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An Example of the Non-adiabatic Motion of a Charged Particle in an External Magnetic Field

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ABSTRACT

In this note it is exhibited that in certain magnetic field varying steeply over a distance of the order of the cyclotron radius two particles of the same charge and with nearly equal initial momentums drift toward opposite directions.

Let us assume that an external magnetic field is normal to a plane which may be specified as the (x, y) plane in a right hand system (i.e. $z=0$) and that the normal component, $H_z)_{z=0}$, takes the three, positive, constant values H_A, H_C and H_B in the increasing order (i.e. $H_A < H_C < H_B$) in the respective regions A, C and B settled as follows: (c.f. Fig. 1)

The region B is the narrow strip of the width a stretching along the x axis. The regions A and C are the upper and the lower half planes divided by the strip B, respectively.

Particles are injected at a point, say P, in B in the positive y direction consequently the motion takes place in the (x, y) plane. