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K. Itoh, M. Yagi, S.-I. Itoh, A. Fukuyama,
H. Sanuki and M. Azumi

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Anomalous Transport Theory for Toroidal Helical Plasmas


National Institute for Fusion Science, Nagoya 464-01 Japan
* Japan Atomic Energy Research Institute, Ibaraki 311-01, Japan
**Research Institute for Applied Mechanics, Kyushu University 87, Kasuga 816, Japan
†Faculty of Engineering, Okayama University, Okayama 700, Japan
Abstract

Anomalous transport coefficients in toroidal helical plasmas are studied, based on the innovative theoretical method. The self-sustained turbulence is analyzed by balancing the nonlinear growth due to the current diffusivity with the nonlinear damping by the ion viscosity and thermal conductivity. Interchange and ballooning mode turbulence is investigated, and the geometrical dependence of the anomalous transport coefficient is clarified. Variation of transport owing to the geometrical difference in toroidal helical plasmas is illustrated. The mechanism for confinement improvement is searched for.

To verify the nonlinear destabilization and the self-sustained state, the nonlinear simulation of the interchange mode turbulence is performed in a sheared slab. It is demonstrated that the nonlinear enhancement of the growth rate occurs when the fluctuation amplitude exceeds the critical level. In the saturation stage, the fluctuation level becomes higher associated with the enhanced nonlinear growth.

Keywords: Anomalous transport, Stellarator, Heliotron, Torsatron, Improved confinement, Nonlinear simulation, Current diffusivity, Self-sustained turbulence
1. Introduction

Anomalous transport is the dominant mechanism in determining the plasma confinement in toroidal helical devices [1]. It is worth while to understand how the differences in geometry affect the anomalous transport. This is a crucial task, because the improvement of the energy confinement is the most important issue for the toroidal helical systems. It is the fundamental issue of the plasma physics as well. In this article, we present the anomalous transport theory based on the self-sustained turbulence approach [2]. The impact of the geometry is clarified. The nonlinear numerical simulation is also performed to confirm this theoretical framework.

2. Transport Coefficient

2.1 Theory of the L-mode Confinement

A new theoretical approach, i.e., the method of the self-sustained turbulence has been developed [2,3]. It is found that the current diffusivity $\lambda$, which is enhanced by the fluctuations, can further enhance the growth of the mode. The stationary state is realized by the balance between this nonlinear destabilization effect and the nonlinear stabilization effect due to ion viscosity $\mu$ and thermal conductivity $\chi$. By use of a renormalization on the reduced set of equations and the mean field approximation, the analytic expressions for the self-sustained turbulence and $\chi$ are obtained as [2-4]

$$\chi_L = G \left(-R\beta\right)^{3/2} \left(\psi/\alpha_p\right)^2 v_\lambda R^{-1}$$

(1)

where $G$ represents the geometrical factor. Other notation: $R$ is the major radius, $\beta$ is the ratio of plasma pressure to magnetic pressure, $'$ denotes the derivative with respect to minor radius, $c$ is the light velocity, $\alpha_p$ is the electron plasma frequency, $v_\lambda$ is the Alfvén velocity, and the suffix L denotes the L-mode.

The analysis on the geometrical factor $G$ requires the specification of the magnetic geometry. We choose the average magnetic curvature $\kappa$, safety factor $q$, and magnetic shear parameter $s = q/\kappa$. This set of parameters is the simplest one that discriminates Heliotron/torsatron (H/T), classical/advanced stellarators and tokamaks. Figure 1 illustrates the$q$ and $\kappa$ profiles. For the system of average hill (H/T system), we study the interchange modes. For stellarators (magnetic well), we investigate the ballooning modes [5]. Table 1 summarizes the result for Heliotron/torsatron and stellarator ($\alpha = q^2R\beta$). Result of tokamak is also added for a reference. Figure 2 shows the radial shape of the normalized geometrical factor, $G(r)/G(0.7a)$. The example for the H/T system has the magnetic well in the center, so that the geometrical factor is largest near edge. The stellarator has flatter profile.
From these results, following conclusion is derived. First, the increment of $\chi$ as a result of the enhanced pressure gradient is a generic nature of toroidal plasmas, causing the power degradation of the energy confinement time. The effects of the density profile and pressure profile lead to the general trend that $\chi$ increases near the edge. The radial form is affected by the q-profile or the magnetic hill.

