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Lagrangian neoclassical transport theory applied to the region near the magnetic axis

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Neoclassical transport theory around the magnetic axis of a tokamak is studied, in which relatively wide "potato" orbits play an important role in transport. Lagrangian formulation of transport theory, which has been investigated to reflect finiteness of guiding-center orbit widths to transport equations, is developed in order to analyze neoclassical transport near the axis for a low-collisionality plasma. The treatment of self-collision term in Lagrangian formulation is revised to retain momentum conservation property of it. With directly reflecting the orbital properties of all the types of orbits in calculation, the ion thermal conductivity around the axis is found to decrease than from that predicted by conventional neoclassical theory. This result supports recent numerical simulations which show the reduction of thermal conductivity near the magnetic axis.

Keywords : neoclassical transport, potato orbit, finite-width effect

I. INTRODUCTION

Recently, neoclassical transport in the core region of tokamaks again attracts much attention. It is well-known that there appear non-standard guiding-center orbits near the magnetic axis called "potato" orbits [1] Typical orbit width of potato particles is as large as $(q^2\rho^2 R_0)^{1/3}$, where q is the safety factor, ρ is the Larmor radius, and R_0 is the major radius, respectively. In recent tokamak experiments in reversed-shear configuration accompanied by the internal transport barrier (ITB), the measured ion thermal conductivities in the core region sometimes become lower than those predicted by a conventional neoclassical transport theory.[2,3] In fact, the standard neoclassical transport theory[4,5] constructed in the small-orbit-width (SOW) approximation is not applicable to the near-axis region, and the orbital properties of potato particles should be considered in analyzing transport in this region. Then, several transport theories have been presented to include the effect of potato particles.[6-9] and Monte Carlo simulations (so called the δf-method)[10-13] have also been carried out to calculate the ion thermal conductivity χ_i in the near-axis region However, there exist differences in the resultant χ_i's depending on the model used in analytical calculations, and neoclassical transport theory in the near-axis region is not completed yet.

Neoclassical transport theory has usually been discussed in Eulerian representation. Then, the extension of the theory to the near-axis region has also been discussed in an Eulerian manner. However, to include orbital properties in the transport theory, Lagrangian formulation[14-16] was found to be suitable for a collisionless (banana-regime) plasma. In this approach, transport phenomena are described by a reduced drift-kinetic equation in the space of three constants-of-motion (COM) along a collisionless particle orbit in a tokamak. The previous works proved that Lagrangian formulation can reproduce the results obtained from the standard Eulerian formulation built in the SOW limit

The present article is the first application of Lagrangian formulation to the near-axis region in which the finite-orbit-width (FOW) effect becomes really important To utilize Lagrangian transport theory, we improve the treatment of like-particle collision term in the formulation to retain the momentum conservation property. In contrast to the other calculations using some analytical approximations, our calculation reflects quantitatively the properties of all types of particles appearing near the magnetic axis. It is found that the ion thermal conductivity χ_i obtained by Lagrangian transport theory becomes significantly lower than that predicted by conventional Eulerian theory. Our result supports the recent results of both Monte Carlo simulations and experiments in the core region

In Sec. II, analysis of guiding-center orbit is reviewed, and the classification of orbit types in the COM space is presented. The reduced kinetic equation and collision operator in the COM space are derived in Sec. III and the transport equation is obtained by solving the kinetic equation in Sec. IV. We also discuss how to compare the neoclassical flux between Lagrangian and Eulerian representations. Transport coefficients, especially the ion thermal conductivity in the near-axis region is calculated in Sec V.

II. PARTICLE ORBIT NEAR THE MAGNETIC AXIS

As pointed out in recent works,[17,18] the guiding-center orbit near the magnetic axis is not so simple as in conventional analysis, in which particle orbit has been classified as "passing" or "banana". Let us explain key points in analyzing orbit here.

Consider a guiding-center motion projected on the poloidal cross-section (r, θ) of a tokamak. The magnetic field strength is given as $B = B_0[1 - (r/R_0)\cos\theta]$ and q-value is assumed to be constant. The guiding-center velocity of a particle is expressed as $\mathbf{v} = v_\parallel \mathbf{b} + \mathbf{v}_d$, where $\mathbf{b} = \mathbf{B}/B$ and $\mathbf{v}_d$ is the drift velocity in the direction across the magnetic field lines. To investigate orbit topology, the poloidal angular velocity $\dot{\theta}$ is considered. In the low-β approximation, it is given by

$$\begin{aligned}\dot{\theta} &= (v_\parallel \mathbf{b} + \mathbf{v}_d)\cdot\nabla\theta \\ &\simeq \frac{1}{qR_0}\left[v_\parallel - \frac{q}{r\Omega_0}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\cos\theta\right].\end{aligned} \tag{1}$$

where $\Omega_0 = eB_0/R_0$. One can see that the contribution of the second term, which arises from $\mathbf{v}_d \cdot \nabla\theta$, is proportional to r^{-1} and then it becomes significant when a particle is approaching the magnetic axis. This fact means that orbit topology cannot be classified as in a usual way by counting only the turning points of $v_\parallel$. We have shown[19] that the proper way of classifying orbit is to count both the turning points of $\sigma_\parallel = v_\parallel/|v_\parallel|$ and $\sigma_\theta = \dot{\theta}/|\dot{\theta}|$ along a particle orbit.

The criterion of classifying orbit types is shown in Table I, and examples of orbits are shown in Figs. 1 and 2. Here, we use some new names of orbit types which are characteristic of the near-axis region. Outer-, and inner-circulating particles are localized on either side of the magnetic axis, though they do not change $\sigma_\parallel$ like passing particles. Kidney orbits[18] encircle the axis, though they are trapped in the magnetic mirror like bananas. We distinguish concave-kidney orbit from kidney orbit, according to the turning points of σ_θ.

Such a detailed classification as above is needed to identify particle orbit in the $(\mathcal{E}, \mu, \langle r\rangle)$ space, in which we will discuss Lagrangian formulation from the next section. Here, $\mathcal{E}$ is the energy, μ is the magnetic moment, and $\langle r\rangle$ is the radial position averaged over one poloidal period. The region of each orbit type in the $(\langle r\rangle, \lambda_0)$ plane for $\mathcal{E} = 10$keV hydrogen ions is shown in Figs. 3 and 4. Here, $\lambda_0 = \mu B_0/\mathcal{E}$ is the normalized magnetic moment. The circle in Fig. 3 corresponds to the fattest banana orbit B in Fig. 1, of which width is $2(2q^2\rho^2 R_0)^{1/3}$ where $\rho = v/\Omega_0$. Potato orbits, of which typical width becomes $\Delta_r \sim (q^2\rho^2 R_0)^{1/3}$, correspond to those appearing around

$$(\langle r\rangle, \lambda_0) \sim \left((q^2\rho^2 R_0)^{1/3}, 1 \pm (q\rho/R_0)^{1/3}\right) \tag{2}$$

in Figs. 3 and 4. For the convenience of notation, we introduce a typical small radius

$$r_p = 2(2q^2\rho_{i0}^2 R_0)^{1/3}, \tag{3}$$

where $\rho_{i0} = v_{thi}/\Omega_{i0}$ is the Larmor radius of thermal ions. In this article, we use "potato particles" to describe those which appear $0 < r < r_p$ with their orbit width being $\Delta_r \sim r_p$.

One of the important features in Figs. 3 and 4 is that there are some overlaps in regions of orbit types around the solid-line part of the boundary $l2$. In overlapped regions, particle orbit cannot be identified only by the value $(\mathcal{E}, \mu, \langle r\rangle, \sigma_\parallel)$. Therefore the criterion in Table I should be adopted to identity orbits.

As approaching $l2$, a banana (concave-kidney) orbit bifurcates into a kidney and a counter-passing (inner-circulating) orbit as shown in Fig. 5. Such barely-transit particles are almost stagnated at $(r, \theta) = (\langle r\rangle, \pi)$. We call the solid-line part of $l2$ "the transition boundary" hereafter. There is the other type of stagnated particles appear on the boundary $l1$. They are outer-circulating particles stagnated at $(r, \theta) = (\langle r\rangle, 0)$ and move only in the toroidal direction. Conventionally, such particles have been regarded as banana particles in the limit $v_\parallel = 0$, but in fact stagnation occurs when $\dot{\theta}$ becomes zero on $Z = 0$ plane, and stagnated particles have finite $v_\parallel$. Note that on the dashed-line part of $l2$ (the left side from the triangle mark in Fig. 3), bifurcation of orbit types does not occur. This boundary corresponds to inner-circulating orbits with zero-width like outer-circulating particles at $l1$.

One advantage in using $(\mathcal{E}, \mu, \langle r\rangle$ (or $\langle v\rangle))$ as a set of COM variables is that, in the collisionless limit, the position-like variable $\langle r\rangle$ changes continuously when crossing the transition boundary. On the other hand, γ_0 used in Ref. 15, which is the minor-radius of one of two crossing points of orbit with $Z = 0$ plane, changes discontinuously at the transition boundary. It is also practical to choose $\langle r\rangle$ as the position-like variable in that $\langle r\rangle$ is the most suitable value to represent the lowest-order approximation of the particle position.

Finiteness of orbit width appears on the region of each orbit type. In the zero-width limit, banana particles exist in the range $1 - \langle\epsilon\rangle \le \lambda_0 \le 1 + \langle\epsilon\rangle$, where $\langle\epsilon\rangle = \langle r\rangle/R_0$. In reality, however, this simple

analysis is not valid for the region $0 < \langle r \rangle \lesssim r_p$ in which potato orbits appear. Moreover, because of the finiteness of orbit width, there are no particle exist at $\langle r \rangle < q\rho$. The existence of outer- and inner-circulating particles can be found only if the finiteness of orbit width is considered, and they have not been treated in the conventional neoclassical transport theory, nor in the recent studies treating the near-axis region. However, it will affect transport around the axis because some of them have large orbit width $\Delta_r \sim r_p$.

In the forthcoming sections we will derive Lagrangian formulation of neoclassical transport theory which can include the orbit properties near the magnetic axis discussed here.

III. KINETIC EQUATION IN LAGRANGIAN FORMULATION

A. Reduction of the kinetic equation

Consider an axisymmetric configuration. We use the magnetic coordinate system (ψ, θ, ζ), where ψ, θ, ζ is the poloidal flux, the poloidal angle, and the toroidal angle, respectively. The electromangnetic field is represented as $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$ and $\mathbf{E} = -\nabla\Phi(\psi)$, where $I = RB_t$ and we assume that the field is time-independent. Three constants of motion in an axisymmetric configuration are

$$\mathcal{E} = \frac{m_a v^2}{2} + e_a\Phi = \frac{m_a v_\parallel^2}{2} + \mu B - e_a\Phi, \tag{4a}$$

$$\mu = \frac{m_a v_\perp^2}{2B}, \tag{4b}$$

$$P_\zeta = \psi - \frac{m_a R^2}{e_a}\mathbf{v}\cdot\nabla\zeta. \tag{4c}$$

where $v_\parallel = \mathbf{v}\cdot\mathbf{b}$, $v_\perp = |\mathbf{v} - v_\parallel\mathbf{b}|$, and the subscript a denotes particle species. The start point of Lagrangian formulation of neoclassical transport theory is the drift kinetic equation in an Eulerian representation in the $(\mathbf{x}, \mathcal{E}, \mu)$ space.

$$\frac{\partial}{\partial t} f_a(\mathbf{x}, \mathcal{E}, \mu, t) + \dot{\mathbf{x}}\cdot\frac{\partial f_a}{\partial \mathbf{x}} = C_{ab}. \tag{5}$$

where $\dot{} = d/dt$. $\mathbf{x}$ is the guiding-center position and C_{ab} is a collision operator. Note that eq. (5) is independent of the gyrophase ϕ. We change the independent variables in eq. (5) into three constants of motion in the collisionless limit (z_1, z_2, z_3), and the other three variables $(\tilde{z}_4, \tilde{z}_5, \tilde{z}_6)$. One can choose an arbitrary set of independent variables $(\mathbf{z}, \tilde{\mathbf{z}})$. In this paper, we choose $(\tilde{z}_4, \tilde{z}_5, \tilde{z}_6) = (\theta, \zeta, \phi)$, while $z_1 = \mathcal{E}$, $z_2 = \mu$, and $z_3 = \langle\psi\rangle$ instead of P_ζ. $\langle\psi\rangle$ represents the averaged radial position of a particle orbit. The orbit average operator for any function $a(\mathbf{z}, \tilde{\mathbf{z}})$ is defined as

$$\langle a\rangle \equiv \frac{1}{4\pi^2\tau_p}\oint\frac{d\theta}{\dot{\theta}}d\zeta d\phi\, a(\mathbf{z}, \tilde{\mathbf{z}}). \tag{6}$$

where

$$\tau_p \equiv \oint\frac{d\theta}{\dot{\theta}} \tag{7}$$

is the poloidal period of an particle orbit. Note that the integral is carried out along one poloidal circuit of the particle orbit. Note also that we can use $z_3 = \langle r\rangle$ instead of $\langle\psi\rangle$ when it is convenient. By using the set of variables $(\mathbf{z}, \tilde{\mathbf{z}})$, eq. (5) is transformed into

$$\frac{\partial}{\partial t} f_a(\mathbf{z}, \theta, t) + \dot{\theta}\frac{\partial f_a}{\partial\theta} = C_{ab}. \tag{8}$$

where the property $\partial/\partial\phi = \partial/\partial\zeta = 0$ is used.

