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On Turbulent Transport in Burning Plasmas

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Abstract: The change of the transport coefficient due to the fusion energy source is studied. The scale invariance property of the reduced set of equations is investigated in the presence of the self-heating term due to the fusion reaction. The pressure gradient as well as the fusion power are the free energy sources that dictate the turbulent transport. It is shown that the burning transport coefficient can have a form with much wider variety, and that the transport property could be different owing to the self-heating by the fusion reactions.

Keywords: turbulent transport, burning plasma, scale invariance, strong turbulence, gradient

One of the key issues for the high temperature plasmas is the turbulence-driven transport of burning plasmas which are confined in toroidal magnetic confinement devices. In order to study the plasma properties near the fusion-ignition conditions, a design study like ITER has been performed [1]. In the design study, the transport property is estimated based on an extrapolation of the empirical formula of energy confinement time [2]. Detailed analysis on the present database of energy confinement time has been performed, and the projected value is obtained with a window of uncertainty. Such a projection has been done by assuming that the qualitative nature of turbulence-driven transport is not altered by the strong heating due to the nuclear fusion reaction. The validity of this assumption could be tested only after the transport property of ignited plasmas will be measured; nevertheless the investigation would be necessary from a theoretical view point.

The assumption, i.e., that the transport property in ignited plasmas will be the same as those observed today, is often based on the arguments that the empirical formula of confinement time has been satisfied being independent of the heating methods (e.g., radio frequency wave heating, neutral beam injection heating, etc.) However, this argument does not suffice. First, the confined plasmas are subject to bifurcations [3], and the characteristics of the bifurcation change in ignited plasmas owing to the nonlinear coupling between the fusion output, plasma profile, impurity and ash evolution and the electro-magnetic fields (in other words, the radial electric field and current profile) [4]. Second, the energy confinement scaling law has been established under the condition that the energy input is externally fixed and controlled. In burning plasmas, the constraint of energy input becomes different. In the far nonequilibrium matter (like inhomogeneous and confined plasmas), the transport property can change depending on the constraints of the energy input.
For instance, the transport nature can be different whether the energy input is given and fixed or the gradient is given and fixed. This is in contrast to the systems near thermodynamical equilibrium, in which the transport coefficient is not altered by the variation of the energy input. The simple extrapolation of empirical confinement law to the burning plasmas is often performed, but does not necessarily have a firm basis.

The method of scale invariance has been applied to the problem of turbulent transport. From the scale-invariance property of the basic equations, several features of turbulence-driven transport have been derived [5-8]. For instance, two types of anomalous transport coefficients, Bohm-like diffusion and gyro-Bohm-like diffusion have been derived. For the gyro-Bohm type transport, the linear and nonlinear analyses have been developed and deductive models of transport coefficients have been given. Concerning the Bohm-like diffusion, a relevant linear instability has not been discussed so far, but its presence has been widely recognized. For instance, the experiments on D-III D have shown that the L-mode confinement is composed of both the Bohm-like and gyro-Bohm-like transports [9]. In addition, the analysis on the transient response of the plasma profile has suggested that there is a component of energy flux which is controlled by fluctuations with very long correlation lengths [10]. The scale invariance method is useful to understand the characteristic change of transport properties, even if the deductive theory for a particular plasma dynamics is not given. We here study the scale invariance property of the reduced set of equations in the presence of the self-heating term due to the fusion reaction. The change of the transport coefficient due to the fusion energy source is discussed. It is shown that the transport coefficient can have a form with much wider variety owing to the self-heating, and the transport property could be different in fusion plasmas.

Let us study current diffusive interchange mode turbulence [11], which is analyzed either by an analytic theory or by a direct numerical simulation in previous work [12-14]. The anomalous transport model based on this theory captures some essential elements in experimental observations on the L-mode as well as various improved confinement modes. It is relevant to study the change of this transport model in the presence of fusion reactions.

We choose a model equation of Chapter 14 of [3] which consists of the vorticity equation (equation of motion), Ohm's law and energy balance equation. In the presence of the fusion ignition, the energy conservation relation is written as

$$\frac{d}{dt}P = \frac{1}{3} P_f$$  \hspace{1cm} (1)$$

where $P_f$ is the perturbation component of the heating power density by fusion process. The electron temperature could be higher than ion temperature in ignited plasma, because the fusion alpha particles dominantly heat electrons. We in this article do not distinguish the electron energy and ion energy, and the plasma energy density is simply estimated by $3\rho$. Noting that the perturbation of fusion power $P_f$ depends on the local pressure perturbation, and we symbolically
write \( P/\beta = F p \) i.e., \( F = 3 \cdot 1\partial P/\partial \rho \). A set of
equations describes the nonlinear dynamics of
perturbations of the static potential \( \Phi \), current \( j \)
and pressure \( p \) as

\[
\frac{d}{dt} \begin{pmatrix} \nabla \phi \int_j \left( \begin{array}{c} \phi \\ j \\ p \end{array} \right) + \left( \begin{array}{ccc} 0 & -ik & ik \cdot G_0 \\ ik & 0 & 0 \\ ik \cdot F & 0 & F \end{array} \right) \left( \begin{array}{c} \phi \\ j \\ p \end{array} \right) = 0 \end{pmatrix}
\]

(2)

where

\[
\frac{d}{dt} = \frac{d}{dt} + \left[ \phi, \ldots \right]
\]

is the Lagrange derivative which includes the nonlinearities owing to the \( E \times B \) motion, and \( G_0 \) is the normalized pressure gradient being divided by the gradient length of magnetic field. The Cartesian coordinate is used, and the \( x \) - axis is in the direction of the pressure gradient, \( y \) - axis is on the magnetic surface and is perpendicular to the magnetic field. \( k_\parallel \) is in the mode number in the direction of the main magnetic field.

Dimensionless form is employed, and normalization is explained in [6]. In Eq. (2) the term \( F p \), i.e., the variation of the fusion energy source, is added, noting the fact that the fusion energy source is dependent on the local plasma pressure.

In order to clarify the arguments, collisional dissipation terms are neglected. In Eq.(2), the matrix includes the inhomogeneity parameter \( G_0 \) and the term \( F \) which represents the fusion reaction.

Let us consider a transformation

\[
x \rightarrow \lambda_s x \quad \text{and} \quad t \rightarrow \lambda_2 t
\]

(4)

Field quantities are transformed as [6]

\[
\begin{pmatrix} \phi \\ j \\ p \end{pmatrix} \rightarrow \begin{pmatrix} \lambda_2 \lambda_2^{-1} \phi \\ \lambda_2^{-1} j \\ \lambda_2^{-1} p \end{pmatrix}
\]

(5)

In the equation of energy balance, two terms \( d p / dt \) and \( F p \) follow the same transformation.

That is, the transformation

\[
F \rightarrow \frac{1}{\lambda_2} F
\]

(6)

holds. From the equation of motion (vorticity equation), a transformation

\[
G_0 \rightarrow \frac{1}{\lambda_2^2} G_0
\]

(7)

is required. From Eqs.(6) and (7), we can choose an arbitrary parameter \( \tau \) and a combination of parameters \( G_0^{\eta \gamma} F^{1-n} \) obeys the scale transformation

\[
G_0^{\eta \gamma} F^{1-n} \rightarrow \frac{1}{\lambda_2} G_0^{\eta \gamma} F^{1-n}
\]

(8)

Case of finite \( k_\parallel \) is studied first. In this case, the term \( k_\parallel j \) remains in the equation of motion. Requiring that this term has the same scale-invariant property as other terms, one has a relation [6]

\[
\lambda_2 = \frac{1}{\lambda_2}
\]

(9)

By use of Eq.(9), one finds that the turbulent transport coefficient, \( D \), which has the dimension
of \( D = \text{length}^2 \text{time}^{-1} \), obeys the transformation relation

\[
D \rightarrow \frac{1}{\lambda_2} D.
\]  
(10)

Combining Eqs. (8) and (10), we have the relation that the quantity

\[
\frac{D}{\left( G_0^{\eta_2} F^{1 - \eta} \right)^3}
\]  
(11)

is invariant under the scale transformations Eq.(4). In other words, one obtains that the transport coefficient has the dependence like

\[
D \propto \left( G_0^{\eta_2} F^{1 - \eta} \right)^3
\]  
(12)

This result is the generalization of the transport coefficient. In the absence of the fusion power, the dependence

\[
D \propto G_0^{\eta_2}
\]  
(13)

has been derived, the explicit form of which was given as the CDIM transport coefficient.

Equation (13) is a special case of Eq.(12) with \( \eta = 1 \). In the presence of fusion reaction, more general form Eq.(12) is allowed for the transport coefficient.

Next, the case of vanishing \( k_1 \) is studied. In this case, coupling between the equation of motion and Ohm's law disappears. The dynamical equations consist of vorticity equation and energy balance equation as

\[
\frac{d}{d\tau} \begin{pmatrix} \nabla \cdot \phi \\ p \end{pmatrix} + \begin{pmatrix} 0 & ik_c G_0 \\ ik_c F \end{pmatrix} \begin{pmatrix} \phi \\ p \end{pmatrix} = 0.
\]  
(14)

The transformation Eq.(4) with

\[
\begin{pmatrix} \phi \\ p \end{pmatrix} \rightarrow \begin{pmatrix} \lambda_2^{-1} \lambda_1 \phi \\ \lambda_1 p \end{pmatrix}
\]  
(15)

is considered. Under this transformation, Eq.(14) is invariant with \( \lambda_1 = 1 \) and Eq.(8). From the relation \( \lambda_1 = 1 \), the transport coefficient is found to obey the transformation rule

\[
D \rightarrow \frac{1}{\lambda_2} D
\]  
(16)

Combining Eqs.(8) and (16), one finds that

\[
\frac{D}{G_0^{\eta_2} F^{1 - \eta}}
\]  
(17)

is invariant under this scale transformations. That is, the diffusivity satisfies the relation

\[
D \propto G_0^{\eta_2} F^{1 - \eta}
\]  
(18)

This is a generalization of the result

\[
D \propto G_0^{\eta_2}
\]  
(19)

which was obtained in [6] for the long wavelength-limit (in the absence of fusion power). More general form is allowed to appear in the near-ignition plasmas.

In summary, based on the scale-invariance property of the reduced-set of equations,
the transport coefficient has a wider freedom in ignited plasmas compared to those which are sustained by the external fixed power source. A general form of the transport coefficient in the ignited plasma is obtained as Eqs.(12) and (18). The new formula for the ignited plasmas includes that of the externally-sustained plasmas as a special limiting case. The analysis in this note indicates the possibility that the transport property of burning plasmas could be qualitatively different form those which are observed now and in the past. It should be noted that the scale-invariance method does not provide the quantitative level of transport coefficient. The derived form of transport coefficient suggests that the simple extrapolation of energy confinement time to the ignited plasma might not hold; however, it does not conclude whether the confinement is better or worse in ignited plasmas in comparison with the prediction based on the scaling law.

The dependence of the transport coefficient on the fusion power causes the variation of the power degradation of energy confinement time. Let us take an example that the fusion power depends on the local pressure as \( P = P_0^\mu \) (\( \mu \) being a constant) where \( P_0 \) is the plasma pressure. The coefficient \( F \) is then expressed in terms of the fusion power as 
\[
F = P^{(\mu - 1)\nu}.
\]
The power balance equation
\[
\frac{P}{aR} = D \frac{P_0}{a}
\]  
(20)

and
\[
\nu = 2 \left( 1 - \frac{3(\mu - 1)}{\mu} \frac{1 - \eta}{2 + 3\eta} \right)
\]  
(21b)

Substitution of Eq.(21) into Eq.(20) provides the power degradation of energy confinement time \( \tau_e \) as
\[
\tau_e \approx P^{-\alpha}
\]  
(22a)

with
\[
\alpha = \frac{3}{5} \left( 5\eta + \frac{10}{2 + 3\eta} \frac{\mu - 1}{\mu} (1 - \eta) \right)
\]  
(22b)

When the fusion power has a strong pressure dependence, \( \mu > 5/4 \), the power degradation of \( \tau_e \) become worse, i.e., \( \alpha > 3/5 \), in comparison with the case where the transport coefficient is independent of the fusion power \( \eta = 1 \).

The diffusion coefficient in the limit of \( k_t = 0 \), Eqs.(18) and (19), is much greater than that in the case of \( k_t \neq 0 \) [3]. Usually there is a magnetic shear in toroidal plasmas and the condition \( k_t = 0 \) is satisfied only on the resonant magnetic surface. If it is so, the diffusion formula of \( k_t = 0 \) does not hold. However, there could exist a finite-amplitude global magnetic island in tokamaks which is driven, e.g., by nonlinear tearing instability [16]. Under such circumstance, global perturbation of fusion energy source with the condition of \( k_t = 0 \) could take place and drives a new type of transport phenomena. The analysis
of Eq.(18) indicates the importance of the combined dynamics of the fusion ignition and magnetic island sustainment.

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