2.2 Theory of the H-mode Confinement

Associated with the gradients of the radial electric field, the reduction of the anomalous transport has been discussed [6]. This method allows to search, in a quantitative manner, the geometry and plasma operation in the toroidal helical systems, which provide a reduced thermal conductivity. The formula is given as

$$
\chi = \frac{\chi_0}{\left[ 1 + g \frac{\omega_s}{\omega_E} \right]},
$$

where $\omega_E = E_t \tau_{Ap}/B$, $\tau_{Ap} = qR/v_A$ and the coefficient $g$ is given explicit by the equilibrium plasma parameters. For H/T systems it is given as $g \approx (a/R)^2/K\beta^* [7]$. For stellarators, we have $g \approx 0.8/\alpha$ in the small $\alpha$ limit and $g \approx 1.56\alpha^2$ in the large $\alpha$ limit [8]. The improvement in the confinement is expected when $E_t'$ is in the range of 100V/cm$^2$. The formation of $E_t'$ was analyzed in [9]. The quantitative analysis of the improved confinement in tokamaks is given in [10] and can be applicable to toroidal helical plasmas.

3. Nonlinear Simulation

3.1 Simulation Model

We study the electrostatic interchange mode in a slab plasma (x- and z-axes are taken in the direction of the pressure gradient and main magnetic field, respectively). We focus to investigate the nonlinear mechanism on electrons [3,11]. The Ohm's law which we use is given by

$$
\partial j/\partial t + [\phi, j] = -\nabla \phi + \lambda_c \Delta_{\perp} j
$$

The reduced set of equations [12] is employed. The equation of motion and the energy balance equation are given as $\partial \nabla^2 \phi/\partial t + [\phi, \nabla^2 \phi] = \nabla \cdot \Omega \nabla \phi + \mu_c \nabla^2 \phi$ and $\partial \rho/\partial t + [\phi, \rho] + \nabla \phi = \chi_c \Delta_{\perp} \rho$. In the simulation study, we normalize the length and time by $c/\omega_p$ and $\tau_{Ap}$, respectively, operator $\nabla_x = \partial/\partial y$ denotes the influence of the equilibrium gradient in the x-direction, $\Omega$ is the drive by curvature, and $[\cdot, \cdot]$ is the Poisson bracket. The terms $\lambda_c$, $\mu_c$ and $\chi_c$ denote the transport coefficients due to the Coulomb collisions. (Here we regard them as constant numbers.) Simulation is done.
for the fixed background pressure gradient. Range in the x-direction is L, and M
Fourier modes in k_y are kept. Parameters L=80 (300 grids) and M=64 (k_y=0.10/64
and k_y=0.10) are usually taken for the two-dimensional simulation [13].

3.2 Nonlinear Growth and High Saturation Level

Figure 3 shows the time evolution of the perturbed pressure W_t = \langle p^2 \rangle, for the
case of \lambda_c=0.01. Average \langle \cdot \rangle is defined as \langle p^2 \rangle = (2L)^{-1} \int_0^L \int p(x,k_y)^2 dx M^{-1} \sum |p(x,k_y)|^2 .

The case of the linear Ohm's law (i.e., [\phi, j] term is neglected in Eq.(3) ) is also shown
by the dotted line. (Other parameters: \Omega=0.5, s=0.5, \chi_e = \mu_e = 0.2.) The linear
growth of the mode corresponds to the electron-inertial interchange instability. If the
convective nonlinearity works in the Ohm's law, the growth rate starts to increase when
the fluctuation level exceeds a threshold value. This level of threshold amplitude
coincides with the theoretical prediction of the nonlinear instabilities [2,3]. (The mode
shows a simple nonlinear saturation when the linearized Ohm's law is employed.) The
nonlinear destabilization is confirmed.