We introduce here an ordering parameter δ_c as

$$\delta_c = \nu_c^{eff}\tau_p \ll 1. \tag{9}$$

where ν_c^{eff} is a typical collision frequency. This assumption corresponds to the condition that the plasma is in the collisionless regime, or the banana regime. In eq. (8), $\partial f/\partial t$ and C_{ab} are assumed to be $O(\delta_c)$ so that the variables $(\mathcal{E}, \mu, \langle\psi\rangle)$ can really be the constants of motion through the lowest order in δ_c. Expanding f_a with δ_c, the lowest part becomes

$$\dot{\theta}\frac{\partial f_0}{\partial \theta} = 0, \tag{10}$$

where we omit the subscript a. On the other hand, from the conservation of volume in the phase space, one has

$$\frac{1}{J_z}\frac{\partial}{\partial \mathbf{z}} \cdot \left(J_z \frac{d\mathbf{z}}{dt} \right) + \frac{1}{J_z}\frac{\partial}{\partial \tilde{\mathbf{z}}} \cdot \left(J_z \frac{d\tilde{\mathbf{z}}}{dt} \right) = 0, \tag{11}$$

where $J_z(\mathbf{z}, \theta)$ is the Jacobian of the transform $(\mathbf{x}, \mathbf{v}) \to (\mathbf{z}, \tilde{\mathbf{z}})$. Since $d\mathbf{z}/dt = 0$ to $O(\delta_c^0)$, one obtains

$$J_z|\dot{\theta}| = J_0(\mathcal{E}, \mu, \langle\psi\rangle), \tag{12}$$

$$f_0 = f_0(\mathcal{E}, \mu, \langle\psi\rangle). \tag{13}$$

Next, by using eq. (11), $O(\delta_c^1)$ part of eq. (8) can be written as

$$\frac{\partial}{\partial t} f_0(\mathbf{z}, t) + \frac{1}{J_z}\frac{\partial}{\partial \theta}\left(J_z \frac{d\theta}{dt} f_1 \right) = \frac{1}{J_z}\frac{\partial}{\partial \mathbf{z}} \cdot \left(J_z \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{\Gamma}(f_0) \right). \tag{14}$$

where the right hand side is derived from the fact that the collision term can be written in the divergence form in the velocity space $C(f) = \nabla_{\mathbf{v}} \cdot \mathbf{\Gamma}(f)$. The last procedure is to take the orbit average of eq. (14). It yields

$$\frac{\partial \bar{f}}{\partial t} = \frac{1}{J_c}\frac{\partial}{\partial \mathbf{z}} \cdot \left(J_c \left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{\Gamma}(\bar{f}) \right\rangle \right) = \bar{C}. \tag{15}$$

where $\bar{f} = f_0$ is used to emphasize that $\bar{f}$ is a function of $(\mathcal{E}, \mu, \langle\psi\rangle, t)$, and

$$J_c(\mathcal{E}, \mu, \langle\psi\rangle) \equiv 4\pi^2 J_0 \tau_p \tag{16}$$

is the Jacobian in the $(\mathcal{E}, \mu, \langle\psi\rangle)$ space. The collision term is also averaged over a particle orbit. Thus we obtain a reduced drift-kinetic equation in the $(\mathcal{E}, \mu, \langle\psi\rangle)$ space.

B. Jacobian

Here, we derive the explicit form of the Jacobian J_c defined in eq. (16). First, consider the transform from Cartesian coordinate system $(\mathbf{x}, \mathbf{v})$ to the guiding-center variables $(\psi, \theta, \zeta, \mathcal{E}, \mu, \phi)$. The Jacobian of the transform is[20]

$$J = \frac{1}{(\mathbf{B} \cdot \nabla\theta)} \cdot \frac{B_*}{m^2 |v_\parallel|}. \tag{17}$$

where $B_* = B[1 + (v_\parallel/\Omega)\mathbf{b} \cdot \nabla \times \mathbf{b}]$. By changing ψ to its orbit averaged value $\langle\psi\rangle$, we obtain the set of variables $(\mathbf{z}, \tilde{\mathbf{z}}) = (\mathcal{E}, \mu, \langle v\rangle, \theta, \zeta, \phi)$. Therefore, the Jacobian J_z can be written as $J_z = J|\partial v/\partial\langle v\rangle|$. To determine J_z, we use the conservation of P_ζ (of its gyro-averaged form)

$$P_\zeta = \psi - \frac{I}{\Omega} v_\parallel = const. \tag{18}$$

Taking orbit average of both sides, it becomes

$$\psi - \frac{I}{\Omega} v_\parallel = \langle\psi\rangle - \left\langle \frac{I}{\Omega} v_\parallel \right\rangle. \tag{19}$$

Differentiating both sides by $\langle\psi\rangle$, we have

$$\left[1 - \frac{\partial}{\partial \psi}\left(\frac{I}{\Omega} v_\parallel \right) \right] \frac{\partial \psi}{\partial \langle\psi\rangle} = 1 - \frac{\partial}{\partial \langle\psi\rangle} \left\langle \frac{I}{\Omega} v_\parallel \right\rangle. \tag{20}$$

Note that all partial derivatives in eq. (20) are taken with $\mathcal{E}$, μ, and θ being kept constant. Next, by using the equation of guiding-center motion,[20] one obtains

$$\dot{\theta} = \mathbf{v} \cdot \nabla\theta = \frac{v_\parallel \mathbf{B} \cdot \nabla\theta}{B_*} \left[1 - \frac{\partial}{\partial \psi}\left(\frac{I}{\Omega} v_\parallel \right) \right]. \tag{21}$$

Then, combining eqs (17), (20) and (21) yields

$$J_z = \frac{1}{m^2|\dot{\theta}|}|1-\delta_*|, \tag{22}$$

where

$$\delta_* \equiv \frac{\partial}{\partial\langle\psi\rangle}\left\langle \frac{I}{\Omega}v_\parallel \right\rangle \tag{23}$$

Thus one can confirm that $J_z|\dot{\theta}| = J_0$ in eq. (12) is independent of θ. Finally, combining eqs. (16) and (22), we obtain the Jacobian in the $(\mathcal{E}, \mu, \langle\psi\rangle)$ space

$$J_c = \frac{4\pi^2}{m^2}\tau_p|1-\delta_*|. \tag{24}$$

In a numerical calculation, the poloidal period τ_p can easily be determined. As concerns δ_*, it should be noted that, from eq (19),

$$\left\langle \frac{I}{\Omega}v_\parallel \right\rangle = \left.\frac{Iv_\parallel}{\Omega}\right|_{(\langle\psi\rangle,\theta^*)}, \tag{25}$$

where $(\psi,\theta) = (\langle\psi\rangle,\theta^*)$ is the position at which a particle crosses its averaged flux surface $\psi = \langle\psi\rangle$. We call it "the averaging point" of an orbit. Then, eq. (23) is interpreted as

$$\delta_* = \left(\frac{\partial}{\partial\psi} + \frac{\partial\theta^*}{\partial\langle\psi\rangle}\frac{\partial}{\partial\theta}\right)\left.\frac{Iv_\parallel}{\Omega}\right|_{(\langle\psi\rangle,\theta^*)}. \tag{26}$$

where $\partial\theta^*/\partial\langle\psi\rangle$ represents the displacement of the averaging point. Fortunately, however, we can be estimate that $1-\delta_* \simeq 1$ for almost all particles as shown in Appendix A. Though we retain the term $1-\delta_*$ in the derivation of transport equation hereafter, it is approximated to be unity in the numerical calculations.

An important property of J_c arises from the factor τ_p for particles which are stagnated on $Z=0$ plane. Remember that there are two types of the stagnated orbit. One type is the outer-circulating orbit stagnated at $(\psi,\theta) = (\langle\psi\rangle, 0)$ (on the $l1$ boundary in Fig. 3), and the other is stagnated orbit at $(\psi,\theta) = (\langle\psi\rangle,\pi)$ (on the transition boundary $l2$). Approaching the $l1$ boundary, orbits resemble a pendulum motion in the Z direction with a infinitesimal oscillation. Therefore, τ_p remains finite on $l1$. On the other hand, $\tau_p \to \infty$ when approaching the $l2$ boundary. Then, we have

$$\lim_{\mu\to l1} J_c = \text{finite}. \tag{27a}$$

$$\lim_{\mu\to l2} J_c = \infty. \tag{27b}$$

We have shown that, in the $(\mathcal{E}, \mu, \langle\psi\rangle)$ space, there are some overlaps in regions of orbit types. Then, $J_c(\mathbf{z})$ and $\bar{f}(\mathbf{z})$ are generally multi-valued functions of $\mathbf{z}$ depending on the orbit types. We introduce the sign σ_t to indicate the orbit type of each particle. The notation $J_c(\mathbf{z})$ and $\bar{f}(\mathbf{z})$ implicitly mean that they also depend on σ_t: $J_c = J_c(\mathbf{z},\sigma_t)$, etc.

C. Collision operator

To obtain transport equations in the $(\mathcal{E}, \mu, \langle\psi\rangle)$ space, we need to evaluate the change rate of COM by collisions. First, consider the collision term in Eulerian representation[21]

$$C_{ab} = K_{ab}\frac{\partial}{\partial\mathbf{v}}\cdot\int d^3v' \mathsf{U}(\mathbf{v}-\mathbf{v}')\cdot\left[\frac{\partial f_a(\mathbf{v})}{\partial\mathbf{v}}f_b(\mathbf{v}') - \frac{m_a}{m_b}f_a(\mathbf{v})\frac{\partial f_b(\mathbf{v}')}{\partial\mathbf{v}'}\right], \tag{28}$$

where

$$\mathsf{U}(\mathbf{v}) = \frac{\mathsf{I}}{|\mathbf{v}|} - \frac{\mathbf{vv}}{|\mathbf{v}|^3},$$

$$K_{ab} = \frac{e_a^2 e_b^2 \ln\Lambda}{8\pi\epsilon_0^2 m_a^2}.$$

Substituting eq. (28) into the averaging operator eq. (6), we obtain the exact description of the orbit-averaged collision term[15]

$$\bar{C}_{ab} = \frac{1}{J_c}\frac{\partial}{\partial \mathbf{z}} \cdot \left[J_c \left(\mathbf{A}\bar{f}_a(\mathbf{z}) + \mathsf{D} \cdot \frac{\partial}{\partial \mathbf{z}} \bar{f}_a(\mathbf{z}) \right) \right]. \tag{29}$$

where

$$\mathbf{A} = -K_{ab}\frac{m_a}{m_b} \left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \int d^3\mathbf{v}' \mathsf{U}(\mathbf{v}-\mathbf{v}') \ \frac{\partial}{\partial \mathbf{v}'} \bar{f}_b(\mathbf{z}') \right\rangle, \tag{30a}$$

$$\mathsf{D} = K_{ab} \left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \int d^3\mathbf{v}' \bar{f}_b(\mathbf{z}') \mathsf{U}(\mathbf{v}-\mathbf{v}') \cdot \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \right\rangle, \tag{30b}$$

Note that $\mathbf{z}'$ in eq. (30) is a functional depending on θ

$$\mathbf{z}' = (\mathcal{E}', \mu', \langle \psi \rangle') = (\mathcal{E}', \mu', \langle \psi \rangle + \tilde{\psi}'(\mathbf{z}, \mathcal{E}', \mu', \theta)).$$

where $\tilde{\psi}'$ arises from the finiteness of particle orbit width. In this sense, the averaged collision term eq. (29) has a non-locality. Moreover, since eq. (29) is a integro-differential function, it is not suitable to solve analytically. Therefore, we need some approximation to handle collision operator in Lagrangian formulation.

From here on, we consider only the ion transport because the FOW effect near the magnetic axis is important for ions. We neglect the ion-electron term C_{ie} since it is smaller than the ion-ion term C_{ii} by a factor $\sqrt{m_e/m_i}$. An easy approximation for collision term is the Lorentz operator, which had been used in the fundamental study of Lagrangian formulation by Bernstein and Molvig,[14] but it does not conserve momentum. It is well-known that the momentum conservation property of like-species collisions plays an important role in transport theory. Therefore, we use here a model collision operator which conserves the parallel momentum locally so that the transport equation may reproduce the result obtained from Eulerian formulation in the SOW limit.

