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Energy Transport in the Steady State Plasma

Sustained by DC Helicity Current Drive

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Abstract

Steady state operation of tokamaks which is sustained by the DC helicity current drive near edge is studied. The necessary value of the current diffusivity is obtained. Relation between the current diffusivity and the thermal diffusivity, which are governed by the microscopic turbulence, indicates that this requires too large thermal transport for the parameters in present day experiments.

Keywords: DC Helicity Injection, Current Diffusivity, Steady State Tokamak, Thermal Conductivity
The research on the efficient noninductive method to drive the toroidal current in tokamaks has been one of the key issues of the magnetic fusion research[1-3]. As the perspectives of the conventional scheme of the non-inductive current drive (such as rf momentum injection and neutral beam injection) in applying to reactor-grade plasmas has become clearer[4-8], the research on the innovative current drive method is thought to be more and more important.

One trend in the investigations to find the breakthrough is to utilize the concept of the helicity injection[7-10]. Recent theoretical progress has shown that the rf wave helicity can drive the electro-motive force on plasma, with the expense of little wave momentum[11]. Taking the Alfvén wave or the wave of the ion cyclotron range of frequencies as the example, the current drive efficiency has been discussed[11-14].

The other concept of the helicity current drive has also been developed, namely DC helicity injection[9,10]. In such a scheme, the current which is driven (by helicity injection) near edge is thought to be transported to the core plasma. We in this article study the steady state of the plasma in which the resistive dissipation of the current is sustained by the cross field transport of the current. In order to satisfy the WHD stability, the internal inductance cannot be too low. Relation between the current diffusivity and the thermal diffusivity is discussed. The requirement on the current profile imposes too large thermal transport for the parameters in present day experiments.
We study a cylindrical tokamak model for the simplicity. The cylindrical coordinates \((r, \theta, z)\) are used, where \(r=0\) corresponds to the magnetic axis. The Ohm's law is written as

\[
\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} - \lambda \Delta \mathbf{J}
\]  

(1)

where \(\mathbf{E}\) is the electric field, \(\mathbf{B}\) is the magnetic field, \(\mathbf{V}\) is the flow velocity, \(\mathbf{J}\) is the current, \(\eta\) is the resistivity, and \(\lambda\) is the current diffusivity. \(\lambda\) is the coefficient for the transport process of the longitudinal current across the equilibrium magnetic surface. We consider the stationary state without equilibrium flow, and have

\[
\eta \mathbf{J} - \lambda \Delta \mathbf{J} = 0.
\]

(2)

In this expression, \(\mathbf{J}\) represents the stationary and global current, and the fluctuating current, which can exist in stationary states, are included through the contribution to the transport coefficients. This relation (2) indicates that the dissipated helicity is sustained by the radial transport of the helicity. Figure 1 illustrates the schematic drawing. The current is driven externally near the edge (such as the parallel current in the scrape-off layer or injection of energetic electrons, etc.), and the current diffuses into the core region.

Equation (2) yields the current profile in the stationary state. For the simplicity, we look for the solution with the
cylindrical symmetry. [Extension to asymmetric states is straightforward and does not change the conclusion in the following.] For the purpose to obtain the analytic insight of the problem, we assume that coefficient $\lambda/\eta$ is constant in space.

The solution is given as

$$J(r) = J(0)I_0(\sqrt{\eta/\lambda} r),$$  \hspace{1cm} (3)

where $J(0)$ is the current density at the axis, and $I_0$ is the zeroth order modified Bessel function of the first kind. The shape of the current is dictated by the parameter

$$\alpha = a\sqrt{\eta/\lambda},$$  \hspace{1cm} (4)

where $a$ is the minor radius of the plasma. Figure 2 illustrates the internal inductance $L_1$ as a function of $\alpha$. Here the internal inductance is defined by

$$L_1 = \frac{2\int_0^a B_y(r)^2 rdr}{a^2 B_y(a)^2}$$  \hspace{1cm} (5)

For the actual application, the current density cannot be too large at the edge to avoid the violent WKB activity. The internal inductance must be of the order of $1/2$. This condition, $L_1 \ll 1$, requires that $\alpha$ has to be of the order of unity as is seen from Fig.2. For the conventional operation regime where the safety factor is around 5, $L_1$ must be larger than 0.4. (See, for
instance. Fig. 6 of Ref. [15].) This condition requires

$$\alpha < 2.$$  \hfill (6)