The saturation stage is also investigated. Figure 4 illustrates the result of the
longer time evolution: (a) nonlinear Ohm's law with \lambda_c = 0.01, (b) linear Ohm's law
with \lambda_c=0, and (c) nonlinear Ohm's law with \lambda_c = 0.2. In the time asymptotic limit, the
saturation is realized. The saturation level for nonlinear instability, (a) and (c), is much
higher than the case of the linear Ohm's law (b). It is confirmed that the convective
nonlinearity in the electron dynamics gives rise to the nonlinear acceleration of the
growth rate and the enhanced saturation level. By comparing (a) and (c), we see that the
enhanced saturation level is not influenced by the linear growth rate much. The
theoretical model of the self-sustained turbulence is confirmed. (Also confirmed is the
argument based on the scale invariance method [9].) The k_y spectrum in the case of (a)
is shown in Fig.5. In the phase of the nonlinear growth (t = 60), the growth takes
place associated with the normal cascade of the spectrum. This cascade works as the
effective dissipation of electron motion, leading to the nonlinear instability. When
amplitude grows enough, the inverse cascade takes place (t = 80 for (a)). In the
stationary state (t = 200), the largest amplitude component appears in the long-wave
length region, though the nonlinear excitation is stronger for the higher k_y components.

4. Summary

The influence of the magnetic geometry on the transport coefficient is
investigated for the L-mode as well as for the H-mode-like plasmas, and is summarized
as the geometrical factor G. The intrinsic feature of the anomalous transport, \chi \propto
(-R \beta)^{3/2} (c/\omega_p)^2 v_A R^{-1} , is also clarified. The nonlinear simulation is performed on the
interchange mode by keeping the electron nonlinearity in the Ohm's law. The
theoretical model of the self-sustained turbulence is confirmed. These studies provided
the basis for the future progress in improving the toroidal plasma confinement based on
the understanding of them.

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References

Figure Captions

Figure 1  Typical spatial profiles of the safety factor (a) and magnetic curvature (b). For Heliotron/torsatron (H/T), we take $q(a)=1$, $q(0)=2$, $a/R = 1/7$ and magnetic well for $r/a < 0.5$.

Figure 2  Radial shape of the normalized geometrical factor $G(r)/G(0.7a)$. Solid line for Heliotron/torsatron system and dashed line for stellarators. The case of tokamaks (dotted line) is also shown for the reference.

Figure 3  Temporal evolution of perturbed pressure energy, showing the nonlinear growth. Nonlinear Ohm's law (a), and linearized Ohm's law (b) are used.

Figure 4  Temporal evolution and saturation of the interchange mode turbulence. (a) nonlinear Ohm's law with $\lambda_c = 0.01$, (b) linear Ohm's law with $\lambda_c = 0$, and (c) nonlinear Ohm's law with $\lambda_c = 0.2$.

Figure 5  $k_y$ spectrum of the fluctuations for the case (a) of Fig.4 in the nonlinear growth phase ($t = 60$, dotted line) and in the saturation phase ($t = 200$, solid line).
<table>
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<tr>
<th></th>
<th>H/T Systems</th>
<th>Stellarators</th>
<th>Tokamaks</th>
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<tbody>
<tr>
<td><strong>Average</strong></td>
<td>bad</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td><strong>Curvature</strong></td>
<td>(except center)</td>
<td></td>
<td>(if (q&gt;1))</td>
</tr>
<tr>
<td><strong>Magnetic Shear</strong></td>
<td>strong</td>
<td>weak</td>
<td>strong</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>interchange</td>
<td>ballooning</td>
<td>ballooning</td>
</tr>
<tr>
<td><strong>Formula of G</strong></td>
<td>((kR/a)^{3/2}q^2s^{-2})</td>
<td>(q^2f(s,\alpha)^{-1})</td>
<td>(q^2f(s,\alpha)^{-1})</td>
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**Table 1** *Summary of the turbulence mode and form factor of \(\chi\) (Explicit form of \(f(\alpha,s)\) is given in [3]).*
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