The model collision operator is given in the following form[22]

$$C_i(f_i) = \frac{\nu_i}{2}\frac{\partial}{\partial \mathbf{v}} \cdot (v^2 \mathsf{I} - \mathbf{vv}) \cdot \frac{\partial}{\partial \mathbf{v}} f_i + \nu_i \frac{m_i v_\parallel u_{i\parallel}}{T_i} f_{iM}. \tag{31}$$

where $u_{i\parallel}$ is a functional of f_i, and f_{iM} is a local Maxwellian. Collision frequency ν_i is defined as

$$\nu_i = \frac{3\sqrt{\pi}}{4\tau_i c_i^3} y(c_i). \tag{32}$$

where $c_i = v/v_{thi}$ and

$$\tau_i^{-1} = \frac{n_i Z_i^4 e^4 \ln \Lambda}{3\pi^{3/2}\epsilon_0^2 m_i^2 v_{thi}^3}. \tag{33a}$$

$$y(c) = \left(1 - \frac{1}{2c^2}\right)\Psi(c) + \frac{1}{2c}\Psi'(c). \tag{33b}$$

$$\Psi(c) = \frac{2}{\sqrt{\pi}} \int_0^c dx e^{-x^2}. \tag{33c}$$

Here, $u_{i\parallel}$ is determined to conserve the parallel momentum

$$\int d^3v v_\parallel C_i(f_i) = 0. \tag{34}$$

Then, substituting eq. (31) into this equation yields

$$u_{i\parallel} = \frac{\tau_i}{2n_i \mathcal{K}_1} \int d^3v \nu_i v_\parallel f_i. \tag{35}$$

where

$$\mathcal{K}_n \equiv \int_0^\infty dc e^{-c^2} c^n y(c). \tag{36}$$

It is convenient to rewrite eq. (31) in the divergence form, by noting $\partial f_M/\partial \mathbf{v} = -(m\mathbf{v}/T)f_M$,

$$C_l(f_l) = \frac{\nu_l}{2}\frac{\partial}{\partial \mathbf{v}}\cdot\left[\left(\mathsf{V}(\mathbf{v})\cdot\frac{\partial}{\partial \mathbf{v}}f_l - \frac{m_l u_{l\parallel}}{T_l}\mathbf{w} f_{lM}\right)\right], \tag{37}$$

where $\mathsf{V}(\mathbf{v}) = v^2\mathsf{I} - \mathbf{vv}$ and $\mathbf{w} = v^2\mathbf{b} - v\,\mathbf{v}$. Since $\mathsf{V}\cdot\mathbf{v} = 0$ and $\mathbf{w}\cdot\mathbf{v} = 0$, the model collision operator eq. (37) also conserves particle number and energy.

Finally, by taking the orbit average of eq. (37), we obtain the orbit-averaged model collision operator

$$\bar{C}_l(\bar{f}_l) = \frac{1}{J_l}\frac{\partial}{\partial \mathbf{z}}\cdot J_l\frac{\nu_l(\epsilon_l,\langle\psi\rangle)}{2}\left[\left\langle\frac{\partial \mathbf{z}}{\partial \mathbf{v}}\cdot\mathsf{V}\cdot\frac{\partial \mathbf{z}}{\partial \mathbf{v}}\right\rangle\frac{\partial \bar{f}_l}{\partial \mathbf{z}} - \left\langle\frac{m_l u_{l\parallel}}{T_l}\frac{\partial \mathbf{z}}{\partial \mathbf{v}}\cdot\mathbf{w} f_{lM}\right\rangle\right]. \tag{38}$$

We neglect here the variation of ν_l along an ion orbit because, though typical ion orbit width becomes as large as $\Delta_l \sim r_p$ there, experiments show that density and temperature profiles near the axis are flat. Note also that we neglect the variation of v along a particle orbit in averaged collision terms. To ensure this approximation, it it assumed that

$$\mathcal{E} \gg \left|\Delta_l e_l \frac{d\Phi}{d\langle r_l\rangle}\right| \tag{39}$$

and v is evaluated as

$$v = \sqrt{\frac{2}{m_l}\left[\mathcal{E} - e_l\Phi(\langle\psi\rangle)\right]} \tag{40}$$

In eq. (38), we need to evaluate $\partial\mathbf{z}/\partial\mathbf{v}$. It is immediately shown that

$$\frac{\partial \mathcal{E}}{\partial \mathbf{v}} = m\mathbf{v}$$
$$\frac{\partial \mu}{\partial \mathbf{v}} = \frac{m\mathbf{v}_\perp}{B}$$

We also need the expression of $\partial\langle\psi\rangle/\partial\mathbf{v}$. This factor is important in Lagrangian formulation, because it measures the rate of excursion in average radial position of a particle by scattering in the velocity space. $\partial\langle\psi\rangle/\partial\mathbf{v}$ can be obtained by taking partial derivative on both sides of eq. (19). It gives

$$\begin{aligned}\frac{\partial\langle\psi\rangle}{\partial \mathbf{v}} &= -\frac{I}{\Omega}\mathbf{b} + \frac{\partial}{\partial \mathbf{v}}\left\langle\frac{Iv_\parallel}{\Omega}\right\rangle \\ &= -\frac{I}{\Omega}\mathbf{b} + \left(\frac{\partial \mathcal{E}}{\partial \mathbf{v}}\frac{\partial}{\partial \mathcal{E}} + \frac{\partial \mu}{\partial \mathbf{v}}\frac{\partial}{\partial \mu} + \frac{\partial\langle\psi\rangle}{\partial \mathbf{v}}\frac{\partial}{\partial\langle\psi\rangle}\right)\left\langle\frac{Iv_\parallel}{\Omega}\right\rangle.\end{aligned} \tag{41}$$

In a previous work,[16] this factor is treated approximately as a step-function

$$\begin{cases}\dfrac{\partial\langle\psi\rangle}{\partial \mathbf{v}} = -\dfrac{I}{\Omega}\mathbf{b} & \text{(Banana)}, \\ \dfrac{\partial\langle\psi\rangle}{\partial \mathbf{v}} = 0 & \text{(Passing)}.\end{cases} \tag{42}$$

in the SOW limit. However, the estimation above needs some explanations. Note that $\partial\langle\psi\rangle/\partial\mathbf{v}$ can be decomposed as $a_1\mathbf{v} + a_2\mathbf{b}$. Since $\mathsf{V}\cdot\mathbf{v} = \mathbf{w}\cdot\mathbf{v} = 0$, we need to retain only the $\mathbf{b}$ component of the derivative. Noting eqs. (23) and (25), we obtain

$$(1-\delta_*)\frac{\partial\langle\psi\rangle}{\partial \mathbf{v}} = -\frac{I}{\Omega}(1-\Delta_l)\mathbf{b}, \tag{43}$$

where

$$\Delta_l = -\frac{ev_\parallel}{I}\frac{\partial}{\partial \mu}\left\langle\frac{Iv_\parallel}{\Omega}\right\rangle \tag{44}$$

For the convenience of notation, we introduce a factor γ as follows

$$\frac{\partial\langle\psi\rangle}{\partial \mathbf{v}} = -\gamma(\mathbf{z},\theta,\sigma_l)\frac{I}{\Omega}\mathbf{b} \tag{45}$$

Later, it is shown that we do not need the explicit form of Δ_l to calculate transport coefficients as pointed out in Ref. 14, and the estimation in eq. (42) works well in the SOW limit.

IV. NEOCLASSICAL FLUXES

A. Derivation of transport equation

We now expand the reduced kinetic equation (15) by a small ordering parameter

$$\delta_b \equiv \frac{\Delta_r}{L} \ll 1, \tag{46}$$

where Δ_r is a typical orbit width and L is a typical gradient scale length of plasma pressure. Though Δ_r for ions becomes large in the near-axis region, the condition (46) can be satisfied since L also becomes large there.

The orderings we put in eq. (15) is as follows. $\partial \bar{f}/\partial t$ is assumed to be $O(\delta_b^2)$ as is often called "transport-ordering". For the parallel flow, we put a plausible ordering that $u_{i\parallel}/v_{thi} \sim O(\delta_b)$. Concerning partial derivatives $\partial/\partial \mathbf{z}$. $\partial/\partial\mathcal{E}$ and $\partial/\partial\mu$ are treated as $O(\delta_b^0)$. while $\partial/\partial\langle\psi\rangle \sim O(\delta_b^1)$. Collision operator is then expanded in δ_b as $\bar{C} = \bar{C}^{(0)} + \delta_b \bar{C}^{(1)} + \delta_b^2 \bar{C}^{(2)} \cdots$. With the expansion $\bar{f} = \bar{f}_0 + \delta_b \bar{f}_1 + \delta_b^2 \bar{f}_2 \cdots$, the $O(\delta_b^0)$ part of eq. (15) for ion becomes

$$\bar{C}_i^{(0)}(\bar{f}_{i0}) = \frac{1}{J_c}\frac{\partial}{\partial\mu}\frac{J_c\nu_i}{2}\left[\left\langle \frac{\partial\mu}{\partial\mathbf{v}}\cdot\mathsf{V}\cdot\frac{\partial\mu}{\partial\mathbf{v}}\right\rangle \cdot \frac{\partial \bar{f}_{i0}}{\partial\mu}\right] = 0. \tag{47}$$

Because $(\partial\mathcal{E}/\partial\mathbf{v})\cdot\mathsf{V} = 0$. only μ-derivative appears in eq. (47). Then. any distribution function $\bar{f}_{i0}$ independent of μ is the solution of this equation. However. we adopt here the averaged collision operator of its exact form shown in eq. (29) for $\bar{C}_i^{(0)}$. Then the solution of $\bar{f}_{i0}$ becomes the local Maxwellian[16]

$$\bar{f}_{i0} = \bar{n}_i \left(\frac{m_i}{2\pi T_i}\right)^{3/2} \exp\left[-\frac{\mathcal{E} - e_i\Phi}{T_i}\right]. \tag{48}$$

where $\bar{n}_i$. T_i and Φ are defined as functions of $\langle\psi\rangle$.

The definition of $\bar{n}_i$ is not equal to the flux-surface averaged density $n_i(\psi)$. First. consider the particle number per unit $\langle\psi\rangle$ as follows

$$\mathcal{N}_i(\langle\psi\rangle) = \sum_{\sigma_i}\int d\mathcal{E}d\mu J_c \bar{f}_{i0}(\mathcal{E},\mu,\langle\psi\rangle;\sigma_i). \tag{49}$$

Then. $\bar{n}_i(\langle\upsilon\rangle)$ is defined as

$$\bar{n}_i = \left.\frac{d\upsilon}{dV}\right|_{\upsilon\ \langle\psi\rangle} \mathcal{N}_i(\langle\upsilon\rangle)\lambda_n(\langle\psi\rangle), \tag{50}$$

where V is the volume enclosed by a $\upsilon = const$ surface. The numerical factor λ_n is given by

$$\lambda_n(\langle\psi\rangle) = \frac{dV}{d\upsilon}\left(\frac{2\pi T_i}{m_i}\right)^{3/2}\left(\sum_{\sigma_i}\int d\mathcal{E}d\mu J_c \exp\left[-\frac{\mathcal{E}-e_i\Phi}{T_i}\right]\right)^{-1}. \tag{51}$$

If $\langle\psi\rangle$ is away from the magnetic axis and the orbit width is narrow. $\lambda_n \to 1$ and $\bar{n}_i(\langle\psi\rangle) \simeq n_i(\psi)$. When approaching $\langle\upsilon\rangle \to 0$. however. λ_n becomes large because the integral region in the $(\mathcal{E}.\mu)$ plane is small there. Note that $\mathcal{N}_i$ is nearly proportional to λ_n^{-1} near the axis. assuming that the density profile in the real space is flat in the range $r < r_p$. This assumption is valid when considering the core region with ITB. Then. it is a simple and plausible assumption that $\bar{n}_i(\langle\psi\rangle) \simeq n_i(\psi = \langle\psi\rangle)$. even in the near-axis region.

Before proceeding to the $O(\delta_b^1)$ equation. let us consider the order expansion of the momentum-restoring term in eq. (38) Since $u_{i\parallel}$ and f_{iM} vary along a particle orbit. we expand it as

$$\left.\frac{u_{i\parallel}f_{iM}}{T_i}\right|_{(\psi.\theta.\mathbf{v})} \equiv \frac{I}{B}\frac{B_0\overline{u_{i\parallel}}}{I_0 T_i(\langle\psi\rangle)}\bar{f}_{i0}(\mathbf{z}) + \Delta_\parallel(\mathbf{z},\theta)\bar{f}_{i0}(\mathbf{z}). \tag{52}$$

Here. $\overline{u_{i\parallel}}(\langle\psi\rangle)$ is the lowest-order approximation of $u_{i\parallel}$ chosen properly so that $\Delta_\parallel$ becomes $O(\delta_b)$. The reason why we expand it in this way is an analogy with the fact that the neoclassical flux in the radial direction, in Eulerian representation. is proportional not to $\langle u_{i\parallel}\rangle_\psi$. but to $\langle Iu_{i\parallel}/B\rangle_\psi$.[5] where $\langle\cdots\rangle_\psi$ means the flux-surface average. Then, substituting eq. (52) into eq. (38) yields

$$\bar{C}_i(\bar{f}_i) = \frac{1}{J_c}\frac{\partial}{\partial \mathbf{z}} \quad \frac{J_c \nu_i}{2}\left[\left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \vee \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \right\rangle \frac{\partial \bar{f}_i}{\partial \mathbf{z}} - \frac{m_i B_0 \overline{u_i}_{\parallel}}{I_0 T_i}\left\langle \frac{I}{B}\frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{w} \right\rangle \bar{f}_{i0} \right.$$
$$\left. - m_i \left\langle \frac{\partial \mathbf{z}}{\partial \mathbf{v}} \cdot \mathbf{w}\Delta_{\parallel} \right\rangle \bar{f}_{i0}\right]. \tag{53}$$

Now consider the $O(\delta_b^1)$ equation of kinetic equation $\bar{C}_i^{(0)}(\bar{f}_{i1}) = -\bar{C}_i^{(1)}(\bar{f}_{i0})$, or written it down explicitly.

