma})} = - \frac{Z_{0}}{a \sqrt{\pi} J_{n-1}(X_{\sigma})}. \]

Electric field profiles of typical lower order LP\(_{nm}\) modes are graphically plotted in Fig. 2. The direction of the electric field is oriented to Y-direction.

![Fig. 2: Electric field profiles of LP\(_{01}\), LP\(_{11}(\text{odd, even})\), LP\(_{21}(\text{odd, even})\), LP\(_{02}\)-modes](image)

Generally, a propagating mmw in a corrugated waveguide is expressed as a superposition of several eigen modes \( \sigma (= 0 \cdots N) \). The electric field at the position of \((x_{i}, y_{j}, z_{k})\) is described by the following equation;

\[ e_{tot}(x_{i}, y_{j}) = \sum_{\sigma=0}^{N} \sqrt{\rho_{\sigma}} \exp\{j(\phi_{\sigma} - k_{\sigma} z_{k})\}E_{\sigma}(x_{i}, y_{j}) \]

\[ x_{i} = i \times \Delta x \]

\[ y_{j} = j \times \Delta y \]

where \( i, j = 0 \cdots M-1 \) and \( p_{\sigma}, \phi_{\sigma} \) and \( k_{\sigma} \) are the power, phase and wave-number of the propagating mode \( \sigma \), respectively. The evaluation function \( W_{tot} \) for determining mode content is defined by the summation of square value of the difference between the measured \( O \) and theoretical \( T \) functions,

\[ W_{tot}(p_{\sigma}, \phi_{\sigma}) = \sum_{k=0}^{N-1} W(z_{k}), \]

where

\[ W(z_{k}) = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \left[ O(x_{i}, y_{j}, z_{k}) - T(x_{i}, y_{j}, z_{k}) \right]^{2} \]

\[ T(x_{i}, y_{j}, z_{k}) = \frac{|e_{tot}(x_{i}, y_{j}, z_{k})|^{2}}{e_{tot|MAX}} \]

When the mode with \( \sigma = 0 \) is assumed to be the LP\(_{01}\) fundamental mode with the phase \( \phi_{0} = 0 \) and \( \sum_{\sigma=0}^{N} p_{\sigma} = 1 \), each \( p_{\sigma}, \phi_{\sigma} \) can give the ratio of mode-content and the initial phase of each mode \( \sigma \).