L-Mode Confinement Model Based on Transport-MHD Theory in Tokamaks

K. Itoh, M. Yagi, S.-I. Itoh, A. Fukuyama and M. Azumi

(Received – Sep. 22, 1992)

NIFS-192

RESEARCH REPORT
NIFS Series

This report was prepared as a preprint of work performed as a collaboration research of the National Institute for Fusion Science (NIFS) of Japan. This document is intended for information only and for future publication in a journal after some rearrangements of its contents.

Inquiries about copyright and reproduction should be addressed to the Research Information Center, National Institute for Fusion Science, Nagoya 464-01, Japan.
L-Mode Confinement Model
Based on Transport-MHD Theory in Tokamaks

K. Itoh*, W. Yagi†, S.-I. Itoh**, A. Fukuyama††, and M. Azumi†

* National Institute for Fusion Science, Nagoya 464-01, Japan
† Japan Atomic Energy Research Institute, Naka.
Ibaraki 311-01, Japan
** Institute for Applied Mechanics, Kyushu University 87.
Kasuga 816, Japan
†† Faculty of Engineering, Okayama University, Okayama 700, Japan

Abstract

Theory of the L-mode confinement in tokamaks is developed based on the microscopic ballooning instability which is destabilized by the plasma transport below the critical pressure gradient against the ideal MHD instability. The destabilization by the current-diffusivity and the stabilization by the thermal transport and ion viscosity, are analyzed. The least stable mode determines the anomalous transport coefficients. The formula of the thermal transport coefficient is derived, which explains major experimental observations on the L-mode confinement.

Keywords: L-Mode, Anomalous Transport, Current Diffusivity, Tokamak, Ballooning Instability, Prandtl Number

1
The transport of the plasma across the magnetic field in tokamaks is much faster than that by the Coulomb collision. It has been known as the anomalous transport\(^1\). The L-mode confinement, in which the energy confinement time \(\tau_E\) decreases as the heating power \(P\) is increased, is observed in all tokamaks\(^2,3\). The L-mode confinement is a generic nature of tokamak plasmas. The database has been made on how \(\tau_E\) depends on the externally controllable parameters, such as \(P\) and the plasma current \(I_p\)\(^3\). The radial profile of the effective thermal conductivity \(\chi\) (the ratio of the energy flux per particle to the temperature gradient \(\nabla T\)) has been studied\(^4\). It has been confirmed that the microscopic fluctuations play important roles for the anomalous transport\(^4\). The ion viscosity is also enhanced and of the same order of the electron and ion thermal conductivities\(^5\). Ohkawa's model, based on the fluctuations of the scale length of the collisionless skin depth \(\delta\)\(^6\), is one of the few which could explain the large \(\chi\)-value near edge and at high temperature, but does not fully explain the dependences of \(\tau_E\). Theoretical development has been made based on the mixing length estimate\(^7\), scale invariance method\(^8\) or one/two point renormalization methods\(^9\). These gave identical results from the viewpoint of the physics argument\(^10\). No theory has succeeded in explaining the radial shape of \(\chi\) and the scaling \(\tau_E(P, I_p, \cdots)\) simultaneously\(^11\). The understanding of the L-mode confinement is far from satisfactory.

Microscopic modes grow in tokamaks extracting the free energy source of the pressure gradient. The shear-stabilized plasma can become unstable due to the fluctuation-driven trans-
port, and the analysis on the self-sustaining turbulence has been developed\textsuperscript{12-15)}. A model of the subcritical turbulence of the interchange mode was proposed to explain the anomalous transport in helical plasmas\textsuperscript{15}). We apply this method to tokamaks. Below the beta-limit of the ideal magnetohydrodynamic (MHD) mode, the microscopic ballooning mode can be unstable if there is the plasma transport such as the current-diffusivity $\lambda$. We find that $\lambda$ destabilizes the mode while other transport coefficients, $\chi$ and the ion viscosity $\mu$, stabilize it. The unstable microscopic ballooning mode enhances the transport coefficients $\lambda$, $\chi$ and $\mu$. If the anomalous transport is enhanced much, then $\chi$ and $\mu$ stabilize the mode. The transport coefficients are determined by the marginal stability condition for the least stable mode. The result on $\chi$ is compared to the experimental observations and shows agreements.

