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# Modeling of Lyman-$\alpha$ line polarization in fusion plasma due to anisotropic electron collisions

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**Abstract.** We have derived a formula for calculating the polarization state of the Lyman-$\alpha$ line in plasma caused by anisotropy in the electron velocity distribution function. Calculation results under the condition of the magnetically confined fusion plasma show that the longitudinal alignment of 0.01 is caused by approximately 10%–20% difference in the electron temperature in the directions parallel and perpendicular to the magnetic field. It is also found that the polarization decreases with the increasing electron density, $n_e$, due to the collisional population averaging effect over the magnetic sublevels in the range of $n_e > 10^{18}\,\mathrm{m}^{-3}$ and is virtually extinguished at $n_e = 10^{21}\,\mathrm{m}^{-3}$.

## 1. Introduction

Anisotropy in the velocity distribution function (VDF) is thought to play a key role for determining confinement characteristics of electrons in the magnetically confined fusion plasma [1]. However, no reliable method for measuring the anisotropic electron VDF (EVDF) has been established to date. The plasma polarization spectroscopy is a promising technique for that purpose [2].

The Lyman-$\alpha$ of neutral hydrogen can be a target of the observation first because hydrogen is the dominant constituent of the fusion plasma and, hence, high emissivities are expected. Second, because the energy state structures relating to this line are rather simple so that a quantitative analysis with a theoretical model is facilitated.

Dominant line emissions of the hydrogen Lyman-$\alpha$ line are known to be located outside the confined region where magnetic field lines are open [3]. In such regions, confinement characteristics of electrons are dependent on their velocity pitch angle with respect to the magnetic field line. Electrons having a small pitch angle, called passing electrons, are immediately guided to the divertor plates along the magnetic field lines, while electrons having a large pitch angle, which are called the trapped electrons, are confined by the mirror effect in the ripples of the magnetic field strength caused by magnetic coils discretely located in the toroidal direction. As a result, trapped electrons have a longer confinement time than passing electrons, which should lead to an anisotropic EVDF.

This paper describes the outline for calculating the polarization state of the Lyman-$\alpha$ line with anisotropic electron collisions following Fujimoto's methodology [2], and shows some results of the actual calculation under typical plasma conditions of the Large Helical Device (LHD), which is a fusion experiment machine of magnetic confinement.

## 2. Theoretical model

The polarization of an emission line originates in the population imbalance over the magnetic sublevels in the upper state of the transition. The Lyman-$\alpha$ line consists of two fine structure lines, i.e., $1\,^2S_{1/2} - 2\,^2P_{1/2}$ and $1\,^2S_{1/2} - 2\,^2P_{3/2}$. Figure 1 shows all transitions between magnetic sublevels composing the Lyman-$\alpha$ line.
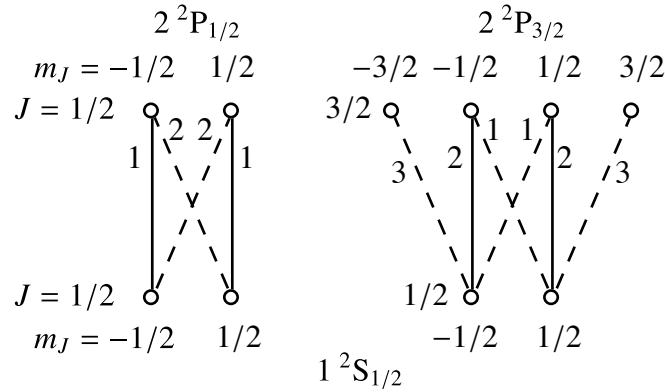


**Figure 1.** Line components included in the Lyman-$\alpha$ line. The solid and dashed lines represent the $\pi$- and $\sigma$-light, respectively. The numbers next to the lines indicate relative values of Einstein $A$ coefficient.

The $\Delta m_J = 0$ and $\Delta m_J = \pm 1$ transitions emit light linearly polarized in the quantization axis direction ($\pi$ light) and circularly polarized on the plane perpendicular to the quantization axis ($\sigma$ light), respectively, where $m_J$ is the magnetic quantum number. If all the magnetic sublevels have the same population, the intensities of $\pi$, $\sigma^+$, and $\sigma^-$ lights are identical, namely, no polarization is observed.