The equations (4) and (6) implies that the current diffusivity must be larger than a critical value. The enhancement of the current diffusivity usually is associated by the increment of the other cross field transport coefficient, such as the thermal diffusivity $\chi$ and ion viscosity. The current diffusivity $\lambda$ has a relation between the electron shear viscosity $\mu_e$ as [16,17]

$$\lambda = \frac{\mu_0 c^2}{\omega_p^2} \mu_e.$$  \hfill (7)

where $c$ is the speed of light and $\omega_p$ is the plasma oscillation frequency. The electron shear viscosity and thermal transport coefficient can be of the same order of magnitude. It is often argued that the current, which is driven near edge by dc helicity current drive, is transferred into the core region by the magnetic braiding. We here study the case that the current diffusion is due to the magnetic braiding.

When the cross field transport coefficient is driven by the magnetic braiding, $\lambda$ and $\chi$ are estimated as [17]

$$\mu_e / \chi = \pi / 6.$$  \hfill (8)

$$\lambda / \chi = \pi \mu_0 c^2 / 6 \omega_p^2.$$  \hfill (9)
and

\[ \lambda = \left[ \sqrt{\pi} c^2 / 2 \omega_p^2 \right] \mu_0 v_{te} D_M. \]  

(10)

where \( v_{te} \) is the electron thermal velocity and \( D_M \) is the diffusion coefficient of the magnetic field line[18]. From the upper bound of the parameter \( \alpha \) and the relation (4) and (9), the electron thermal transport coefficient must be larger than some critical value. [This implies of cause the upper bound for the confinement time.] Using Eq.(4), Eq.(9) is rewritten as

\[ \lambda = \frac{6 \omega_p^2 a^2 \eta}{\pi c^2 \mu_0} \frac{1}{\alpha^2}. \]  

(11)

The condition (8) then requires

\[ \lambda > \frac{3 \omega_p^2 a^2 \eta}{2 \pi c^2 \mu_0}. \]  

(12)

We take the classical parallel resistivity \( \eta \). In the MKSA unit \( \eta \) is estimated as \( \eta \sim 3 \times 10^{-8} T_{\text{keV}}^{-3/2} \) for the pure hydrogen plasma, where \( T_{\text{keV}} \) is the electron temperature measured in the unit of keV. The relation (11) can be rewritten as

\[ \lambda \approx 2 \omega_p^2 a_m^2 n_{20} \eta T_{\text{keV}}^{-3/2} \times 10^5 \quad [m^2/s]. \]  

(13)
where $a_m$ is the minor radius measured in m and $n_{20}$ is the density measured in $10^{20} \text{m}^{-3}$. From this relation we see that the condition on $\alpha$, i.e., $\alpha < 2$, requires the thermal transport coefficient of the order of $10^3 \text{m}^2/\text{s}$ for the parameters of $T=10 \text{keV}$, $a=1 \text{m}$, $n=10^{20} / \text{m}^3$. Eq. (12) gives the lower bound of $\chi_{\text{min}} \sim 1600 \text{m}^2/\text{s}$ for these parameters. This necessary value of the energy transport coefficient is too large for the experimental applications.

In summary, we studied the steady state toroidal plasma where the toroidal current is generated near edge and transferred to the core region by the cross field transport of the plasma current. If the current diffusivity is high, such a scheme for the DC helicity current drive allows enough plasma current at the axis. However, the quantitative estimate on the required value for the current diffusivity shows that the energy transport coefficient must also be high, so that the actual application to high temperature plasmas would be difficult. This indicates the importance of the method to transport the current, for which the rf helicity current drive was shown to work\(^{11}\).

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Figure Caption

Figure 1  Schematic illustration of the current profile. The toroidal current is driven near edge (or in the scrape-off layer) and diffuses into the core plasma. In the core plasma, the diffusion and the resistive decay balance to establish stationary solution. The current drive by DC helicity injection is applied in the hatched area.

Figure 2  Internal inductance $l_i$ as a function of the parameter $\alpha$. 
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