Multi-Objective Optimization of Superconducting Linear Acceleration System for Pellet Injection by Using Finite Element Method*)

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The enhancement of the acceleration performance of a superconducting linear acceleration (SLA) system to inject the pellet container has been investigated numerically. To this end, a numerical code used in the finite element method has been developed for analyzing the shielding current density in a high-temperature superconducting film. In addition, the on/off method and the normalized Gaussian network (NGnet) method have been implemented in the code for the shape optimization of an acceleration coil, and the non-dominated sorting genetic algorithms-II have been used as the optimization method. The results of the computations show that the speed of the pellet container for the current profile of the optimized coil is significantly faster than that for the homogeneous current profile of the coil. However, for the on/off method, the current profile is scattered, whereas the coil shape becomes hollow for the NGnet method. Consequently, the NGnet method is an effective tool for improving the acceleration performance of the SLA system and for obtaining a coil shape that is easy to design.

Keywords: finite element analysis, genetic algorithm, linear accelerator, nuclear fuel, thin film

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1. Introduction

A pellet injection system has been developed using both hydrogen and deuterium pellets to supply fuel to the fusion reactor. These pellets can be accelerated using an air gun with a helium gas and injected into a high-temperature plasma [1,2]. However, the injected pellets melt around the plasma before they reach the core of the plasma owing to insufficient pellet speed, which reduces the fuel efficiency of the fusion reactor.

To resolve this problem, a high-temperature superconducting (HTS) linear acceleration system has recently been proposed for injecting pellets into the plasma core [3]. Hereinafter, this system is referred to as the superconducting linear acceleration (SLA) system. This system can electromagnetically accelerate containers with frozen hydrogen pellets using an HTS film, such as a linear motor car. In the system, two types of films are used for acceleration and levitation. According to the Yanagi and Motojima [3], the estimated required speed of the SLA system to reach the plasma core is more than 5 km/s. However, it is not clear how much pellet speed can be obtained experimentally since the SLA system has not yet been applied to pellet injection.

In recent years, we have simulated an SLA system using the developed numerical code for analyzing a shielding current density in an HTS film [4]. In the present study, we focus on the genetic algorithm (GA) to improve the acceleration performance of the SLA system. The GA is a method that imitates the mechanism of natural selection in biological evolution artificially. Specifically, GA uses selection, crossover, and mutation in the population of the first generation. In the next generation, new individuals are created by the selection of the ones with high fitness. Another individuals are generated by the crossover and the mutation with a probability. By these operations, the individuals with low fitness are deleted as possible. An optimized solution is obtained by repeating these operations until the termination condition is satisfied. The GA has been used for the design optimization of electromagnetic devices because of the recent improvements in computing performance [5, 6]. Applying the GA to the shape optimization of the acceleration coil in the SLA system, we are interested to see if the pellet speed can be improved.

The purpose of the present study is to enhance the acceleration performance for the SLA system numerically. To this end, we implement the GA of the multi-objective optimization in the code of the finite element method for analyzing the shielding current density in an HTS. Furthermore, we use the on/off method and the NGnet method [7]

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for the shape optimization of the acceleration coil.

2. Governing Equations and Equation of Motion

A magnetic flux density $\boldsymbol{B}$ in an SLA system is generated by an acceleration coil. As a result, $\boldsymbol{B}$ is applied to an acceleration HTS, and simultaneously, a shielding current density $\boldsymbol{j}$ flows in the HTS. As is well known, $\boldsymbol{j}$ is closely related to an electric field $\boldsymbol{E}$. The relation can be written as $\boldsymbol{E} = E(|\boldsymbol{j}|)[\boldsymbol{j}/|\boldsymbol{j}|]$. To determine the characteristics of the superconducting properties, we use the power-law $E(j) = E_C[j/j_C]^N$ as a function of $E(j)$. Here, E_C and j_C are the critical electric field and critical current density, respectively. The index N is a constant.

In the SLA system, a disk-shaped HTS film of radius R and thickness b can be accelerated using a rectangular cross-section coil. The coil shape is described in Section 3. Throughout the present study, we simulate the SLA system using the cylindrical coordinate system (r, θ, z) in which the origin O and the z-axis are considered as the centroid of the coil and the symmetry axis, respectively. In addition, the centroid of the acceleration HTS is at the z-axis, and the pellet is moved to the positive direction of z in the SLA system.