We study the circular tokamak with the toroidal coordinates ($r, \theta, \xi$). The reduced set of equations\textsuperscript{16}) is employed. The current diffusivity is due to the electron viscosity and is kept in the Ohm's law as\textsuperscript{17}) $\mathbf{E} \times \mathbf{v} \times \mathbf{B} = \mathbf{J}/\sigma - \mathbf{V}^{2} \lambda \mathbf{J}$ ($\sigma$ is the conductivity). The equation of motion is given as $n_{i} m_{i} \{d(\mathbf{V}^{2} \phi)/dt - \mu \mathbf{V}^{4} \phi\} = B_{//} \mathbf{J}_{//}$ $+ \nabla \times \nabla (2 r \cos \theta / R)$ and the energy balance equation $\mathbf{d}\rho/\mathbf{d} t = - \mathbf{V}^{2} \rho$ is employed, where $m_{i}$ is the ion mass, $n_{i}$ is the ion density, $\phi$ is the stream function, $B$ is the main magnetic field, $\rho$ is the plasma pressure, and $\mathbf{J}$ is the current.

The ballooning transformation is employed as\textsuperscript{18}) $\phi (r, \theta, \xi) = \sum_{n} \exp(-i m \phi + i n \xi) \phi(\eta) \exp(i m \eta - i n \eta) d\eta$, ($q$ is the safety factor) since we are interested in the microscopic modes. The linearized
equation is reduced to the ordinary differential equation as \(^{19}\)

\[
\frac{d}{d\eta} \frac{F}{\tau + \Sigma F + \alpha F^2} \frac{d\phi}{d\eta} + \frac{\alpha[\kappa + \cos(\eta - \alpha \sin \eta) \sin \eta]}{\tau + \Sigma F} = 0 \quad (1)
\]

We use the normalizations \( r/a \rightarrow r, \ t/\tau_{Ap} \rightarrow t, \ \tau_{Ap}/a^2 \rightarrow \tau, \ \mu \tau_{Ap}/a^2 \rightarrow \mu, \ \tau_{Ap}/\mu_0 a^2 \rightarrow 1/\alpha, \ \lambda \tau_{Ap}/\mu_0 a^4 \rightarrow \lambda, \ \tau_{Ap} = \sqrt{\mu_0 \rho m_1 \beta_p}/\tau_{Ap}, \ \tau_{Ap} \rightarrow \tau. \) Notations: \( \hat{\tau} = n^2 q^2 / \beta, \ \Lambda = \tau n^2 q^4, \ X = \tau n^2 q^2, \ M = \mu n^2 q^2, \ \gamma \) is the growth rate, \( \sigma = r(dq/dr)/q, \ F = 1 + (s \eta - \alpha \sin \eta)^2, \ k = -(r/R)(1 - 1/q^2) \) (average well), \( B_p = Br/qR, \ \alpha - q^2 \beta' / \epsilon, \ \epsilon = r/R, \ a \) and \( R \) are the major and minor radii, \( \beta \) is the pressure divided by the magnetic pressure, and \( \beta' = d\beta/d(r/a). \) If we neglect \( \Lambda, \ \tau \) and \( \mu, \) Eq. (1) is reduced to the resistive ballooning equation. The ideal WHD mode equation is recovered by further taking \( 1/\alpha = 0, \) Since \( \kappa \) is small but negative, the interchange mode is stable and the ballooning mode is the most unstable. Equation (1) predicts that the current-diffusive ballooning mode has a large growth rate. We take \( 1/\alpha = 0 \) for simplicity. (This is appropriate as is shown at the end of this article.) The growth rate of the short wave length mode, driven by the \( \tau \) term, is first estimated by the Wentzel-Kramers-Brillouin (WKB) method by neglecting \( \Lambda \) and \( \mu \) terms. We have