We assume axisymmetry with respect to the magnetic field direction which is taken as the quantization axis. In this case, the population distribution has "mirror symmetry," namely, the populations of $m_J$ and $-m_J$ sublevels are the same in each state. Because of this restriction, the line corresponding to the $1\,^2S_{1/2} - 2\,^2P_{1/2}$ transition is never polarized. We first focus on deriving the polarization state of the line $1\,^2S_{1/2} - 2\,^2P_{3/2}$, and then incorporate the influence of the unpolarized $1\,^2S_{1/2} - 2\,^2P_{1/2}$ line into the final result.

Under an axisymmetric system, the spherical coordinate representation of the density matrix of an excited state $p$, $\rho(p)$, can be expanded as

$$\rho(p) = \rho_0^0(p) T_0^{(0)}(p) + \rho_0^2(p) T_0^{(2)}(p), \tag{1}$$

where $T_q^{(k)}(p)$ is the the so-called irreducible tensor operator [4]. The coefficients $\rho_0^0(p)$ and $\rho_0^2(p)$ correspond respectively to the population and the alignment, the latter of which represents the inhomogeneity over the magnetic sublevels in that excited state. We hereafter use $a(p)$ instead of $\rho_0^2(p)$ for simplicity. The conventional population is given as $n(p) = \sqrt{2J_p + 1}\rho_0^0(p)$, where $J_p$ is the total angular momentum quantum number of the state $p$.

We consider a simple atomic model for a quantitative calculation of $n(p)$ and $a(p)$. Under the corona equilibrium, the population is determined by the balance between the electron impact excitation and the spontaneous radiative decay, which can be expressed as

$$C^{0,0}(1,p) n_e n(1) = \sum_s A(p,s) n(p), \tag{2}$$

where $C^{0,0}(1,p)$ is the rate coefficient of the electron impact excitation, $A(p,s)$ is the Einstein $A$ coefficient of the transition from $p$ to a lower state $s$, and $n_{\mathrm{e}}$ is the electron density. The equilibrium condition of $a(p)$ is similarly expressed as

$$C^{0,2}(1,p)n_{\mathrm{e}}n(1) = \left[\sum_s A(p,s) + C^{2,2}(p,p)n_{\mathrm{e}}\right] a(p),\tag{3}$$

where $C^{0,2}(1,p)$ is the alignment creation rate coefficient and $C^{2,2}(p,p)$ is the alignment destruction rate coefficient. The population $n(p)$ and the alignment $a(p)$ are then derived as

$$n(p) = \frac{C^{0,0}(1,p)n_{\mathrm{e}}}{\sum_s A(p,s)} n(1),\tag{4}$$

$$a(p) = \frac{C^{0,2}(1,p)n_{\mathrm{e}}}{\sum_s A(p,s) + C^{2,2}(p,p)n_{\mathrm{e}}} n(1).\tag{5}$$

We assume observation of the line intensity with a linear polarizer from a direction perpendicular to the quantization axis. The longitudinal alignment $A_{\mathrm{L}}$ is defined as [2]

$$A_{\mathrm{L}} = \frac{I_\pi - I_\sigma}{I_\pi + 2I_\sigma},\tag{6}$$

where $I_\pi$ and $I_\sigma$ are the linearly polarized light intensity in the parallel and perpendicular directions to the quantization axis, respectively. For the transition from the state $p$ to $s$, $A_{\mathrm{L}}$ is expressed with $a(p)/n(p)$ as [2]

$$A_{\mathrm{L}}(p,s) = (-1)^{J_p+J_s}\sqrt{\frac{3}{2}}(2J_p+1)\left\{\begin{array}{ccc} J_p & J_p & 2 \\ 1 & 1 & J_s \end{array}\right\}\frac{a(p)}{n(p)},\tag{7}$$

where $\{\cdots\}$ is the 6-$j$ symbol.