In the present study, we assume that a shielding current density can hardly flow in the z-direction because the thickness of the HTS is sufficiency less as compared with the radius. This is referred to as the thin layer approximation [8], and the shielding current density $\boldsymbol{j}$ containes the θ-component for the axisymmetric model.

Under these assumptions, the shielding current density in an HTS is expressed as $\boldsymbol{j} = (2/b)(\nabla S \times \boldsymbol{e}_z)$, where $\boldsymbol{e}_z$ is a unit vector in the z-direction. $S(r, t)$ is a scalar function, and its function is governed by the following integro-differential equation:

$$\mu_0 \partial_t \int_0^R Q(r, r') S(r', t) r' dr' + \frac{2}{b} S = -\partial_t \langle B_z \rangle - \frac{1}{r} \partial_t (r E_\theta). \quad (1)$$

Here, μ_0 is the permeability of vacuum. B_z and E_θ are the z-component of an applied magnetic flux density $\boldsymbol{B}$ and the θ-component of $\boldsymbol{E}$. A bracket $\langle\ \rangle$ is an average operator over the film thickness. The function $Q(r, r')$ is denoted as follows:

$$Q(r, r') = -\frac{2}{\pi b^2 \sqrt{r r'}} \sum_{m=0}^{1} (-1)^m k_m K(k_m). \quad (2)$$

Here, $K(x)$ is a complete elliptic integral of the first kind and its parameter k_m is defined by

$$k_m^2 \equiv \frac{4 r r'}{(r + r')^2 + m b^2}. \quad (3)$$

We assume that the initial and boundary conditions are $S(r, 0) = 0$ and $S(R, t) = 0$, respectively.

To simulate the SLA system, it is necessary to solve the equation of motion. For this purpose, we adopt Newton's equation of motion as follows

$$\frac{d^2 Z}{dt^2} = \frac{4\pi}{m} \int_0^R \frac{\partial S}{\partial r} \langle B_r \rangle r dr. \quad (4)$$

Here, $Z(t)$ is the position of the HTS, and B_r is the r-component of the applied magnetic flux density $\boldsymbol{B}$. Furthermore, m is the total mass of the pellet container and the HTS film.

The initial boundary conditions are given by $Z = Z_0$ and $v = 0$ m/s at $t = 0$, where v and z_0 denote the velocity of the pellet container and an initial position of the HTS, respectively. The initial-boundary-value problem of Eqs. (1) and (4) is discretized with respect to space using the finite element method, and subsequently, it is reduced to simultaneous ordinary differential equations. As a result, we can obtain both the time evolution of the shielding current density and the dynamic motion of the HTS film. We adopt the Runge-Kutta method with an adaptive step-size algorithm [9] to achieve the numerical stability. Accordingly, the interval $[0, R]$ of the film is divided equally into n finite elements in the present study.

In the following section, we describe the improvement of the acceleration performance of the SLA system using a shape optimization problem for a GA with multi-objective optimization. The optimized design space is the rectangular cross-section of the acceleration coil.

3. Optimization of Current Profile of Coil

3.1 Generic Alghorithm

Equation (4) clearly shows that the pellet speed of the SLA system improves as the magnetic flux density $\boldsymbol{B}$ increases, and the shape of the acceleration coil affects the strength of $\boldsymbol{B}$. In the present study, we change the coil shape by determining the current profile of the coil to increase the pellet speed. In the present study, the coil shape is determined using the GA.

In the optimization problem using the GA, an objective function is maximized or minimized under certain constraints, and the variable of the objective function is a gene formed by a binary digit or a real vector. In addition, when there exist two or more objective functions, this becomes a multi-objective optimization problem. In the present study, we solve this problem with two objective functions using the non-dominated sorting genetic algorithms-II (NSGA-II) [10]. The NSGA-II is a highly effective stable algorithm for multi-objective optimization problems, and it can be implemented for various numerical codes using pymoo [11], a multi-objective optimization framework in Python.