$$\frac{\nu_i}{J_c}\frac{\partial}{\partial \mu}\left[J_c \mu \left\langle \frac{m_i v_{\parallel}^2}{B} \right\rangle \frac{\partial \bar{f}_{i1}}{\partial \mu}\right] = -\frac{\nu_i}{J_c}\frac{\partial}{\partial \mu} J_c \mu \left[\left\langle \frac{I\gamma}{\Omega_i} v_{\parallel} \right\rangle \frac{\partial \bar{f}_{i0}}{\partial \langle \psi \rangle} \right.$$
$$\left. + \frac{m_i \Omega_{i0} \overline{u_i}_{\parallel}}{I_0 T_i}\left\langle \frac{I v_{\parallel}}{\Omega_i} \right\rangle \bar{f}_{i0}\right]. \tag{54}$$

To solve eq. (54) for $\bar{f}_{i1}$, it is rewritten in terms of driving forces F_k as

$$\bar{C}_i^{(0)}(\bar{f}_{i1}) = \frac{\bar{f}_{i0}}{J_c}\sum_{k=1}^{3} F_k \frac{\partial \alpha_k}{\partial \mu}. \tag{55}$$

where

$$F_1 = \frac{d \ln \bar{n}_i}{d\langle \psi \rangle} + \frac{e_i}{T_i}\frac{d\Phi}{d\langle \psi \rangle}. \tag{56a}$$
$$F_2 = \frac{d \ln T_i}{d\langle \psi \rangle}. \tag{56b}$$
$$F_3 = \frac{m_i \Omega_{i0} \overline{u_i}_{\parallel}}{I_0 T_i}, \tag{56c}$$

and

$$\alpha_1 = -J_c \nu_i \mu \left\langle \frac{I\gamma}{\Omega_i} v_{\parallel} \right\rangle. \tag{57a}$$
$$\alpha_2 = -\left(\epsilon_i^2 - \frac{3}{2}\right) J_c \nu_i \mu \left\langle \frac{I\gamma}{\Omega_i} v_{\parallel} \right\rangle. \tag{57b}$$
$$\alpha_3 = -J_c \nu_i \mu \left\langle \frac{I v_{\parallel}}{\Omega_i} \right\rangle. \tag{57c}$$

Introducing the perturbed distribution function $g_k(\mathcal{E}, \mu, \langle \psi \rangle)$ which satisfies

$$\bar{C}_i^{(0)}(g_k) = \frac{\bar{f}_{i0}}{J_c}\frac{\partial \alpha_k}{\partial \mu}. \tag{58}$$

$\bar{f}_{i1}$ can be expressed as

$$\bar{f}_{i1} = \sum_{k=1}^{3} g_k F_k. \tag{59}$$

Thus the first order equation is found to have a similar form to that by Bernstein *et al.*[11] though we successfully include the momentum-restoring term by introducing an additional driving force F_3.

Next, consider $O(\delta_b^2)$ part of the reduced kinetic equation

$$\frac{\partial \bar{f}_{i0}}{\partial t} = \bar{C}_i^{(2)}(\bar{f}_{i0}) + \bar{C}_i^{(1)}(\bar{f}_{i1}) + \bar{C}_i^{(0)}(\bar{f}_{i2})$$
$$\Leftrightarrow \frac{\partial \bar{f}_{i0}}{\partial t} - \frac{1}{J_c}\frac{\partial}{\partial \langle \psi \rangle} J_c \nu_i \frac{\mu}{e_i}\left[\left\langle \frac{I^2}{\Omega_i}\gamma^2 \right\rangle \frac{\partial \bar{f}_{i0}}{\partial \langle \psi \rangle} + \frac{m_i \Omega_{i0} \overline{u_i}_{\parallel}}{I_0 T_i}\left\langle \frac{I^2\gamma}{\Omega_i} \right\rangle \bar{f}_{i0} \right.$$
$$\left. + \left\langle \frac{I\gamma}{\Omega_i} v_{\parallel} \right\rangle e_i \frac{\partial \bar{f}_{i1}}{\partial \mu}\right] - \frac{1}{J_c}\frac{\partial}{\partial \mu} J_c \nu_i \mu \left[\left\langle \frac{I\gamma}{\Omega_i} v_{\parallel} \right\rangle \frac{\partial \bar{f}_{i1}}{\partial \langle \psi \rangle} \right.$$
$$\left. + \left\langle \frac{m_i}{B} v_{\parallel}^2 \right\rangle \frac{\partial \bar{f}_{i2}}{\partial \mu} - m_i \left\langle v_{\parallel} \Delta_{\parallel} \right\rangle \bar{f}_{i0}\right] = 0. \tag{60}$$

By taking a moment with $\mathcal{E}$ and μ, we obtain the particle transport equation in the $\langle\psi\rangle$ direction

$$\frac{\partial}{\partial t}\mathcal{N}_i + \frac{\partial}{\partial\langle\psi\rangle}J_1^i = 0, \tag{61}$$

where the particle flux J_1^i is given as

$$J_1^i = \sum_{k=1}^{3}(S_{1k}^{ex} + S_{1k}^{im})F_k. \tag{62}$$

An important point is that the transport coefficients are separated into the explicit and implicit parts. S_{jk}^{ex} and S_{jk}^{im}, respectively.[14] They are given as follows

$$S_{11}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2}{\Omega_i}\gamma^2\right\rangle\cdot\bar{f}_{i0}\right\}, \tag{63a}$$

$$S_{12}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2}{\Omega_i}\gamma^2\right\rangle\cdot\left(c_i^2-\frac{3}{2}\right)\bar{f}_{i0}\right\}. \tag{63b}$$

$$S_{13}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2\gamma}{\Omega_i}\right\rangle\cdot\bar{f}_{i0}\right\}. \tag{63c}$$

$$S_{1k}^{im} = \left\{\frac{\alpha_1}{J_c}\cdot\frac{\partial g_k}{\partial\mu}\right\}. \tag{63d}$$

where the inner-product is defined as $\{a,b\}\equiv\sum_{\sigma_i}\int d\mathcal{E}d\mu J_c\,ab$.

To obtain the energy transport equation, the moment to be taken is $\int d\mathcal{E}d\mu J_c W$, where $W = \mathcal{E} - e_i\Phi(\langle\psi\rangle)$. Define here

$$\mathcal{Q}_i \equiv \sum_{\sigma_i}\int d\mathcal{E}d\mu J_c W\bar{f}_{i0}. \tag{64}$$

which is the sum of the kinetic energy of particles with the same $\langle\psi\rangle$. The use of partial integrals yields the energy transport equation

$$\frac{\partial}{\partial t}\mathcal{Q}_i + \frac{\partial}{\partial\langle\psi\rangle}\left[J_2^i + \frac{3}{2}J_1^iT_i\right] = -e_iJ_1^i\frac{d\Phi}{d\langle\psi\rangle}. \tag{65}$$

where J_2^i represents the conductive ion heat flux

$$\frac{J_2^i}{T_i} = \sum_{k=1}^{3}(S_{2k}^{ex} + S_{2k}^{im})F_k. \tag{66}$$

where

$$S_{21}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2}{\Omega_i}\gamma^2\right\rangle\cdot\left(c_i^2-\frac{3}{2}\right)\bar{f}_{i0}\right\}. \tag{67a}$$

$$S_{22}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2}{\Omega_i}\gamma^2\right\rangle\cdot\left(c_i^2-\frac{3}{2}\right)^2\bar{f}_{i0}\right\}. \tag{67b}$$

$$S_{23}^{ex} = -\left\{\nu_i\frac{\mu}{e_i}\left\langle\frac{I^2\gamma}{\Omega_i}\right\rangle\cdot\left(c_i^2-\frac{3}{2}\right)\bar{f}_{i0}\right\}. \tag{67c}$$

$$S_{2k}^{im} = \left\{\frac{\alpha_2}{J_c}\cdot\frac{\partial g_k}{\partial\mu}\right\}. \tag{67d}$$

The right hand side of eq. (65) describes work done by the radial ion current.

B. Properties of transport coefficients

An important property of the transport coefficients S_{jk} is the symmetry of the implicit part $S_{jk}^{im} = S_{kj}^{im}$. This can be shown by as follows. First, integrating both sides of eq. (58) by μ yields

$$\alpha_j = \frac{J_c \nu_i}{\bar{f}_{i0}} \left\langle \frac{m v_\parallel^2}{B} \right\rangle \mu \frac{\partial g_j}{\partial \mu}. \tag{68}$$

where the integral constant is zero from the boundary condition. Then, one obtains

$$\begin{aligned} S_{jk}^{im} &= \sum_{\sigma_i} \int d\mathcal{E} d\mu \alpha_j \frac{\partial g_k}{\partial \mu} \\ &= \sum_{\sigma_i} \int d\mathcal{E} d\mu J_c \frac{\nu_i}{\bar{f}_{i0}} \left\langle \frac{m v_\parallel^2}{B} \right\rangle \mu \frac{\partial g_j}{\partial \mu} \frac{\partial g_k}{\partial \mu}. \end{aligned} \tag{69}$$

This equation is symmetric in j and k, therefore implicit coefficients are symmetric.

One can also find that the explicit part has a symmetry $S_{12}^{ei} = S_{21}^{ei}$. Then, it is natural to seek the third flux which satisfies the symmetry $S_{3k}^{ei} = S_{k3}^{ei}$.

$$J_3^i \equiv \sum_{k=1}^{3} (S_{3k}^{ei} + S_{3k}^{im}) F_k. \tag{70}$$

Now we show that the proper definition of the third flux is

$$J_3^i \equiv \frac{2\bar{n}_i \mathcal{K}_1}{\tau_i} \left\langle \frac{I u_{i\parallel}}{\Omega} \right\rangle_\psi \frac{dV}{d\psi}. \tag{71}$$

Noting that $\langle \cdots \rangle_\psi$ means the flux-surface average, we have

$$\begin{aligned} J_3^i &= 4\pi^2 \int_0^{2\pi} \frac{d\theta}{\mathbf{B}\cdot\nabla\theta} \frac{I}{B} \int d\mathcal{E}' d\mu' \frac{B_*}{m_i^2 |v_\parallel'|} \nu_i(v') v_\parallel' f_i \Big|_{\psi \quad \langle\psi\rangle} \\ &= \frac{4\pi^2}{m_i^2} \int_0^{2\pi} \frac{d\theta}{\mathbf{B}\cdot\nabla\theta} \frac{I}{B} \int d\mathcal{E}' d\mu' dv' \delta(\langle\psi\rangle - \psi') \sigma_\parallel' B_* f_i \frac{\partial}{\partial \mu'} (\nu_i \mu') \\ &= -\frac{4\pi^2}{m_i^2} \int_0^{2\pi} \frac{d\theta}{\mathbf{B}\cdot\nabla\theta} \frac{I}{B} \int d\mathcal{E}' d\mu' d\langle v\rangle' \left| \frac{\partial\psi}{\partial\langle\psi\rangle} \right| \sigma_\parallel' B_* \nu_i \mu' \frac{\partial}{\partial \mu'} f_i. \end{aligned}$$

Next, distribution function is expanded as

$$\frac{\partial f_i}{\partial \mu} \simeq \frac{\partial}{\partial \mu} \left[\bar{f}_{i0} - \Delta_\psi \frac{\partial \bar{f}_{i0}}{\partial \langle\psi\rangle} + \bar{f}_{i1} \right] = \frac{I\gamma}{e_i v_\parallel} \frac{\partial \bar{f}_{i0}}{\partial \langle\psi\rangle} + \frac{\partial \bar{f}_{i1}}{\partial \mu}. \tag{72}$$

where

$$\Delta_\psi \equiv \psi - \langle\psi\rangle = \frac{I v_\parallel}{\Omega_i} - \left\langle \frac{I v_\parallel}{\Omega_i} \right\rangle \tag{73}$$

represents the deviation from an instantaneous particle position to its averaged flux-surface, and we use the relation $\partial\langle\psi\rangle/\partial\mathbf{v} = -\partial\Delta_\psi/\partial\mathbf{v}$. Using eqs. (20) and (21), and changing the order of integrals, we have

$$\begin{aligned} J_3^i &= -\sum_{\sigma_i} \int d\mathcal{E} d\mu \frac{4\pi^2 \tau_p}{m_i^2} |1 - \delta_*| \frac{\nu_i \mu}{\tau_p} \oint \frac{d\theta}{\dot{\theta}} \frac{I v_\parallel}{B} \frac{\partial}{\partial \mu} \bar{f}_i \\ &= -\sum_{\sigma_i} \int d\mathcal{E} d\mu J_c \left[\frac{\nu_i \mu}{e_i} \left\langle \frac{I^2 \gamma}{\Omega_i} \right\rangle \frac{\partial \bar{f}_{i0}}{\partial \langle\psi\rangle} + \nu_i \left\langle \frac{I v_\parallel}{\Omega_i} \right\rangle \mu \frac{\partial f_{i1}}{\partial \mu} \right]. \end{aligned} \tag{74}$$

Note here that the implicit coefficients S_{3k}^{im} are written as follows, by its definition,

$$S_{3k}^{im} \equiv \left\{ \frac{\alpha_3}{J_c}, \frac{\partial g_k}{\partial \mu} \right\} = -\sum_{\sigma_i} \int d\mathcal{E} d\mu J_c \nu_i \left\langle \frac{I v_\parallel}{\Omega_i} \right\rangle \mu \frac{\partial g_k}{\partial \mu} \tag{75}$$

Then, comparing eqs. (74) and (75), one finds that the explicit coefficients for J_3^i are given as follows

$$S_{31}^{ex} = -\left\{\nu_i \frac{\mu}{e_i}\left\langle\frac{I^2\gamma}{\Omega_i}\right\rangle \bar{f}_{i0}\right\} = S_{13}^{ex}, \tag{76a}$$

$$S_{32}^{ex} = -\left\{\nu_i \frac{\mu}{e_i}\left\langle\frac{I^2\gamma}{\Omega_i}\right\rangle, \left(c_i^2 - \frac{3}{2}\right)\bar{f}_{i0}\right\} = S_{23}^{ex}, \tag{76b}$$

$$S_{33}^{ex} = 0. \tag{76c}$$

Thus we obtain the 3×3 symmetric coefficients for both explicit and implicit parts.