\[
\frac{\pi}{4} = \int_0^{\eta_C} d\eta \sqrt{1 + \alpha F^2 \tau / \tau_{Ap}^2} \sqrt{\alpha(\cos \eta - (s \eta - \alpha \sin \eta) \sin \eta)} \tau^2 \beta
\]

where the kernel of the integral vanishes at \( \eta = \eta_C \) and the well
term is neglected. For the analytic insight, we take the short wave length limit, \( A/\tau \gg 1 \), which yields \( \sqrt{1/F+AE/F} = \sqrt{AF}/\tau \). By approximating \( dF/d\eta \approx 2s/\tau \), Eq. (2) is reduced to \( \pi/4 = [\sqrt{A/\tau}/2s] \sqrt{\alpha F} \), where \( F_c = F(\eta_c) \). We also consider the case that the inertia term determines \( \eta_c \), having the estimate \( F_c = \alpha/\tau^2 \). Using these limiting approximations, the dispersion relation (2) is written as \( \pi/4 = [\sqrt{A/\tau}/2s] \alpha^{3/2}/\tau^2 \), or

\[
\tau \sim \lambda^{1/5} \alpha^{3/5} s^{-2/5}.
\]  

(3)

Since the exponent to \( \tau \) is 1/5, even the very small current diffusivity gives rise to the ballooning instability. The condition \( A/\tau \gg 1 \) requires \( \lambda n^4 q^4 \alpha^{3/4}/\sqrt{s} \).

This large growth rate is confirmed for a wide range of parameters by the numerical calculation\(^{19}\). Figure 1 illustrates \( \alpha \) vs \( \tau \) and \( \lambda \) vs \( \tau \), keeping \( \lambda/\tau \) and \( \tau/\mu \) constant. The analytic estimation for small \( \lambda \) is confirmed. As the transport coefficients are increased much, the stabilizing effects by \( \tau \) and \( \mu \) overcome the destabilizing effect of \( \lambda \).

The stability boundary is derived. Setting \( \tau = 0 \) in Eq. (1), we have the eigenvalue equation, which determines the relation between \( \tau \), \( \lambda \) and \( \mu \). We here study the case that the ballooning mode is destabilized by the normal curvature, not by the geodesic curvature, i.e., \( 1/2 + \alpha > s \). For the strongly localized mode, \( s^2 \eta^2 < 1 \) and \( \eta^2 < 1 \), this eigenvalue equation is approximated by the Weber type equation as
\[ d^2 \phi / d \eta^2 + (\alpha n^2 q^2 / \tau)(1 - (1/2 + \alpha - s) \eta^2) - \alpha n^2 q^2 (1 + 3 s^2 \eta^2) = 0, \]  

and we have the stability boundary as

\[ \alpha^{3/2} \tau^{-3/2} \rho^{-1/2} = N^{-2}(1-N^4)^{-2}((1/2+\alpha-s) + 3s^2N^4) \]  

where \( N \) is the normalized mode number \( N = nq(\tau \mu / \alpha)^{1/4} \). This result is confirmed by the numerical computation as is shown in Fig.2.

It is shown that, when \( (\tau, \lambda, \rho) \) increase, the mode is stabilized by the enhanced transport coefficient. The stabilization is possible when both \( \tau \) and \( \rho \) are finite. The lower boundary of \( \alpha \) for the stability is calculated by obtaining the minimum of the RHS of Eq. (5). When \( N^{-2} \) and the term in \( \{ \) determines the minimum, it is estimated as \( \sqrt{0} \) (neglecting \( \alpha \)-s for the simplicity). In the limit \( s = 0 \), the minimum is given as \( 25 \sqrt{5}/32 \). We have the stability limit of \( \alpha \) for the least stable mode as

\[ \alpha^{3/2} = f(s) \sqrt{\tau}^{3/2} \lambda^{-1}, \]  

where \( f(s) = \sqrt{0} \) or 1.7 (\( s \to 0 \)).

Based on the stability analysis, we can derive the formula for the anomalous transport coefficient. When the mode amplitude and the associated transport coefficients are small, Eq.(1) gives the instability. In the self-sustained state, the enhanced transport coefficients satisfy Eq.(6) for the given pressure gradient \( \alpha \). This state is thermodinamically stable: The excess
growth of the mode and enhanced transport coefficients lead to the damping of the mode. When the mode amplitude and transport coefficients are small, the mode continues to grow until Eq. (6) is satisfied.

From Eq. (6) \( t \) is expressed in terms of the Prandtl numbers \( \beta/\tau \) and \( \lambda/\tau \). (Note that the current-diffusivity \( \lambda \) is originated from the electron viscosity\(^{17}\).) We have

\[
\tau = a^{3/2}(\lambda/\tau)\sqrt{\lambda/\beta} f(s). \tag{7}
\]

In enhancing \( (\lambda, \beta, \tau) \) by fluctuations, the ratio \( \lambda/\tau \) and \( \beta/\tau \) are given to be constant. The relations \( \lambda/\tau = \delta^2/a^2 \) and \( \beta/\tau \approx 1 \) hold for electrostatic perturbations\(^{17,20}\). The formula of \( \lambda \) is obtained in an explicit form as

\[
\lambda = f(s)^{-1} q^2 (RB'/r)^{3/2} \delta^2 v_A/R. \tag{8}
\]

We here note that the usual method for the estimation of \( \lambda \) by \( \tau/k^2 \) for the most unstable mode gives the same results as \( \tau \sim a^{3/2} (\delta/a)^2 \).

This form of \( \lambda \) is consistent with experimental results known for the L-mode. In the following, we compare the theoretical prediction to observations by choosing \( n_i = n_e \) and \( T_i = T_e \).

(i) The dimensional dependence of \( \lambda \) is \([T]^{1.5}/[a][B]^2\). (ii) Not the local beta value but the gradient of \( \beta \) generates \( \lambda \) so that the density and \( q \) profiles governs the radial profile of \( \lambda \). Equation (8) indicates that \( \lambda \) increases towards the edge for the
usual plasma profiles in the L-mode. Experiments on $\alpha$ is reported that $\alpha \approx B_p^{-y} s^{-2}$ with $1 < y < 2$ and $0 < s < 1$ \cite{21}, which is consistent with Eq. (6). (iii) The point model argument of the energy balance, $\tau_\xi = a^2 / \alpha$ and $2\pi^2 a^2 \kappa_n T = \tau_\xi \tau_p$, provides the scaling law

$$\tau_\xi = C a^{0.4} \lambda_p^{1.2} I_p^{0.8} \alpha^{0.6} \Delta^{0.5} \eta^{0.4} \left\{ n_e \lambda_p / \sqrt{A} \right\}^{0.6}, \quad (9)$$

where $C$ is a numerical coefficient, $A$ is the ion mass number, and $\lambda_p$ is the gradient scale length $(nT)/|V(nT)|$. This result is consistent with the L-mode scaling law, including the dependences on $a$, $R$, $I_p$, $P$ and favourable dependences on the ion mass and magnetic shear \cite{22,23}. Slight difference is seen in the the final term in the parenthesis ( ), which is discussed later. (iv) $T_e(r)$ profile is predicted from Eq. (6). The peaking parameter, $T(a)$, only weakly depends on the location of the peak of the power deposition, but depends strongly on $q(a)$ \cite{19}. This explains the 'profile resilience'. (v) Since $\alpha \approx |V(nT)/n|^{1.5}$, the thermal diffusion coefficient deduced from the pulse propagation, $\chi_{HP}$, is larger than that evaluated by the power balance $\alpha$. If $|\nabla_n n_i| \lesssim |\nabla T|/T$ holds, for the simplicity, we have $\chi_{HP} = 2.5 \alpha$. (vi) The typical perpendicular wave number of the most unstable mode satisfies

$$k_{\perp 6} \approx 1/\sqrt{\alpha}. \quad (10)$$

It should be noticed that though the dimensional relation $k_{\perp} \approx [B]/[\sqrt{T}]$ holds, $k_{\perp}$ does not scale with the local gyroradius. The
collisionless skin depth is the more relevant length. The correlation time $\tau_c$ is estimated as $1/\tau = \sqrt{3} \pi \sigma T A^p$. (vii) The estimation $\mathcal{E}/p \sim 1/k_\perp l_p$ shows that the mode amplitude is larger near edge and larger for the high heating power.

In summary, we developed the stability theory of the microscopic ballooning mode in tokamaks under the influence of the anomalous transport coefficients $\chi$, $\lambda$ and $\mu$. It is found that this mode is unstable when the transport is small and finite, but can be stabilized by the thermal transport and ion viscosity. The stability boundary is obtained, and the transport coefficients are derived. The important role of the collisionless skin depth, pointed out in Ohkawa's model, is confirmed: the consistent calculation for stability gives the further explicit dependence on $\mathcal{E}^*$ and the geometrical factor. The form of $\chi$ is compared with experiments. Major part of the observations on L-mode can be explained by this model simultaneously.

Since $\chi$ is dimensionally independent of the density, $\tau_E$ derived in Eq. (9) includes the density dependence (see, e.g., [23]). However, the density dependence is offset by the gradient scale length. In L-mode plasmas, the density gradient profile is often steeper than the temperature near edge, and $l_p$ in Eq. (9) would be replaced by $l_n^* = |n_i/Vn_i|$. The high density plasma has more steeper edge density profile; Tsuji found that $n_e(0)/\bar{n}_e - 1$ is a decreasing function of $n_e$ and $l_n n_e$ is a weak function of the density[24]). For some dataset of JT-60, Takizuka reported[25] the dependence as $\tau_E(\text{thermal}) \propto n_e^{0.5}$, suggesting that the classification of the dataset by the profile is necessary.
We here take various simplifications for the analytic insight. The resistivity is neglected. The similar analysis can be done for the resistive plasma to have the instability boundary $\alpha = \pi \delta/2$. The resistive modes$^{26}$ give higher stability limit of $\alpha$ than the current diffusive modes if $\delta \tau^{1/3} > (\tau/\chi)^{2/3}$ holds. This condition is usually satisfied and supports the simplification. Other extension is necessary for the case that the mode is driven by the geodesic curvature. For such a case, the results in this article are also confirmed, and will be reported elsewhere$^{19}$. The present result (9) is obtained except for the numerical factor. Nonlinear simulation would give this coefficient and examine the validity of the ansatz of $\tau$ that the transport coefficients affecting the microscopic mode is equated to that for the global quantity. Also necessary is the study of the effects such as the diamagnetic drift for kinetic corrections. These research are open for future study.

Authors would like to acknowledge useful discussions with Drs. T. Takizuka, S. Tsuji and K. Ida. This work is partly supported by the Grant-in-Aid for Scientific Research of Ministry of Education Japan and collaboration program between universities and JAERI on fusion.
References


Nuclear Fusion Research 1990 (IAEA, Vienna, 1991) Vol.1,


Figure Captions

Fig.1  (a) Growth rate of the current-diffusive ballooning mode as a function of $\alpha$ for various values of $\lambda$. Dashed line shows the ideal MHD limit.  (b) Growth rate as a function of $\lambda$ for $\beta=0.8\%$. Parameters are: $s=0.4$, $\gamma/\alpha=1000$, $\mu=1$, $q=3$, $r/\lambda_p=0.6$, $\varepsilon=1/8$, $n=30$, and $1/\delta=0$.

Fig.2  Stability boundary as a function of the mode number $n$. Solid line indicates the analytic formula (5), and dashed line is obtained by the numerical calculation. Parameters are $s=0.4$, $\gamma=1.7\times10^{-5}$, $\gamma/\alpha=1000$, $\mu=1$, $q=3$, $r/\lambda_p=0.6$, $\varepsilon=1/8$, and $1/\delta=0$. 
Fig. 1

(a) 

\[ \gamma \tau_{Ap} \]

\[ \hat{\chi} = 4.4 \times 10^{-5} \]

\[ \hat{\chi} = 10^{-9} \]

\[ \hat{\chi} = 0 \]

\[ \beta \]

(b) 

\[ \gamma \tau_{Ap} \]

\[ 10^{-6} \rightarrow 10^{-5} \rightarrow 10^{-4} \]

\[ \hat{\lambda} \]

\[ 10^{-9} \rightarrow 10^{-8} \rightarrow 10^{-7} \]
Fig. 2
Recent Issues of NIFS Series


NIFS-163 K. Itoh, *A Review on Application of MHD Theory to Plasma*
Boundary Problems in Tokamaks; Aug. 1992


O. Kaneko, T. Kawamoto, S. Kubo, R. Kumazawa, K. Matsuoka, 
S. Morita, O. Motojima, T. Mutoh, N. Nakajima, N. Noda, M. Okamoto, 
T. Ozaki, A. Sagara, S. Sakakibara, H. Sanuki, T. Saki, T. Shoji, 
F. Shimbo, C. Takahashi, Y. Takeiri, Y. Takita, K. Toi, K. Tsumori, 
M. Ueda, T. Watari, H. Yamada and I. Yamada, Heating Experiments 
Using Neutral Beams with Variable Injection Angle and ICRF 
Waves in CHS ; Sep. 1992

NIFS-185 H. Yamada, S. Morita, K. Ida, S. Okamura, H. Iguchi, S. Sakakibara, 
K. Nishimura, R. Akiyama, H. Arimoto, M. Fujiwara, K. Hanatani, 
S. P. Hirshman, K. Ichiguchi, H. Idei, O. Kaneko, T. Kawamoto, 
S. Kubo, D. K. Lee, K. Matsuoka, O. Motojima, T. Ozaki, 
V. D. Pustovitov, A. Sagara, H. Sanuki, T. Shoji, C. Takahashi, 
Y. Takeiri, Y. Takita, S. Tanahashi, J. Todoroki, K. Toi, K. Tsumori, 
M. Ueda and I. Yamada, MHD and Confinement Characteristics in the 
High-\(n\) Regime on the CHS Low-Aspect-Ratio Heliotron / Torsatron 
; Sep. 1992

NIFS-186 S. Morita, H. Yamada, H. Iguchi, K. Adati, R. Akiyama, H. Arimoto, 
M. Fujiwara, Y. Hamada, K. Ida, H. Idei, O. Kaneko, K. Kawahata, 
T. Kawamoto, S. Kubo, R. Kumazawa, K. Matsuoka, T. Morisaki, 
K. Nishimura, S. Okamura, T. Ozaki, T. Seki, M. Sakurai, 
S. Sakakibara, A. Sagara, C. Takahashi, Y. Takeiri, H. Takenaga, 
Y. Takita, K. Toi, K. Tsumori, K. Uchino, M. Ueda, T. Watari, 
I. Yamada, A Role of Neutral Hydrogen in CHS Plasmas with 
Reheat and Collapse and Comparison with JIPP T-IIU Tokamak 
Plasmas ; Sep. 1992

NIFS-187 K. Itoh, S.-I. Itoh, A. Fukuyama, M. Yagi and M. Azumi, Model of the 
L-Mode Confinement in Tokamaks ; Sep. 1992

NIFS-188 K. Itoh, A. Fukuyama and S.-I. Itoh, Beta-Limiting Phenomena in 
High-Aspect-Ratio Toroidal Helical Plasmas; Oct. 1992

NIFS-189 K. Itoh, S.-I. Itoh and A. Fukuyama, Cross Field Ion Motion at 
Sawtooth Crash ; Oct. 1992

NIFS-190 N. Noda, Y. Kubota, A. Sagara, N. Ohyabu, K. Akaiishi, H. Ji, 
O. Motojima, M. Hashiba, I. Fujita, T. Hino, Y. Yamashina, T. Matsuda, 
T. Sogabe, T. Matsumoto, K. Kuroda, S. Yamazaki, H. Ise, J. Adachi and 
T. Suzuki, Design Study on Divertor Plates of Large Helical Device 
(LHD) ; Oct. 1992

NIFS-191 Y. Kondoh, Y. Hosaka and K. Ishii, Kernel Optimum Nearly-Analytical 
Discretization (KOND) Algorithm Applied to Parabolic and 
Hyperbolic Equations : Oct. 1992