Calculations of the coefficients $C^{0,0}(1,p)$ and $C^{0,2}(1,p)$ are carried out under a certain EVDF. We assume that the EVDF is axisymmetric with respect to the quantization axis and is expressed by two temperatures which are in the directions parallel ($T_\parallel$) and perpendicular ($T_\perp$) to the quantization axis or the magnetic field, respectively. Such EVDFs are explicitly given as [2]

$$f(v,\theta) = 2\pi\left(\frac{m}{2\pi k}\right)^{3/2}\left(\frac{1}{T_\perp^2 T_\parallel}\right)^{1/2}\exp\left[-\frac{mv^2}{2k}\left(\frac{\sin^2\theta}{T_\perp} + \frac{\cos^2\theta}{T_\parallel}\right)\right],\tag{8}$$

where $v$ is the absolute value of the velocity, $\theta$ is the pitch angle of the velocity regarding the magnetic field, and $m$ and $k$ are the electron mass and the Boltzmann constant, respectively.

The rate coefficients $C^{0,0}(1,p)$ and $C^{0,2}(1,p)$ are evaluated as [2]

$$C^{0,0}(1,p) = \int Q_0^{0,0}(1,p)4\pi f_0(v)v^3 \mathrm{d}v,\tag{9}$$

$$C^{0,2}(1,p) = \int Q_0^{0,2}(1,p)\left[4\pi f_2(v)/5\right]v^3 \mathrm{d}v,\tag{10}$$

where $Q_0^{0,0}(1,p)$ and $Q_0^{0,2}(1,p)$ are the excitation and alignment creation cross sections, respectively, for the corresponding transition, and $f_0(v)$ and $f_2(v)$ are the coefficients of the expansion of $f(v,\theta)$ by the Legendre polynomials $P_K(\cos\theta)$ as

$$f(v,\theta) = \sum_K f_K(v)P_K(\cos\theta).\tag{11}$$

The coefficient $f_K(v)$ is explicitly given as

$$f_K(v) = \frac{2K+1}{2} \int f(v, \theta) P_K(\cos \theta) \sin \theta \, \mathrm{d}\theta. \tag{12}$$

The quantity $Q_0^{0,2}(1, p)$ is obtained with $Q_0^{0,0}(1, p)$ as [2]

$$Q^{0,2}(1, p) = (-1)^{J_p + J_s} \sqrt{\frac{2}{3}} (2J_p + 1)^{-1} \left\{ \begin{array}{ccc} J_p & J_p & 2 \\ 1 & 1 & J_s \end{array} \right\}^{-1} A_{\mathrm{L}}(p, 1) Q_0^{0,0}(1, p). \tag{13}$$

Here, $A_{\mathrm{L}}$ is the value for the case where the excitation takes place with mono-energetic beam collisions. We adopt the data by Bray [5] for $Q_0^{0,0}$ and the compiled data of James [6] for $A_{\mathrm{L}}$. Figure 2 shows these elemental quantities relating to the present transition.
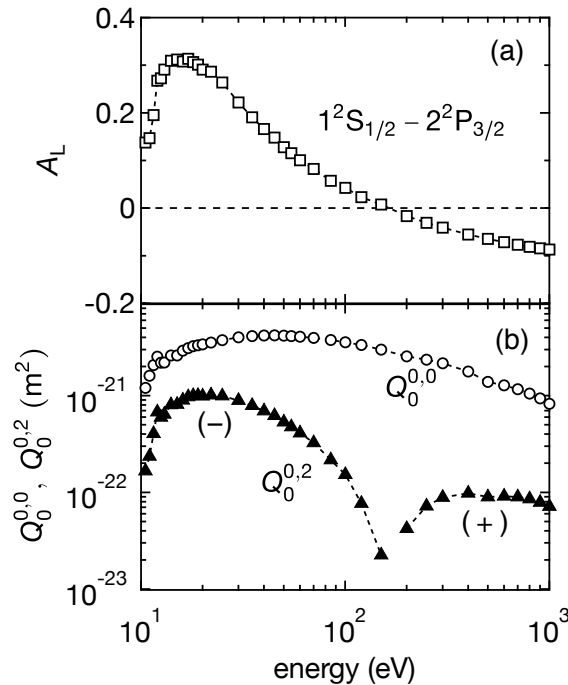


**Figure 2.** $A_{\mathrm{L}}$ values under an assumption of mono-energetic beam collision experiment (a) and $Q_0^{0,0}$ and $Q_0^{0,2}$ (b) for the $1\,^2\mathrm{S}_{1/2} - 2\,^2\mathrm{P}_{3/2}$ transition. The actual $Q_0^{0,2}$ indicated with $(-)$ take negative values.

