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A Model of Major Disruption in Tokamaks

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Abstract

A mechanism for the onset of the major disruption is proposed which considers the effect of magnetic stochasticity on the growth of the $m=2$ tearing mode. The toroidicity can cause stochasticity near the separatrix of the $m=2$ magnetic island which enhances the current viscosity, resulting in explosive growth. The threshold condition and the time scale of rapid growth are studied. The role of the toroidal coupling to the $m=1$ component is also discussed.

Keywords: Major disruption, Stochasticity, Current diffusivity, Tokamak, Toroidicity, $m=2$ tearing mode, Explosive Growth
Major disruptions in tokamaks, which suddenly terminate the plasma discharge, have been known from the beginning of tokamak research\(^1\). Experimental investigation has clarified that the growth of low \((m,n)\) helical deformations \((m\) and \(n\) are poloidal and toroidal mode numbers, respectively\), brought about by various operating conditions, can cause a major disruption\(^1-7\). The intrinsic feature of the major disruption is the occurrence of a sudden central temperature collapse (thermal quench), followed by a rapid change of the internal inductance (redistribution of the current). These abrupt changes cause a strong interaction of the plasma with the wall/limiter, resulting in an enhanced impurity radiation which leads to the final collapse of the plasma energy and current (current quench). The essential features of the phenomena are the thermal quench and the redistribution of the internal flux. The \(m=2/n=1\) tearing mode\(^8\) has been a candidate in theoretical modelling of a major disruption, and the mode has been known in experiments to be an important component\(^2-5\). The nonlinear theory\(^9\) of the single \(m=2\) tearing mode, however, has shown that this mode only grows with the slow resistive diffusion time, \(\tau_T = \mu_0 r_2^2/\eta\) (where \(r_2\) is the minor radius of the \(q=2\) rational surface, \(q\) is the safety factor \(r_Bp/\widehat{RBp}\), \(R\) is the major radius, subscripts \(p\) and \(t\) denote poloidal and toroidal directions, respectively, and \(\eta\) is the resistivity). Other nonlinearities such as the effect on the current shape\(^10-12\) and the effect of the interaction with modes of different helicity\(^13\) have been proposed. Particular emphasis was put on the study of the interaction between the \(m/n=2/1\) and \(3/2\) modes using computa-
tional codes. The computations\textsuperscript{13-17} have shown that the nonlinear destabilization takes place when the magnetic islands of m/n=2/1 and 3/2 modes overlap, stimulating the development of the theory of nonlinear growth with modified resistivity\textsuperscript{18,19}. More detailed computations, however, have shown that the interaction between the 2/1 and 3/2 modes does not cause the destruction of the central part of the plasma, and thus does not fully explain a major disruption\textsuperscript{20}. The additional important role of the m=1 mode, in experiments, has recently been recognized\textsuperscript{4-7}.

In this note we outline a model of the major disruption, including the development of stochasticity from the nonlinear interaction of the m/n=2/1 mode with the toroidicity. This extends our previous investigation on the role of magnetic stochasticity on sawtooth activity\textsuperscript{21}. Explosive growth of the mode is found due to the enhanced current diffusivity, when the mode amplitude exceeds a certain criterion that is a function of the magnetic shear. We also discuss the effect of the toroidal coupling to the m=1 internal mode\textsuperscript{22,23}. The mechanisms described in this article are qualitative, used for obtaining physical insight.

The magnetic field structure is described by the Hamiltonian formulation\textsuperscript{24}. Introducing the Hamiltonian $H(J, \theta, \phi) (J={r^2}/2, \theta$ and $\phi$ are the poloidal and toroidal angles, respectively), the field line follows $\partial J/\partial \phi = -\partial H/\partial \theta$, $\partial \theta/\partial \phi = \partial H/\partial J$. The unperturbed Hamiltonian is given as $H_0 = \int \mathcal{L}(J) dJ$, where $\mathcal{L}$ is the rotational transform, $I/q$. In the presence of the m/n=2/1 perturbation, the
\[ m=2 \text{ island is approximately described by the pendulum Hamiltonian around } q=2 \text{ rational surface as} \]

\[ K_0[\hat{A}_0, \hat{\theta}] = G(\hat{A}_0)^2/2 + F \cos(\hat{\theta}) \]  

