§ 16. Cascade and Inverse Cascade in Multiple-Scale Turbulence

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The numerical simulation is performed to investigate the cascade and inverse cascade in multiple-scale turbulence. We consider the neighboring turbulence such as ITG and the short wavelength ITG mode\(^\text{10}\). The kinetic-fluid equations for the ion density \(n_i\) and the electrostatic potential \(\phi_k\), which are relevant to describe ITG and the short wavelength ITG mode in sheared slab geometry are given by

\[
\frac{\partial n_{k}}{\partial t} - i\omega n_{k} Z_{e} Z_{c} \Gamma_{0k} \phi_{k} + \nabla (1 + \zeta_{c} Z_{c}) \frac{\partial \phi_{k}}{\partial t} = 0
\]

\[
- m_{i} \rho_{i} \chi_{e} \Gamma_{0k} \phi_{k} - (1 + \zeta_{c} Z_{c} + \lambda_{c}^{2} k^{2}) \frac{\partial \phi_{k}}{\partial t} + \frac{\pi}{2} \sum_{k = k + k'} z \cdot k' \times k \Theta_{k} \phi_{k'} = 0
\]

\[
\frac{\partial \phi_{k}}{\partial t} - i \omega \phi_{k} Z_{e} \phi_{k} - (1 + \zeta_{c} Z_{c} + \lambda_{c}^{2} k^{2}) \frac{\partial \phi_{k}}{\partial t} - m_{i} \rho_{i} \chi_{e} \phi_{k} - \frac{\pi}{2} \sum_{k = k + k'} z \cdot k' \times k \phi_{k} (n_{k}' - \lambda_{c}^{2} k^{2} \phi_{k}') = 0,
\]

where the linear gyrokinetic response is used as a closure for equations and the ion gyro frequency \(\Omega_{i}\) and ion Larmor radius \(\rho_{i}\) are used for normalizations. \(Z_{e, c}\) is the plasma dispersion function with the argument \(\zeta_{c,i,e} = \omega / (k_{\parallel} \rho_{i, e})\).

\[
\chi_{k} = 2 \chi_{k}' = \frac{2}{(1 - b_{i}) \Gamma_{0k} + b_{i} \Gamma_{1k}} \int_{0}^{\infty} dx x e^{-x} dx J_{0}(kx) J_{0}(k'x) J_{0}(k'x) J_{0}(k'x)
\]

where the numerical simulation is performed using the spectral code. The minimum wavenumbers are \(k_{\parallel 0} = 0.1, k_{\parallel 0} = 0.1, k_{\parallel 0} = 0.01\), respectively and \(8 \times 128 \times 16\) Fourier modes are used. Other parameters are given by \(\tau = 1, m_{i} / m_{e} = 1836, \rho_{i} / L_{n} = 0.2, \Omega_{i}^{2} / \omega = 10\). Figure 1 shows the time evolution of 2D spectra of the potential energy in the case with \(k_{\parallel} \rho_{i} = 0.3\). Time slices at \(Q_{i}f = 100,200,300,350\) are plotted. The inverse cascade occurs in the longest wavelength region. The another peak appears in the short wavelength region.

\[
E_{\phi}(k_{\parallel}) = \sum_{k_{\perp} \neq 0} |\hat{\phi}_{k_{\perp}}|^{2}, E_{\phi}(k_{\parallel} k_{\perp}) = \sum_{k_{\parallel}} |\hat{\phi}_{k_{\parallel}}|^{2}
\]

\[
\Gamma_{nk} = -i \sum_{k = k + k'} \frac{3}{2} \hat{\rho}_{k} \hat{\phi}_{k} \cdot q_{nk}
\]

\[
= -i \sum_{k = k + k'} \frac{3}{2} \hat{p}_{k} \hat{\phi}_{k}, q_{nk} = -i \sum_{k = k + k'} \frac{3}{2} \hat{p}_{k} \hat{\phi}_{k}
\]

where the ion and electron pressure are evaluated by using the linear distribution function such as \(p_{k} = m_{e} / 3 \int d^{3} x v^{2} \hat{\mu} \hat{\phi}_{k}\), then \(p_{k} = -\tau + \kappa / 3 \hat{\phi}_{k}\).

It is interesting to compare these results with ITG and short wave length ITG modes in sheared slab geometry. It is left for future work.

References