3.2 Coil shape optimization by on/off method

We adopt two types of shape optimization methods for determining the optimal coil shape. In this section, let us

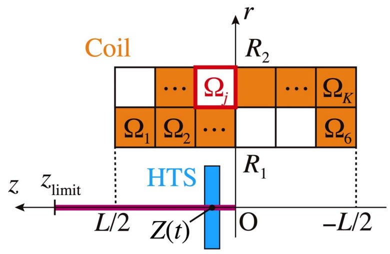

Fig. 1 A schematic view of an acceleration coil that is divided into finite elements.

describe the on/off method and the NGnet method.

In the on/off method, a specified design space is divided into finite elements. Throughout the present study, the design space is the rectangular cross-section $D = \{(z, r)| - L/2 \leq z \leq L/2, R_1 \leq r \leq R_2\}$ of the coil (see Fig. 1). Here, L denotes the height of the coil, and R_1 and R_2 are the inner and outer radii, respectively.

In Fig. 1, we show a schematic view of an acceleration coil that is divided into finite elements. To represent of the shape of the coil, the elements are assigned two types of values for the on-state and the off-state. Thus, the coil is a set of K filaments, and Ω_j denotes the cross-section of the jth filament. In Fig. 1, the on-state and the off-state of the filament are denoted by the symbols ■ and □, respectively. For the on-state, a current is applied to the filament. The on/off states in the filaments are expressed as follows:

$$\Omega_j \leftarrow \begin{cases} \text{on} & : x_j = 1 \\ \text{off} & : x_j = 0 \end{cases}. \tag{5}$$

Here, the gene $\boldsymbol{x}$ is defined as $\boldsymbol{x} \equiv (x_1, x_2, \cdots x_K)^T$, and it is expressed in binary digits.

A jth current in a filament is expressed as follows:

$$I_j(t, Z) \equiv \begin{cases} (\alpha t/K)x_j & (0 \leq Z \leq Z_{\rm limit}) \\ 0 & (\text{otherwise}) \end{cases}. \tag{6}$$

In addition, $Z_{\rm limit}$ and α are the limit of the acceleration region and the increasing rate of the filament current, respectively. We observe from Eq. (6) that the pellet is accelerated within the range $0 \leq Z \leq Z_{\rm limit}$.

The multi-objective functions for the on/off method are defined as follows

$$\text{Maximize} : f_{\rm v}(\boldsymbol{x}) \equiv v_{\rm N}(\boldsymbol{x})/v_{\rm h}, \tag{7}$$

$$\text{Minimize} : f_{\rm on}(\boldsymbol{x}) \equiv K_{\rm on}(\boldsymbol{x})/K. \tag{8}$$

Equations (7) and (8) show the speed ratio and the on ratio, respectively. Here, $K_{\rm on}$ denotes the number of filaments for the on-state. $v_{\rm h}$ and $v_{\rm N}$ are the maximum pellet velocity for the homogenous current profile (that is, $f_{\rm on} = 1$) and the maximum velocity evaluated using the shape optimization method, respectively.

3.3 NGnet method

The normalized Gaussian network (NGnet) is a method for optimizing the shape of a specified design space using Gaussian functions. It should be noted that the Gaussian functions must be superimposed on the design spaces so that there are no gaps. In many cases using the NGnet method, the weighting coefficients become the gene, and its gene is the real vector.

Although the NGnet method is based on the on/off method, the on/off state of the filament is determined as follows: the on-state or off-state in Ω_j is expressed as $g(\boldsymbol{y}_j) \geq 0$ or $g(\boldsymbol{y}_j) < 0$, respectively. The function $g(\boldsymbol{y})$ is defined by $g(\boldsymbol{y}) \equiv \sum_{i=1}^{M} w_i b_i(\boldsymbol{y})$. Here, w_i is the weighting coefficient and b_i is defined as follows: $b_i(\boldsymbol{y}) \equiv G_i(\boldsymbol{y})/\sum_{k=1}^{M} G_k(\boldsymbol{y})$, where M is the number of Gaussian functions $G_i(\boldsymbol{y})$. The function G_k is expressed as $G_k(\boldsymbol{y}) = (2\pi)^{-1}|\sum_k|^{-1/2} \exp\left[-2^{-1}(\boldsymbol{y} - \boldsymbol{z}_k)^T {\sum_k}^{-1}(\boldsymbol{y} - \boldsymbol{z}_k)\right]$ Here, $\boldsymbol{z}_k$ is the kth center position for the function G_k, and $\sum^{-1}$ is the covariance matrix. Throughout the present study, the center position $\boldsymbol{z}_k$ of $G_k(\boldsymbol{y})$ is located on the $n_r \times n_z$ grids of the coil cross-section D to obtain the number M of the position $\boldsymbol{z}$. Here, n_r and n_z are the number of divisions in the r-axis and z-axis, respectively. The value of M is calculated as $M = (n_r + 1)(n_z + 1)$.