(1)

where \( \hat{A}_0 = J/2 - J_0/2 \), \( \xi(J_0) = 1/2 \), \( \hat{\theta} = 2\theta - \Phi \), \( G = 4\alpha L/4J [at J = J_0] \), \( F = J_0 B_n \), and \( B_n \) is the normalized amplitude \( B_n = \tilde{B}_r / r B_\Phi \) of \( m/n = 2/1 \) mode perturbation. The island size in the J space is given by \( \hat{A}_0^2 = 2\sqrt{F/|G|} \), and the rotational number around the O-point of the island is given as \( \omega_0 = \sqrt{F/|G|} \). \( B_n \) has a J dependence since \( \tilde{B}_r \) depends on \( r \). In the spirit of obtaining analytic insight into the phenomena, we here take the value at \( J = J_0 \) and treat \( F \) and \( G \) as constant.

The nonlinear interaction of the main island with the toroidicity causes secondary islands to appear, which overlap near the separatrix leading to stochasticization of the flux surfaces24). The thickness of the stochastic layers near the O-point, \( \delta J \), and the x-point, \( \delta J_x \), can be evaluated in terms of \( F \) and \( G \)21,24), and \( \delta J_x = \sqrt{\hat{A}_0^2 \delta J} \). Figure 1 illustrates the schematic drawing of the stochastic layer near the island separatrix. The growth of the stochastic layer is exponential in the perturbation strength. We find that \( \delta J_x \) becomes thicker than the characteristic current layer width associated with tearing mode when \( \omega_0 > 1/15 \), and that \( \delta J \) can be of the order of the distance between major island chains when \( \omega_0 > 1/6 \). The condition \( i) \hat{A}_0^2 \sim J_0/2 \) means that the \( m=2 \) magnetic island reaches the axis, and is sufficient to reconnnect the magnetic flux inside of the q=2 surface. The condition
(ii) $\omega_0 t/6$ corresponds to the stochastic layer reaching the central part of the plasma, i.e., the rapid heat loss resulting in the energy quench. The condition (iii) $\omega_0 t/15$ allows the rapid growth of the mode as seen in the following.

Figure 2 illustrates these conditions in the space of magnetic shear, $s = r(d\xi/dr)/\xi$ at the $q=2$ rational surface, and the normalized magnetic perturbation amplitude $B_n$. [There are three cases to be considered: (I) a monotonic $q$-profile with $q>1$ on axis; (II) a monotonic $q$-profile with $q<1$ on axis; and (III) a hollow $q$-profile with $q=1$ at some internal radius. Figure 2 corresponds to case(I), but is qualitatively the same for other cases.] When the shear is strong, $s>0.3$, there appears a region where the stochasticity plays a dominant role (condition (iii)), i.e., the region between lines (1) and (3) in Fig. 2. It is also noted that the stronger the shear parameter $s$, the lower the threshold amplitude.

Explosive growth is possible when stochasticity appears near the separatrix. Then current diffusivity appears in Ohm's law, $E+\nabla B = \eta j - \lambda \nabla^2 j$ ($j$ being the current density), and if the diffusive term is greater than the resistive term, the growth becomes:

$$\sqrt{\frac{\partial B_n}{\partial t}} = \frac{2}{3} C^{3/2} s^{3/2} \frac{\lambda}{\mu_0 r_2^4} \Lambda' r_2$$

(2)

Here $C$ is a numerical constant, $C=0.36$, $\Lambda'$ is the parameter for the tearing mode stability, and $t$ is the time after entering the Rutherford regime. The local shear parameter $s$ also changes
due to the current viscosity, which was evaluated as\textsuperscript{24)

\[
\frac{\partial s}{\partial t} = -9s \frac{\lambda}{\mu_0 r_2^4}.
\]

Combining Eqs. (2) and (3), we have the trajectory in the \((s, B_n)\) plane as

\[
B_n^{3/2} + \left[2c_3^3/2\Lambda' r_2/27\right]s^{3/2} = \text{const.}
\]