To close transport equations, we must eliminate the additional driving force $\bar{F}_3$ introduced to include the parallel momentum balance. For this purpose, we choose $\overline{u_{i\parallel}}$ as follows

$$\overline{u_{i\parallel}} \equiv \frac{\Omega_{i0}}{I_0\langle h^2\rangle_\psi}\left\langle\frac{Iu_{i\parallel}}{\Omega}\right\rangle_\psi. \tag{77}$$

The factor $\langle h^2\rangle_\psi$, where $h \equiv B_0/B$, is needed to retain the ambipolarity in the SOW limit, as shown in Appendix C. By using this definition, eq (70) can be solved for F_3. It yields

$$F_3 = -\beta(S_{13}F_1 + S_{23}F_2). \tag{78}$$

$$\beta = -\left(\frac{\bar{n}_i\mathcal{K}_1}{\tau_i}I_0^2\rho_{i0}^2\frac{dV}{d\psi}\langle h^2\rangle_\psi - S_{33}^{nm}\right)^{-1}, \tag{79}$$

where $S_{jk} = S_{jk}^{ex} + S_{jk}^{nm}$ is the total transport coefficients. Finally, neoclassical fluxes are rewritten in the following form

$$\begin{bmatrix} J_1^i \\ J_2^i/T_i \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}. \tag{80}$$

where

$$A_{11} = S_{11} - \beta S_{13}^2, \tag{81a}$$

$$A_{12} = A_{21} = S_{12} - \beta S_{13}S_{23}. \tag{81b}$$

$$A_{22} = S_{22} - \beta S_{23}^2. \tag{81c}$$

Thus the resulting transport matrix A_{jk} in Lagrangian formulation is shown to be Onsagar-symmetry as same as in Eulerian formulation.

Next, let us calculate the total coefficients S_{jk}. The perturbed distribution functions g_k are needed to calculate the implicit part. From eq. (68), we obtain

$$\frac{\partial g_1}{\partial\mu} = -\frac{\langle I\gamma v_\parallel/\Omega_i\rangle}{\langle m_i v_\parallel^2/B\rangle}\bar{f}_{i0}. \tag{82a}$$

$$\frac{\partial g_2}{\partial\mu} = -\left(c_i^2 - \frac{3}{2}\right)\frac{\langle I\gamma v_\parallel/\Omega_i\rangle}{\langle m_i v_\parallel^2/B\rangle}\bar{f}_{i0}. \tag{82b}$$

$$\frac{\partial g_3}{\partial\mu} = -\frac{\langle Iv_\parallel/\Omega_i\rangle}{\langle m_i v_\parallel^2/B\rangle}\bar{f}_{i0}. \tag{82c}$$

From here on, we use the approximation $I(\psi) = I_0 = R_0B_0$, which corresponds to the low-β plasma. For $j, k = 1$, or 2, using eq. (44) yields,

$$\begin{aligned} -S_{jk} &= \frac{I_0^2}{e_i\Omega_{i0}}\left\{\frac{\nu_i\mu}{(1-\delta_*)^2}\left(\langle h(1-\Delta_c)^2\rangle - \frac{\langle hv_\parallel(1-\Delta_c)\rangle^2}{\langle hv_\parallel^2\rangle}\right), \left(c_i^2 - \frac{3}{2}\right)^{j+k-2}\bar{f}_{i0}\right\} \\ &= \frac{3\pi I_0^2\bar{n}_i\rho_{i0}^2qR_0}{8\tau_iB_0}\sum_{\sigma_i}\int dxd\lambda_0\frac{\bar{\tau}_p}{(1-\delta_*)}c_i\left(c_i^2 - \frac{3}{2}\right)^{j+k-2}y(c_i)\lambda_0\left(\langle h\rangle - \frac{\langle hv_\parallel\rangle^2}{\langle hv_\parallel^2\rangle}\right) \\ &\equiv \frac{3\pi I_0^2\bar{n}_i\rho_{i0}^2qR_0}{8\tau_iB_0}\bar{S}_{jk}. \end{aligned} \tag{83}$$

where $x = \exp(-\mathcal{E}/T_i)$, $\lambda_0 = \mu B_0/\mathcal{E}$, and $\bar{\tau}_p = \tau_p v_{thi}/qR_0$, respectively. We call $\bar{S}_{jk}$ the normalized transport coefficients. In the equation above, the terms which are proportional to Δ_c and Δ_c^2 are exactly canceled. In a similar way, one has

$$S_{j3} = -\sum_{\sigma_i} \int d\varepsilon d\lambda_0 \bar{\tau}_p c_i \left(c_i^2 - \frac{3}{2}\right)^{j-1} y(c_i)\lambda_0 \left(\langle h\rangle - \frac{\langle h v_\parallel\rangle^2}{\langle h v_\parallel^2\rangle}\right). \tag{84}$$

$$\bar{S}_{33}^{un} = \sum_{\sigma_i} \int d\varepsilon d\lambda_0 \bar{\tau}_p c_i y(c_i)\lambda_0 \frac{\langle h v_\parallel\rangle^2}{\langle h v_\parallel^2\rangle} \tag{85}$$

Note that eq. (83) differs from eq. (84) only by the factor $(1 - \delta_*)^{-1} \simeq 1$. Therefore, this difference is neglected in the final calculations, and the approximation $S_{13} = S_{11}$ and $S_{23} = S_{12}$ are used. As a complement, we define

$$\bar{\beta} = -\left(\frac{8B_0\mathcal{K}_1}{3\pi q R_0}\frac{dV}{d\psi}\langle h^2\rangle_a - \bar{S}_{33}\right)^{-1} \tag{86}$$

so that eq. (78) can be rewritten as $F_3 = -\bar{\beta}(\bar{S}_{13}F_1 + \bar{S}_{23}F_2)$.

In the SOW limit, $\langle h v_\parallel\rangle^2 = 0$ for banana particles, while $\langle h v_\parallel\rangle^2 \simeq \langle h v_\parallel^2\rangle$ for passing ones. Therefore transport coefficients other than S_{33} are determined mainly by the banana part. Because of this separation of the contribution to transport between banana and passing, the approximation of $\partial\langle\psi\rangle/\partial\mathbf{v}$ as in eq. (42) used in Ref. 16 ends up in the same result as the SOW limit of eqs. (83) to (85). However, by using the exact solution for S_{jk}, we can include the contribution for neoclassical transport not qualitatively, but quantitatively, from all the orbit-types of particles appearing in the near-axis region. The magnitude of contribution of each particle is evaluated by the factor $\langle h\rangle - \langle h v_\parallel\rangle^2/\langle h v_\parallel^2\rangle$.

In the present analysis, we improved the treatment of collision term to retain the momentum conservation low. Since we consider only the ion-ion self collisions here, J_1^i must vanish in the SOW limit. It is shown in Appendix C that A_{11} and A_{12} become zero in this limit, and therefore J_1^i vanishes intrinsically. Thus Lagrangian transport theory applied to the region away from the axis, where potato particles do not appear and the SOW limit is valid, reproduces conventional Eulerian transport theory.

C. Comparison with Eulerian transport theory

The representation of fluxes in eqs. (61) and (65) from Lagrangian formulation are different from those in the standard neoclassical transport theory based on Eulerian representation. The former describes the change in $\mathcal{N}_i$ and $\mathcal{Q}_i$ which are functions of averaged particle position $\langle\psi\rangle$, while the latter describes the change in n_i and $p_i = n_i T_i$ through radial fluxes averaged on a magnetic surface ψ. Then, the comparison of neoclassical flux between these two representations is not straightforward. Let us consider this problem here.

As a preparation, we introduce a normalizing factor for $\mathcal{Q}_i$ as follows

$$\begin{aligned}\lambda_q(\langle\psi\rangle) &\equiv \frac{3}{2}\bar{n}_i T_i V' \mathcal{Q}_i^{-1} \\ &= \frac{3T_i}{2}\left(\frac{2\pi T_i}{m_i}\right)^{3/2} V'\left(\sum_{\sigma_i}\int d\mathcal{E}d\mu J_i W^{-1}\exp\left[-\frac{W}{T_i}\right]\right)^{-1}.\end{aligned} \tag{87}$$

where $V' = dV/d\psi(\psi = \langle\psi\rangle)$. λ_q has a similar property to λ_n in eq. (51). The particle flux and the heat flux are redefined as

$$\Gamma_i \equiv J_1^i/V'. \tag{88a}$$

$$q_i \equiv J_2^i/V'. \tag{88b}$$

so that they represent fluxes per unit cross-section. Note that q_i differs from its general definition by Γ_i since we adopt $F_2 = d\ln T_i/d\langle\psi\rangle$ as a driving force rather than the pressure gradient. Then transport equations are rewritten as follows

$$\frac{\partial\bar{n}_i}{\partial t} + \frac{\lambda_n}{V'}\frac{\partial}{\partial\langle\psi\rangle}(V'\Gamma_i) = 0. \tag{89}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\bar{n}_i T_i\right) + \frac{\lambda_q}{V'}\frac{\partial}{\partial\langle\psi\rangle}\left[V'\left(q_i - \frac{3}{2}\Gamma_i T_i\right)\right] = -\lambda_q e_i \Gamma_i \frac{d\Phi}{d\langle\psi\rangle}. \tag{90}$$

where we assume that the time variation of λ_n and λ_q is slow compared with that of $\mathcal{N}_i$ and $\mathcal{Q}_i$. These transport equations have the same dimensions as those of a standard Eulerian representation.[4] In the SOW limit, λ_n and λ_q becomes unity, and eqs. (89) and (90) reduce to the Eulerian representation.

As will be shown in Sec. V C , λ_n and λ_q is nearly unity around the region $\langle r \rangle \sim r_p$ though the finiteness of the potato width significantly affects the transport coefficients there. Only in the region $\langle r \rangle \lesssim q\rho_{i0}$ they become much larger than unity. Then, the qualitative difference in Γ_i and q_i between two representations are not so much significant as far as we are interested in the neoclassical transport around $\langle r \rangle \sim r_p$.

V. CALCULATION OF THERMAL CONDUCTIVITY

A. Definition of collisionless regime

In this section, we calculate the ion thermal conductivity in the near-axis region. As a preparation, let us reconsider the definition of the collisionless regime in which Lagrangian approach is valid.

Usually, the collisionless (or banana) regime is defined as follows

$$\tau_b \sim \frac{qR_0}{v_{thi}\sqrt{\epsilon}} \ , \ \nu_c^{eff} \sim \frac{\nu_i}{\epsilon},$$
$$\Rightarrow \ \delta_i \sim \frac{\nu_i qR_0}{v_{thi}\epsilon^{3/2}} \ll 1. \tag{91}$$

since banana particles exist in the range $|v_\parallel| < v\sqrt{\epsilon}$. In Eulerian representation, collisions cause diffusion only in the velocity space, and then the collisionless regime can be defined as above. In Lagrangian representation, however, collisions bring about diffusion directly in the $\langle \psi \rangle$ direction through the factor $\partial \langle v \rangle / \partial v_\parallel$. This fact means that the effect of scattering on each particle differs according to this factor.