In the NGnet method, the multi-objective functions are defined as follows:

$$\text{Maximize} : f_{\rm v}(\boldsymbol{y}) \equiv v_{\rm N}(\boldsymbol{y})/v_{\rm h}, \tag{9}$$

$$\text{Minimize} : f_{\rm on}(\boldsymbol{y}) \equiv K_{\rm on}(\boldsymbol{y})/K, \tag{10}$$

$$\text{Subject to} - 1 \leq w_i \leq 1. \tag{11}$$

It should be noted that the gene becomes M weighting coefficients.

4. Numerical Results

In this section, we optimize the current profile of the acceleration coil to enhance the pellet speed on the basis of the on/off and NGnet methods described in Section 3. The physical and geometrical parameters are fixed as follows: $R = 4$ cm, $b = 1$ mm, $m = 10$ g, $Z_0 = 1$ mm, $N = 20$, $E_{\rm C} = 1$ mV/m, $j_{\rm C} = 1$ MA/cm^2, $R_1 = 5$ cm, $R_2 = 7$ cm, $L = 10$ cm, and $\alpha = 20$ kA/cm^2. Further, the parameters used for the shape optimization of the acceleration coil are $n_r = 3$, $n_z = 19$, and $K = 500$. The coil is divided into 50 and 10 in the z- and r-direction, respectively. Therefore, the cross-section of the filament is square. In addition, the number of generations and population are set to 300 and 40, respectively.

Let us first investigate the pellet velocity for the cases of the on/off and NGnet methods. In Fig. 2, we show the time dependencies of the pellet velocity v. The figure shows that the pellet velocities estimated by the two types of shape optimization methods are much higher than the value of v obtained by the homogeneous current profile.

In Figs. 3 (a) and (b), we show the current profiles of the filaments for the on/off and NGnet methods. We observe from these figures that, for the on/off method, the filaments in the on-state are scattered. However, the filaments in the off-state for the NGnet method are concen-

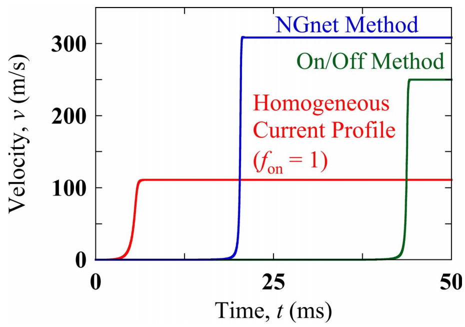

Fig. 2 Time dependence of the pellet velocity v.

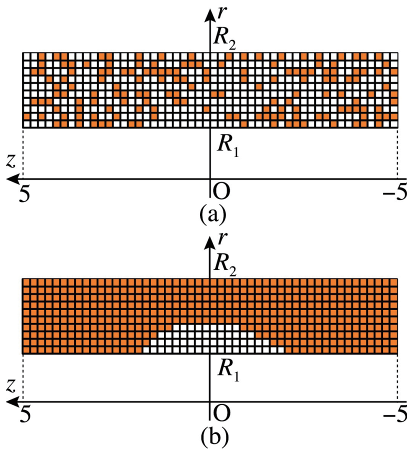

Fig. 3 Current profiles of the coil for (a) the on/off method and (b) the NGnet method.

trated in the range around $r = R_1$ and $-2\,\mathrm{cm} \lesssim z \lesssim 2\,\mathrm{cm}$, and the coil shape becomes a hollow profile. This is because the on/off method has a high degree of freedom in selecting the on/off state. From this result, we concluded that it is difficult to design a coil based on the current profile of the filament obtained by the on/off method. Consequently, the NGnet method is an effective tool for improving the acceleration performance of the SLA system. In the following, we simulate the SLA system using the NGnet method.