For the parameter of interest, i.e., \(s \approx 0\) and below line (1), change of \(s\) along the trajectory is negligible for \(2c_3^3/2\Lambda' r_2/27\) >\(4 \times 10^{-3}\) (or \(\Lambda' r_2 > 0.5\)). If \(\Lambda' r_2 < 0.2\), the trajectory is bent and would not hit line (2). In the following, we consider the large \(\Lambda' r_2\) case, and consider \(s\) to be constant. The magnetic stochasticity near the x-point enhances the current diffusivity, which we have estimated as\textsuperscript{21)}

\[
\frac{\lambda}{\mu_0 r_2^4} = \frac{v_A}{R_0 B_n} \frac{2}{\Gamma_0} \frac{D_M}{D_{QL}}
\]

where \(v_A\) is the Alfvén velocity, \(D_M\) is the diffusion coefficient of the magnetic field lines\textsuperscript{26),} and \(D_{QL}\) is its quasilinear value. The coefficient \(\Gamma_0\) is given by \(\Gamma_0 = \pi^3/2 (c/\omega_p r_2)^2 v_e/v_A\) (\(\omega_p\) is the plasma frequency and \(v_e\) is the electron thermal velocity) and is of the order of \(10^{-4}\) for parameters like the JET tokamak. This coefficient is larger for smaller devices. Combining equations
(2) and (4), we find an explosive growth of the mode when stochastic diffusion switches on, which, in the limit of $D_M / D_{QL} = 1$, gives

$$B_n = \frac{B_{n0}}{(1 - \sqrt{B_{n0} \tau \tau})^2} \quad (6)$$

where $\tau = (Cs)^3 / 2 \Gamma_0 \tau_A \tau_2 / \tau_A^{-1} / 3$. and the origin of time is taken at the time that the growth due to the current diffusivity equals that by the the resistivity, which occurs at the amplitude of $B_n = B_{n0}$. The characteristic time for the explosive growth is $\Delta t = 1 / \sqrt{B_{n0} \tau}$, which is fast and independent of the resistivity. The current diffusivity prevails over the resistivity if $32 \lambda / \omega_2^2 > \eta$ where $\omega_2$ is the width of the $m=2$ island$^{25}$. Substituting the expression for $\lambda$, we have

$$B_{n0} = \left[ \frac{D_{QL}}{D_M} \right] \tau_A / 2 \Gamma_0 \tau_2 \quad (7)$$

If we consider the values $\Gamma_0 \sim 10^{-4}$, $\tau_2 / \tau_A \sim 10^{7-8}$, and $D_M \sim D_{QL}$ for $B_n > 10^{-2}$, we find that $B_n > B_{n0}$ is easily satisfied once the stochasticity switches on; that is, once the condition that $\omega_0 > 1/15$ is satisfied.

For case (II), $q(0) < 1$, the condition that the mode growth is stochastically enhanced is quite similar to that given by curve (3) in Fig.2, and thus the results in Eqs.(6) and (7) still apply. In this regime, the additional condition that stochasticity join the $m/n=2/1$ and $m/n=1/1$ modes needs to be satisfied, which can be
expressed in action variables as \textsuperscript{24}) \((\Delta_{1}^{2} + \Delta_{2}^{2})/\delta J_{12} > 2/3\), where 
\(\Delta_{1}^{2}\) and \(\Delta_{2}^{2}\) are obtained from Hamiltonians of the form (1),
applied to \(m/n=1/1\) and \(m/n=2/1\) islands, respectively, and \(\delta J_{12}\) is
the distance in action space between the \(q=1\) and \(q=2\) surfaces.
Since previous calculations\textsuperscript{21}) indicate that the \(m/n=1/1\) island
will stochasticize field lines to the axis, subject to a
condition on the ion viscosity, we would expect this situation
also leads to a major disruption if the \(m/n=2/1\) island grows
sufficiently to satisfy the \(\omega>1/15\) explosive growth condition.