Remember here that, as mentioned in Sec. IV B, transport coefficients can be obtained by using the estimation that

$$\frac{\partial \langle v \rangle}{\partial v_\parallel} \sim \begin{cases} -\dfrac{I}{\Omega} & (\text{banana}), \\ 0 & (\text{well-passing}). \end{cases} \tag{92}$$

In the near-axis region, potato particles can be assumed to have $\partial \langle \psi \rangle / \partial v_\parallel \sim -I/\Omega$ like bananas. The transition of orbit topology of a potato particle occurs when their averaged radial position changes as large as its orbit width $\Delta_r \sim r_p$, or

$$\Delta_\psi \sim \frac{I}{\Omega_0}\left(\frac{q\rho_{i0}}{R_0}\right)^{1/3}. \tag{93}$$

To change $\langle \psi \rangle$ as large as Δ_w, pitch-angle scattering of the magnitude

$$\Delta v_\parallel \sim \Delta_w \Big/ \frac{\partial \langle \psi \rangle}{\partial v_\parallel} = \left(\frac{q\rho_{i0}}{R_0}\right)^{1/3} v_{thi}$$

is needed. Therefore, on the analogy of eq. (91), the effective collision frequency for potato ions can be defined as

$$\nu_c^{eff} = \frac{1}{\tau_i}\left(\frac{R_0}{q\rho_{i0}}\right)^{2/3}. \tag{94}$$

On the other hand, τ_p for potato particles is estimated as

$$\tau_p^{pot} \sim \frac{qR_0}{v_{thi}}\left(\frac{R_0}{q\rho_{i0}}\right)^{1/3}. \tag{95}$$

From eq. (94) and (95), we obtain

$$\delta_c^{pot} = \nu_c^{eff}\tau_p^{pot} \ll 1$$
$$\Leftrightarrow \ T_i \gg \frac{Z_i^2(\bar{n}_i B_0 R_0^2)^{2/5}}{(m_i/m_p)^{1/5}} \tag{96}$$

where T_i (keV), $\bar{n}_i$ ($10^{20}\mathrm{m}^{-3}$) and m_p is the mass of proton. For example, if $B_0 = 4\mathrm{T}$, $R_0 = 4\mathrm{m}$ and $\bar{n}_i = 1 \times 10^{20}\mathrm{m}^{-3}$ for hydrogen ion, then $T_i \gg 5\mathrm{keV}$ is needed for the collisionless assumption. Note that

the criterion eq. (96) corresponds to that from the usual definition of δ_c in eq (91) evaluated at $r \simeq r_p$ as mentioned in Ref. 13.

In reality, barely-transit potato particles have much longer τ_p than the estimation in eq. (95) Then, particles around the transition boundary $l2$ in Fig 3 break the collisionless-assumption $\delta_c \ll 1$. Treating these collisional particles in Lagrangian transport theory like the banana-plateau transition in the standard Eulerian theory[4] is not considered here. Therefore, our calculation corresponds to the collisionless limit of neoclassical transport

B. Ion heat flux

In Eulerian transport theory, the ion heat flux is expressed as follows

$$\frac{q_i}{T_i} = -n_i \chi_i^r \frac{d}{dr} \ln T_i. \tag{97}$$

where χ_i^r is the ion thermal conductivity in the r direction. Here, q_i in the form as above is the result of (i) neglecting ion-electron collisions, and (ii) $\Gamma_i = 0$ because of the momentum conservation in ion-ion self collisions. In Lagrangian approach, however, the condition $\Gamma_i = 0$ is not intrinsic. In reality, the momentum-restoring term $S_{13}F_3$ in J_1^i cannot exactly cancel $S_{11}F_1 + S_{12}F_2$, especially in the region near the magnetic axis. This is because of the non-local nature of Lagrangian formulation, and partially because only the lowest order expansion of $\overline{u_i}_{\|}$ is included In the present calculation, we rewrite the transport equation in eq. (80) to eliminate F_1

$$\frac{q_i^{\langle r \rangle}}{T_i} = \frac{\bar{S}_{12}}{\bar{S}_{11}} \Gamma_i^{\langle r \rangle} - \bar{n}_i \chi_i^{\langle r \rangle} \frac{d}{d\langle r \rangle} \ln T_i. \tag{98}$$

$$\chi_i^{\langle r \rangle} = -\frac{3q^2 \rho_{i0}^2}{32\pi \langle \epsilon \rangle^2 \tau_i} \left[\bar{S}_{22} - \frac{\bar{S}_{12}^2}{\bar{S}_{11}} \right]. \tag{99}$$

where q, ρ_{i0}, and τ_i are evaluated at $r = \langle r \rangle$, and we change here the radial coordinate from $\langle \psi \rangle$ to $\langle r \rangle$. The ion heat conductivity $\chi_i^{\langle r \rangle}$ defined in this way is compared to χ_i^r, though $\Gamma_i^{\langle r \rangle}$ does not vanish here.

It is well-known that $\chi_i^r \propto q^2 \rho_{i0}^2 / (\epsilon^{3/2} \tau_i)$ in Eulerian theory, while the apparent dependence of $\chi_i^{\langle r \rangle}$ is $q^2 \rho_{i0}^2 / (\langle \epsilon \rangle^2 \tau_i)$. However, $\bar{S}_{jk}$ away from the magnetic axis is proportional to $\sqrt{\langle \epsilon \rangle}$, and $\chi_i^{\langle r \rangle}$ has the same dependency as χ_i^r there.

C. Calculation result and discussion

Numerical calculation of transport coefficients in eq. (83) is implemented by using Monte Carlo integration method. In the calculation, test particles which have a given $\langle r \rangle$ are generated randomly and uniformly in the phase space $(x, \lambda_0; \sigma_t)$, where x and λ_0 are defined in Sec. IV B. And, all the functions in the integrand of $\bar{S}_{jk}$, which we write $F_{jk}(x, \lambda_0, \langle r \rangle; \sigma_t)$ here, are calculated by tracing each particle orbit. Then, transport coefficients at $\langle r \rangle$ are given as

$$\bar{S}_{jk}(\langle r \rangle) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} F_{jk}(x_n, \lambda_{0n}, \langle r \rangle; \sigma_{tn}). \tag{100}$$

where N is total number of test particles and $(x_n, \lambda_{0n}; \sigma_{tn})$ is the position of n-th test particle in the phase space, we can include all the types of orbit to transport coefficients. Note that we approximate $1 - \delta_* = 1$ so that $S_{11} = S_{13}$ and $S_{12} = S_{23}$.

As a example, we calculate the ion thermal conductivity $\chi_i^{\langle r \rangle}$ under the conditions $B_0 = 4\mathrm{T}$, $q = 3$, $T_i = 20\mathrm{keV}$ and $\bar{n}_i = 1 \times 10^{20}\mathrm{m}^{-3}$. The radial electric field $d\Phi/dr$ is neglected. In this case, typical potato particles appear in the region $\langle r \rangle \lesssim r_p = 0.244\mathrm{m}$. According to eq (96), the plasma is well in the collisionless regime. The calculation result of $\chi_i^{\langle r \rangle}$ is plotted in Fig. 6. For a comparison, χ_i^r by a standard Eulerian theory in the banana regime,[4] is also plotted. Note that χ_i^r is obtained by regarding r as $\langle r \rangle$.

A significant reduction in $\chi_i^{\langle r \rangle}$ can be seen in the region $\langle r \rangle \lesssim r_p$. The main reason of this reduction explained in Lagrangian approach is that potato particles, which mainly contribute to the radial transport, cannot exist in the region $\langle r \rangle \lesssim r_p/4$ when observed in the COM space $(\mathcal{E}, \mu, \langle r \rangle)$, as is shown in Fig. 3

The FOW effect is thus included in the calculation by reflecting the real population of potato particles neat the magnetic axis.

In Fig. 7, λ_q defined in eq. (87) is plotted. One can see that λ_q becomes much larger than unity only on the inner-most point $\langle r \rangle = 2q\rho_{i0} = 0.03$m in this case. As mentioned in Sec. IV C, simple comparison of χ_i between Lagrangian and Eulerian formulations is possible as long as $\lambda_q \simeq 1$. Then it can be said that the reduction of the ion thermal conductivity occur not only when it is observed in the $\langle r \rangle$-coordinate, but also in the real space around $r \sim r_p/2$. On the other hand, it will be an underestimation that $\chi_i \sim 0.1$ at $\langle r \rangle = 0.03$m because it is almost the same level as the classical transport $\chi_i \sim \rho_{i0}^2/\tau_i$, and also because of $\lambda_q \gg 1$ there.

In Fig. 7, we also plot A_{11}/A_{22} which is approximately the ratio of Γ_i to q_i. Away from the axis it is almost zero and then Γ_i can be neglected, while it becomes finite around $\langle r \rangle = r_p/2$. It is the FOW effect that cause the finite particle flux by ion-ion collisions. Since the electron particle flux is negligible compared to the ion flux, radial electric field E_r will develops to satisfy ambipolarity $\Gamma_i = 0 + O(\sqrt{m_e/m_i})$.[12] In our present formulation, however, the ambipolar electric field cannot be calculated correctly, because it requires to solve dE_r/dt from the particle flux equation, which in turn affects transport coefficients $A_{jk}(\langle r \rangle, t)$ through the orbit-squeezing effect of potato particles.[23] It is a future work to determine neoclassical E_r in the core region.

Next, to investigate the degree of contribution from each orbit type to transport coefficients, we plot in Fig. 8 the factor $H_\parallel = \langle h \rangle - \langle h v_\parallel \rangle^2 / \langle v_\parallel^2 \rangle$ for particles at $\langle r \rangle = 0.12$m and $\mathcal{E} = 20$keV. Since $\bar{S}_{jk}$ contains $H_\parallel$, one can see that not only banana particles but also all the potato particles, that is, kidney, outer-circulating, and inner-circulating particles appearing around the transition boundary, contribute transport to the same degree. In the numerical calculation, the factor $H_\parallel$ is evaluated without any approximation by using Monte Carlo integration method.

The reductive tendency of χ_i in the near-axis region is the common feature of recent simulation results.[10 13] Our $\chi_i^{\langle r \rangle}$ also shows a similar dependence on $\langle r \rangle$ to these simulations, for example, to the fitting formula by Lin *et al* based on a simple random-walk model. This suggests that the neoclassical transport in the near-axis region can be explained by the random-walk diffusion process of the potato-center $\langle r \rangle$. However, there is a difference between ours and the others in that the reduction of χ_i compared to the standard neoclassical value begins at $r \sim r_p$ in our calculation, while it begins from somewhat more outer position $r \sim 2r_p$ in the other simulations. This may be caused either by the differences of the profiles, of the treatment of collision terms, or of the algorithms used in each calculation. More detailed comparison between Lagrangian formulation and δf simulations will be done in a future work.

VI. SUMMARY

Lagrangian formulation of neoclassical transport theory is applied to the near-axis region $\langle r \rangle \sim r_p$ to include the effect of potato particles in the calculation of ion thermal conductivity. In a collisionless regime plasma, a significant reduction of χ_i compared to the standard neoclassical level is found as shown in Fig. 6. By introducing the factor λ_q, we show that the direct comparison of χ_i between Eulerian and Lagrangian representations is possible at $r \sim r_p$.

Since the reduction of χ_i occurs from $\langle r \rangle \sim r_p \propto q^{2/3}$, the finiteness of potato orbit on neoclassical transport will be important in a reversed-shear configuration in which q-value becomes very high at the near-axis region.[24] The development of ambipolar radial electric field E_r and its effect on χ_i, will be studied in a future work.

In the present article, we show that the Lagrangian transport theory is of practical use in treating transport phenomena in which the orbital property of particles is really important. This approach will also be useful to analyze other problems treating FOW effects, such as the bootstrap current near the magnetic axis, or the neoclassical transport around the internal transport barrier in tokamaks where the gradient scale length of plasma pressure becomes comparable to typical banana width.

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APPENDIX A: ESTIMATION OF δ_*

From the definition of δ_*, we have to evaluate

$$\delta_* = \left(\frac{\partial}{\partial \psi} + \frac{\partial \theta^*}{\partial \langle \psi \rangle} \frac{\partial}{\partial \theta} \right) \alpha_* . \tag{A1}$$

where $\alpha_* = I v_\parallel / \Omega$ evaluated at the averaging point $(\psi, \theta) = (\langle \psi \rangle, \theta^*)$. The difficulty lies in the evaluation of $\partial \theta^* / \partial \langle \psi \rangle$. First, let us consider the case for orbits which have the turning points of $\dot{\theta}$, that is, for banana, outer-circulating, and inner-circulating orbits. The averaging point of these orbits can be approximated by the turning points. Therefore, from the equation of θ in eq. (21), $\partial \alpha_* / \partial \psi = 1$ must be satisfied on the averaging point. Then, we have

$$\frac{\partial \theta^*}{\partial \langle \psi \rangle} \simeq - \frac{\dfrac{\partial}{\partial \psi} \left(\dfrac{\partial \alpha_*}{\partial \psi} \right)}{\dfrac{\partial}{\partial \theta} \left(\dfrac{\partial \alpha_*}{\partial \psi} \right)} . \tag{A2}$$

Substituting eq. (A2) into (A1), we have

$$\delta_* = \frac{m v_\parallel^2 \left(\dfrac{\partial B}{\partial \psi} \dfrac{\partial^2 B}{\partial \psi \partial \theta} - \dfrac{\partial B}{\partial \theta} \dfrac{\partial^2 B}{\partial \psi^2} \right)}{m v_\parallel^2 \dfrac{\partial B}{\partial \psi} \dfrac{\partial^2 B}{\partial \psi \partial \theta} + \mu \dfrac{\partial B}{\partial \theta} \left(\dfrac{\partial B}{\partial \psi} \right)^2 \left(\dfrac{\mu B}{\mu B + m v_\parallel^2} - \dfrac{2 m v_\parallel^2}{\mu B} \right)} \tag{A3}$$

Note that eq. (A3) is evaluated at the averaging point. Since $v_\parallel \simeq 0$ at the turning point of $\dot{\theta}$, one can see that $\delta_* \simeq 0$ for banana and outer-, inner-circulating orbits. As concerns kidney orbits, we can evaluate $\partial \theta^* / \partial \langle \psi \rangle$ by approximating the averaging point by the turning point of $v_\parallel$. Then one also finds that δ_* can be neglected for kidney orbits.

For passing particles, we cannot determine the averaging point θ^* in a simple way as above. However, for well-passing particles, $\partial \theta^* / \partial \langle \psi \rangle$ in eq. (A1) can be negligible, and one can estimate

$$\delta_* \simeq \frac{\partial \alpha_*}{\partial \psi} \sim \alpha_* \frac{1}{B} \frac{\partial B}{\partial r} \sim \frac{q \rho}{\langle r \rangle} . \tag{A4}$$

which is negligible when considering $\langle r \rangle \sim r_p \gg q \rho$.

Thus the approximation $1 - \delta_* \simeq 1$ is ensured for all types of orbit

APPENDIX B: INTEGRAL IN THE $(\mathcal{E}, \mu)$ PLANE

In the derivation of transport equations, integrals in the $(\mathcal{E}, \mu)$ plane appear. We prove some properties used in the integral here. Consider a integral of a function F

$$\sum_{\sigma_t} \int d\mathcal{E} \int d\mu J_c(\mathcal{E}, \mu, \langle \omega \rangle; \sigma_t) F(\mathcal{E}, \mu, \langle \omega \rangle; \sigma_t) . \tag{B1}$$

where J_c and F also depend on the orbit type σ_t. Integral region in the μ direction is shown in Fig. 9. μ_1, μ_2 and μ_3 correspond to the boundaries $l1$, $l2$ and $l3$ in Fig. 3, respectively. The kidney region is laid between μ_2 and μ_3, and it is overlapped with a part of the banana and co-passing regions. The integral path in μ is taken in the direction of arrows in Fig.9.

To use Gauss's theorem to take moments of the reduced kinetic equation (60), some boundary conditions are needed. First, consider the boundary $\mu = 0$. Here, noting that all the integrand having the form $\partial / \partial \mu$ in eq. (60) are proportional to μ. Then the surface integral vanishes there. Second, consider the boundary μ_3, where a co-passing particle moves into a kidney region. This transition occurs continuously, since the difference between kidney and co-passing orbits is only that the former has turning point of $v_\parallel$ and the latter does not. Then, we have

$$\lim_{\mu \to \mu_3} J_c(\mu; \sigma_t = P+) = \lim_{\mu \to \mu_3} J_c(\mu; \sigma_t = K) . \tag{B2}$$

Therefore, the surface integral is canceled at μ_3 between co-passing and kidney, since any physical value in the integrand F is also continuous on the boundary.

On the boundary μ_2, a banana particle bifurcates into a kidney or a counter-passing particle as shown in Fig. 5. At the limit $\mu = \mu_2$, the particle is stagnated at $(r, \theta) = (\langle r \rangle, \pi)$. All the values contained in the integrand F are then evaluated at the stagnation point. On the other hand, Jacobian J_c becomes infinity at μ_2 as is pointed out in Sec. III. Noting that the kidney and counter-passing orbit at the boundary correspond to outer- and inner-sections of a banana orbit, we have

$$\lim_{\mu \to \mu_2} J_c(\mu; \sigma_t = B) = \lim_{\mu \to \mu_2} \left[J_c(\mu; \sigma_t = K) + J_c(\mu; \sigma_t = P-) \right]. \tag{B3}$$

Therefore, the integrand F for each orbit type are required to be continuous on the boundary so that the surface integral on the boundary μ_2 can be canceled between banana, counter-passing and kidney.

The last condition is on the $\mu = \mu_1$ corresponding to stagnated outer-circulating orbits. In eq. (60), one can see that all the terms within the $\partial/\partial\mu$ operator have $v_\parallel$ in the averaged operator $\langle \cdots \rangle$. As mentioned in Sec. II. $v_\parallel$ is not exactly zero for stagnated orbits when particle orbits are solved strictly. Then, there remains small contribution from the surface integral in the μ direction when one takes the moment of eq. (60) to obtain eqs (61) and (65). However, this contribution vanishes in the SOW limit, since stagnated condition is $v_\parallel = 0$ in this limit. Therefore, the contribution from the surface integral to $\partial \mathcal{N}_i/\partial t$ and $\partial \mathcal{Q}_i/\partial t$ is considered to be negligible in this paper.

As a consequence, the perturbed distribution function g_k must have continuous derivative $\partial g_k/\partial \mu$ on the boundary μ_2 and μ_3. This fact is used in eq. (82).

APPENDIX C: TRANSPORT COEFFICIENTS IN THE SOW LIMIT

To show that the particle flux J_1^i vanishes in the SOW limit, consider the transport coefficients $\bar{S}_{11}$ and $\bar{S}_{33}^{nn}$ away from the magnetic axis. We assume a model magnetic field $B = B_0(1 - \epsilon \cos\theta)$ as in Sec. II. Orbit types considered here are passing and banana in a usual sense, and the factor $|1 - \delta_*|$ becomes unity in this limit. From the definition of the normalized transport coefficients eqs. (83) and (85), one has

$$\bar{S}_{33} = \bar{S}_{11} + \sum_{\sigma_t} \int dx d\lambda_0 \bar{\tau}_p c_i y(c) \lambda_0 \langle h \rangle. \tag{C1}$$

Since $\dot{\theta} = v_\parallel \mathbf{b} \cdot \nabla\theta$ in the SOW limit, the integral above can be calculated as

$$\begin{aligned} \sum_{\sigma_t} \int dx d\lambda_0 \bar{\tau}_p c_i y(c) \lambda_0 \langle h \rangle &= \sum_{\sigma_t} \frac{2 v_{thi}}{q R_0} \int_0^\infty dc c_i^2 \exp^{-c_i^2} y(c) \int_0^{1+(\epsilon)} d\lambda_0 \lambda_0 \oint \frac{d\theta}{v_\parallel \mathbf{b} \cdot \nabla\theta} \frac{B_0}{B} \\ &= \sum_{\sigma_\parallel} \frac{2 B_0}{q R_0} \mathcal{K}_1 \int_0^{2\pi} \frac{d\theta}{\mathbf{B} \cdot \nabla\theta} \int_0^h d\lambda_0 \frac{\lambda_0}{\sqrt{1 - \lambda_0/h}}. \end{aligned}$$

Therefore, we obtain

$$\bar{S}_{33} = \bar{S}_{11} + \frac{8 B_0 \mathcal{K}_1}{3\pi q R_0} \frac{dV}{d\psi} \langle h^2 \rangle_\psi. \tag{C2}$$

where we have used the flux-surface average

$$\langle h^2 \rangle_\psi = \frac{1}{2\pi} \frac{dV}{d\psi} \int_0^{2\pi} \frac{d\theta}{\mathbf{B} \cdot \nabla\theta} h^2$$

And, combining eqs. (86) and (C2) yields

$$\bar{\beta} = \bar{S}_{11}^{-1}. \tag{C3}$$

Since $A_{11} \propto \bar{S}_{11}(1 - \bar{\beta}\bar{S}_{11})$ and $A_{12} \propto \bar{S}_{12}(1 - \bar{\beta}\bar{S}_{11})$, the transport matrix A_{jk} other than A_{22}-component vanish intrinsically. Therefore, ion particle flux does not occur in Lagrangian formulation in the SOW limit when only ion-ion collision is considered.

Note here that, in the SOW limit, eq.(83) for $\bar{S}_{jk}$ becomes the same form as eq. (80) in Ref. 25 since $\langle h v_\parallel \rangle = 0$ for bananas in this limit. Then, the ion thermal conductivity obtained from our formulation reproduce the result from Eulerian formulation in the SOW limit. Though this property has already been proved by Bernstein *et* al, we succeed in introducing the momentum-conservation nature in Lagrangian formulation

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TABLE I. Classification of particle orbits.

orbit type	$\sigma_\parallel$, σ_θ[a]	sign of $\sigma_\parallel$
passing	0 , 0	+ or −
banana	2 , 2	±
outer-circulating	0 , 2	+[b]
inner-circulating	0 , 2	−[b]
kidney	2 , 0	±
concave-kidney	2 , 4	±

[a]Numbers of turning points.
[b]For ions. The sign is opposite for electrons.

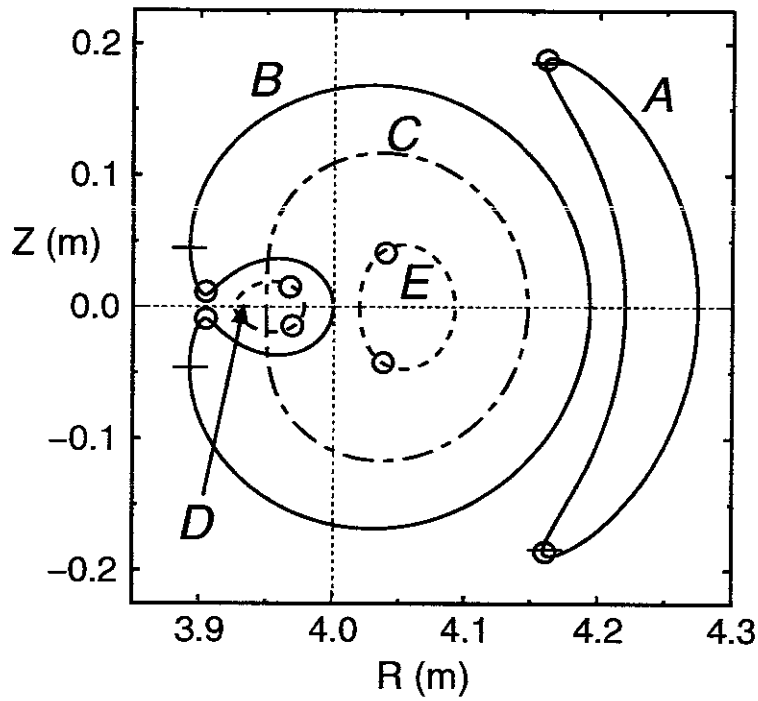

FIG. 1 Examples of particle orbits for $\mathcal{E} = 10$keV ions in the model field $B_0 = 4$T and $q = 3$. The circle and bar marks represent turning points of the signs σ_θ and $\sigma_\parallel$, respectively. Orbit types are A : standard banana, B : the fattest banana, C : passing, D : inner-circulating, and E : outer-circulating, respectively.

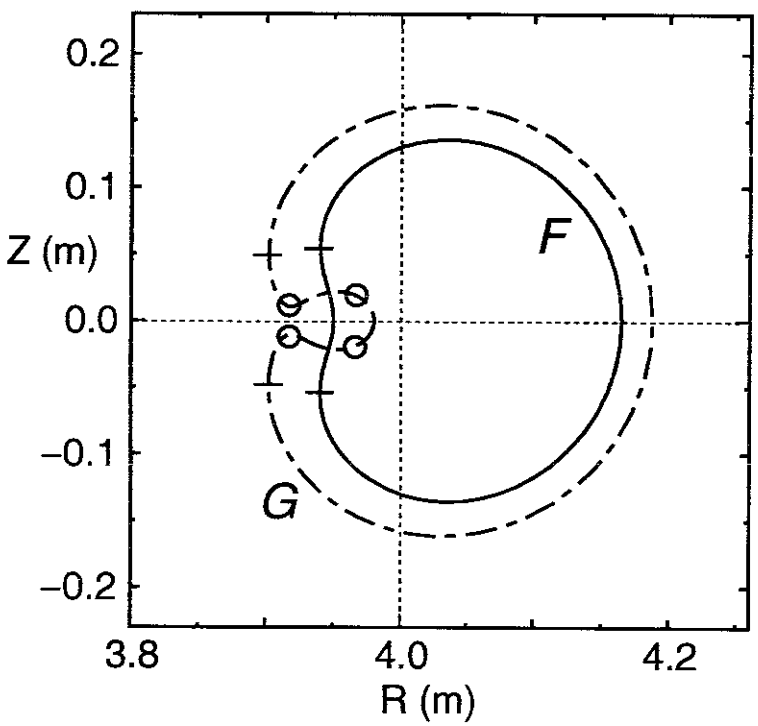

FIG. 2. Examples of kidney (F) and concave-kidney (G) orbits.

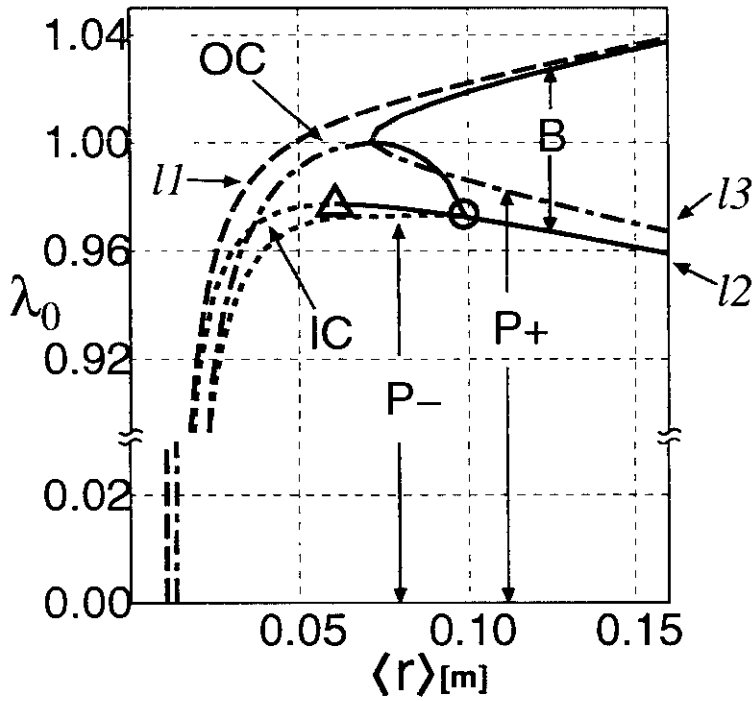

FIG. 3. The regions and the boundaries for each types of orbit in the $(\langle r \rangle, \lambda_0)$ plane for $\mathcal{E} = 10$keV ions. Orbit types are abbreviated as B : banana, P+ : co-passing, P− : counter-passing, OC : outer-circulating, IC : inner-circulating, respectively. The solid-line part of $l2$ up to the triangle mark is the transition boundary. The boundaries $l1$ and the dotted-line part of $l2$ correspond to zero-width outer-circulating and inner-circulating orbits, respectively. $l3$ is the boundary between the kidney and co-passing regions. The circle mark corresponds to the fattest banana orbit.