Let us investigate the reason why the pellet velocity increases significantly. To this end, the radial component of the applied magnetic flux density $B_r(Z, R, t)$ is calculated as the position Z of the HTS film and is depicted Fig. 4. In this figure, we assess B_r for the case with the homogeneous and optimized current profiles. This figure shows that two types of B_r have the maximum as $Z \simeq 5.2\,\mathrm{cm}$. The maximum value for the optimized profile using the NGnet method is larger than that for the homogeneous one. We conclude that this result is a factor that significantly enhances the pellet velocity for the SLA system.

Finally, we investigate the relation between the speed ratio f_v and the on ratio f_{on}. Figure 5 shows the dependence of f_v and f_{on}. We observe from this figure that, for $0.75 \lesssim f_{on} \lesssim 1$, the pellet velocity is higher than that for the

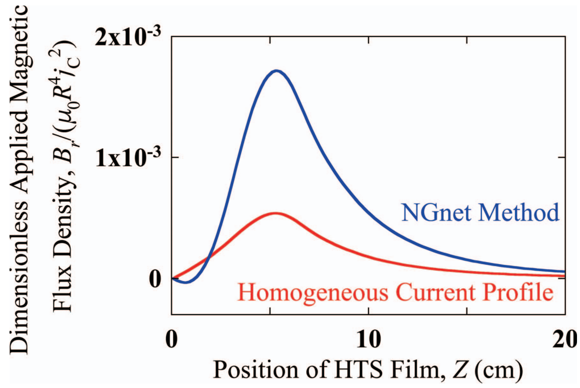

Fig. 4 Dependence of the dimensionless applied magnetic flux density $B_r(Z, R, t)$ on the position of the HTS film Z.

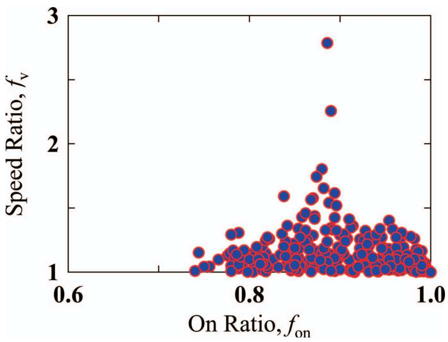

Fig. 5 Dependence of the speed ratio f_v and the on ratio f_{on}.

homogeneous current profile. In particular, it is found that the value of the velocity is more than doubled for the case with $f_{on} = 0.85$ or $f_{on} = 0.88$. These results imply that the material of the coil and the amount of electric power can be reduced.

5. Conclusion

The conclusions obtained in the present study are summarized as follows:

1. The pellet velocity obtained using the two types of shape optimization methods is significantly faster than that of the homogeneous current profile of the acceleration coil. However, for the on/off method, the filaments in the on-state are scattered, whereas the coil shape becomes a hollow profile for the NGnet method. From these results, it is difficult to design a coil based on the current profile of the filament obtained using the on/off method. Consequently, the NGnet method is an effective tool for improving the acceleration performance of the SLA system and for obtaining a coil shape that is easy to design.
2. Even if the on-state of the filament is reduced by approximately 12% from the homogeneous current profile, the pellet speed of the SLA system increases by approximately 2.8 times. This is mainly because of the strength of the applied flux density generated by

the optimized current profile using the NGnet method.

In the present study, the termination condition for NSGA-II to obtain the optimal solution is the maximum number (300) of generations. In order to check whether the above solution is optimal or not, we increase the maximum number of generations from 300 to 5000 or change the random number seed 10 times. As a result, it is found that the same optimal solution is obtained in both cases. In future works, to further investigate the validity of the optimal solution in the present study, we compare it with an optimal solution using another Python Framework for multi-objective optimization such as jMetalpy [12].

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