For case (III), a hollow \(q\)-profile with \(q=1\) at some radius,
a further enhancement of the growth rate is due to the toroidal
coupling of the \(m/n=2/1\) mode to the \(m/n=1/1\) mode\textsuperscript{22,23}). In fact,
it is known that an \(m/n=1/1\) pure MHD instability is possible when
the \(q(r)\) profile is hollow and the difference \(\Delta q=q(0)-1\) is
greater than a critical value \(\Delta q_{c}\)\textsuperscript{27,28}). Using the parameter
\(\Delta q_{c}\), the \(\Delta'\)-value in the toroidal geometry is given as\textsuperscript{23})

\[
\Delta'_{r} = \Delta'_{cyl} r_{2} + D \frac{(\Delta q_{c})^{3/2}}{(\Delta q)^{3/2} - (\Delta q_{c})^{3/2}}
\]  

(8)

where \(\Delta'_{cyl}\) is the \(\Delta'\) parameter calculated in cylindrical
geometry, and \(D\) is a coupling coefficient of the order of \(\varepsilon^{2}\).
This result shows that, by coupling with the \(m=1\) component, the
\(m=2\) tearing mode can have a large \(\Delta'\) value, independent of the
reduction of \(\Delta'_{cyl}\) due to finite mode amplitude. The condition
that \(\Delta q\) reach \(\Delta q_{c}\) is one mechanism for initiating the \(m/n=2/1\)
mode growth until it satisfies the explosive condition and
consequent disruption.

The ion viscosity may stabilize the mode as found in an investigation of m/n=1/1 island growth. To see it, we estimate the typical time scale of viscosity damping by \(1/\tau_{vis} = \mu_i/\omega_2^2\), where \(\mu_i\) is the stochasticity-enhanced ion viscosity. Compared to the typical growth time \(B_n/B_0\) of Eq.(2), the damping time \(\tau_{vis}\) is longer than the growth time if \(\Delta' r_2 > 40\) for the typical parameters; viscous damping can be neglected in case (III) in which \(\Delta' r_2\) is much enhanced. Otherwise, the ion viscosity may be important in the dynamics (at least this reduces the growth rate), and we leave this for future analysis. We also point out that the interaction with them/n=3/2 island also occurs, whether this mode is unstable or not, due to the nonlinear coupling. This particular interaction is not necessary to obtain the explosive growth, but it is part of the general interaction of the m/n=2/1 island that leads to the stochasticity.

We compare these mechanisms to observations. We see that the stochastic heat transport is consistent with the understanding that the thermal quench, following the precursor, is the first among the drastic phenomena connected with a disruption. If the mode growth continues until the full reconnection reaches the axis, the internal inductance and current distribution can flatten rather rapidly inside of the plasma column, due to an electron-viscosity-aided reconnection. The dependence of the critical amplitude on the magnetic shear also agrees well with the experimental observations that high shear at the \(q=2\) surface more easily gives rise to a major disruption.
Toroidal coupling to the $m=1$ component is consistent with the experimental observation, that, in the phase leading to the thermal quench, the $m=1$ deformation has a particular phase relation to the $m=2$ mode$^{4,5}$. At the toroidal angle where the $m=2$ crescent shape island appears on the top and bottom of the torus, the central $n=1$ motion is outward, which is consistent with the toroidal tearing mode$^{29}$. This relation was found on JIPP-TIIU under various conditions$^5$ and was also confirmed on JET$^{30}$. The unique phase relation between $m/n=2/1, m/n=1/1$ shows that the coupling is caused through toroidicity.

It is also noted that the sawtooth often disappears before the major disruption takes place. For instance, Fig. 3 of Ref.[5] indicates that, prior to the major disruption, the inversion radius of the sawtooth converges to the axis and the sawtooth disappears. This observation is consistent with a situation in which the trigger occurs when $Aq$ reaches $Aq_C$. In the case of a low-q disruption, the sawtooth does not necessarily disappear. This implies another mechanism for the initial trigger, such as increasing shear, which allows sufficient stochasticity at the X-point.

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References


Figure Caption

Fig.1  Magnetic island and stochasticity region (hatched region) are illustrated. $\Delta J_M$ and $\Delta J_X$ indicate the width of the main magnetic island and the thickness of the stochastic layer near the X-point, respectively.

Fig.2  Domains of stochasticity and fast growth on the $s-B_n$ plane. The upper solid line (1) indicates the condition that $m=2$ island expands to near axis. The lower solid line (2) denotes the condition that the stochastic region reaches near the axis. The dotted line (3) shows the boundary, above which stochasticity enhances the mode growth. The stochasticity can be important for the high shear region, $s>0.3$. 

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