The alignment destruction process is understood as the relaxation of the population imbalance among the magnetic sublevels. It is known that this process due to electron collisions has some correlation with the Stark broadening of the emission line from that state and its rate coefficient can be approximated by the half width of the Stark broadening [7]. Here, the Stark broadening data for the Lyman-$\alpha$ line by Stehlé [8] are adopted for evaluating $C^{2,2}(p, 1)$.

## 3. Results and discussion
The $A_{\mathrm{L}}$ value for the Lyman-$\alpha$ line is calculated with Eq. (7), for which the typical plasma conditions in LHD are borne in mind. Here, $T_\parallel$ is fixed at $10\,\mathrm{eV}$, which is the typical electron temperature at the location where dominant Lyman-$\alpha$ line emissions are expected, and $T_\perp$ is scanned from $3\,\mathrm{eV}$ to $30\,\mathrm{eV}$.
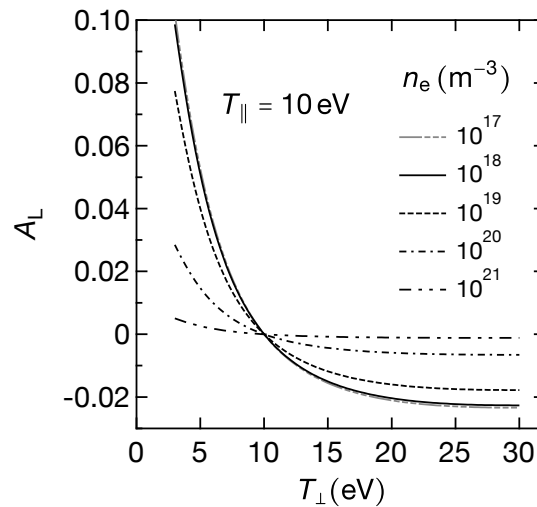
**Figure 3.** Example of the calculation results for $A_\mathrm{L}$ with several $n_\mathrm{e}$ values. $T_\parallel$ is fixed at $10\,\mathrm{eV}$ and $T_\perp$ is scanned from $3\,\mathrm{eV}$ to $30\,\mathrm{eV}$.

Figure 3 shows the results calculated for several $n_\mathrm{e}$ values. It is confirmed that the radiation is unpolarized when $T_\parallel = T_\perp$, and the polarization is relaxed with increasing $n_\mathrm{e}$. Positive $A_\mathrm{L}$ for $T_\perp < T_\parallel$ indicates that the $\pi$-light intensity is larger than that of $\sigma$-light, and negative $A_\mathrm{L}$ for $T_\perp > T_\parallel$ means the opposite condition. These results are inferred from the tendency of the elemental $A_\mathrm{L}$ data in Fig. 2.

The relaxation in $A_\mathrm{L}$ due to increasing $n_\mathrm{e}$ is caused by the collisional averaging over the magnetic sublevels. It is found that this effect is observable in the $n_\mathrm{e}$ range of the LHD plasma, namely, between $n_\mathrm{e} = 10^{18}\,\mathrm{m^{-3}}$ and $10^{21}\,\mathrm{m^{-3}}$. In the actual analysis, it would be necessary to discriminate this effect from the relaxation of anisotropy in EVDF itself because both the effects work for the polarization relaxation.

Finally, it is noted that the detection of $A_\mathrm{L}$ in the order of 0.01 has been realized in the solar atmosphere observation called CLASP (Chromospheric Lyman-Alpha Spectro-Polarimeter) [9]. The present results suggest that if we can detect $A_\mathrm{L} = 0.01$, that corresponds to approximately 10%–20% difference between $T_\parallel$ and $T_\perp$.

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