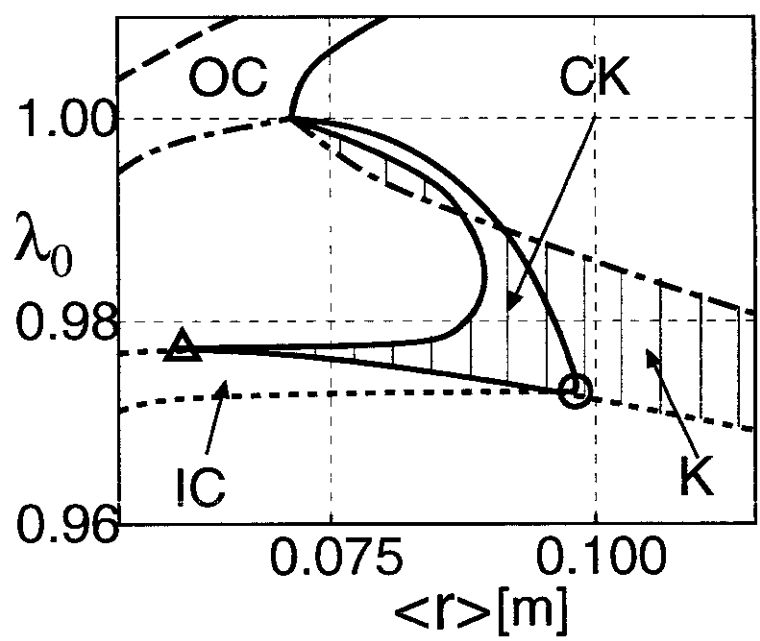

FIG. 4 Regions of K : kidney (shaded) and CK : concave-kidney (enclosed by solid lines) orbits. The kidney region overlaps with the banana, co-passing, and concave-kidney regions. A part of concave-kidney region overlaps with co-passing region.

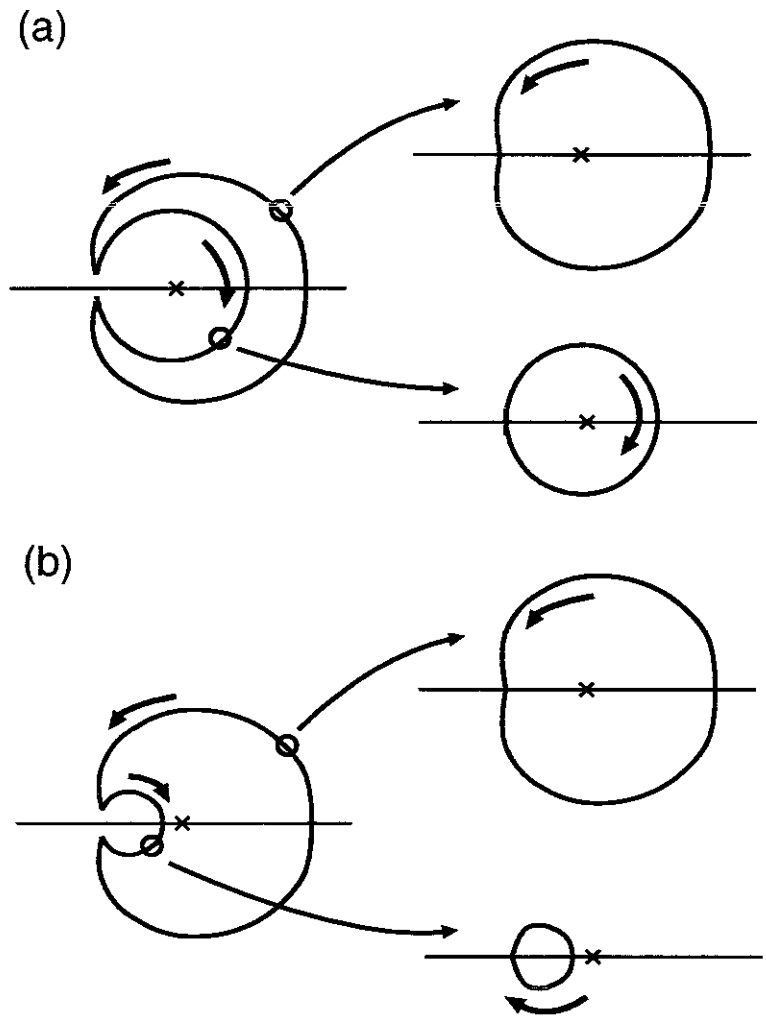

FIG. 5. Transition in orbit types. (a) A banana orbit changes into a kidney or a counter-passing orbit according to the position on which the transition occurs. (b) Similarly, a concave-kidney changes into a kidney or an inner-circulating orbit.

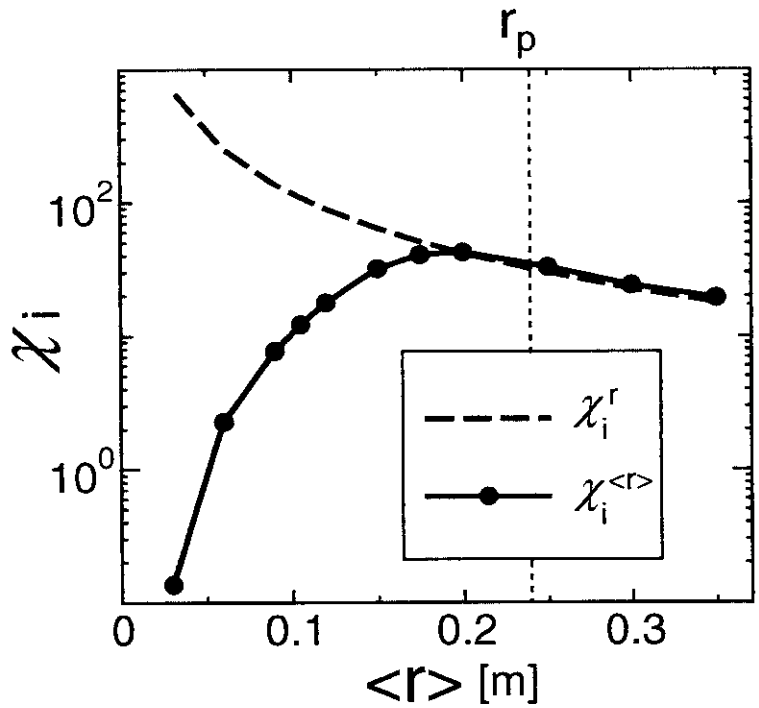

FIG. 6. The ion thermal conductivity normalized by $q^2\rho_{i0}^2/\tau_i$. Solid line is the result from our Lagrangian formulation. Dashed line is from standard Eulerian theory by Hinton and Hazeltine.

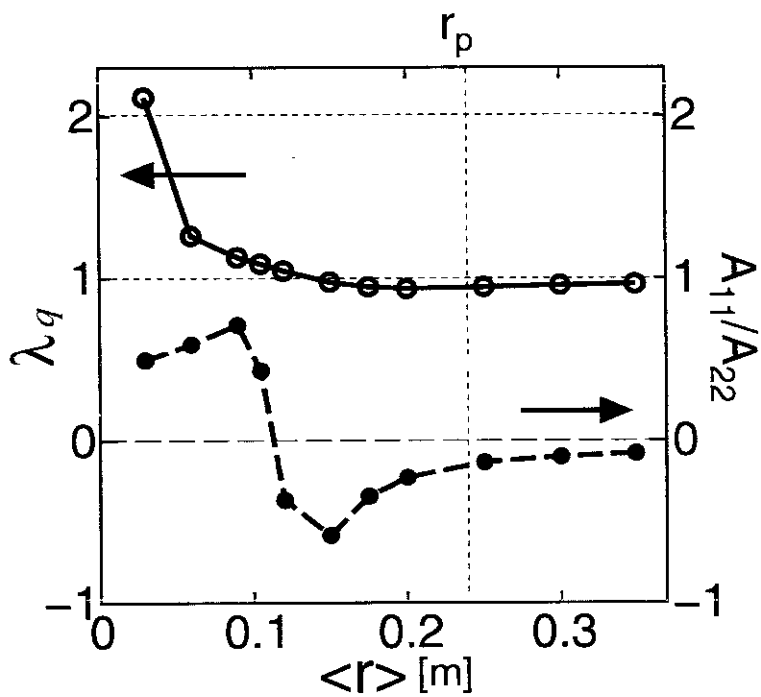

FIG. 7. Dependence of λ_q defined in eq. (87) (solid line) and the ratio of transport coefficients A_{11}/A_{22} (dashed line) on $\langle r \rangle$.

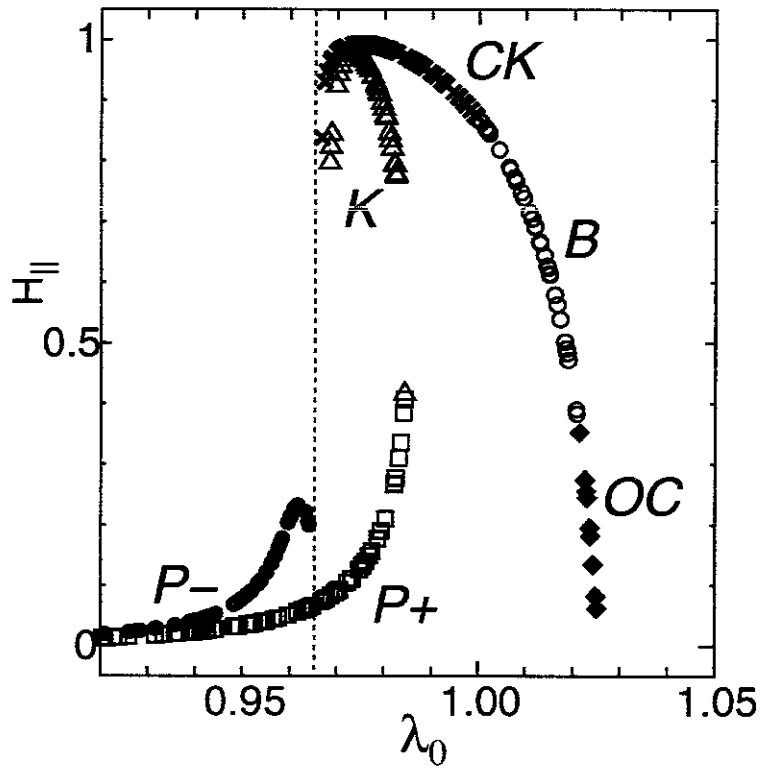

FIG. 8. $H_{\|} = \langle h \rangle - \langle h v_{\|} \rangle^2 / \langle h v_{\|}^2 \rangle$ vs. λ_0 for ions with $\mathcal{E} = 20$keV at $\langle r \rangle = 0.12$m. The transition boundary is at $\lambda_0 = 0.965$. Abbreviations of orbit type are the same as in Figs. 3 and 4.

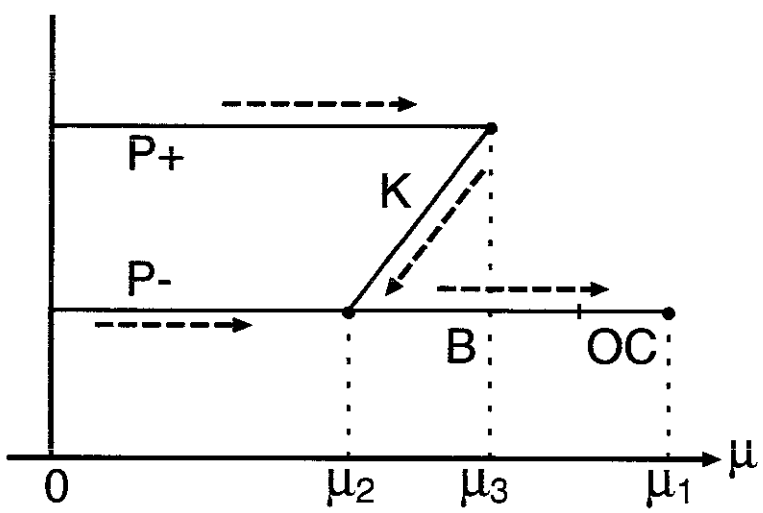

FIG. 9. Integral way in the μ direction. Abbreviations of orbit types are the same as used in Figs. 3 and 4.

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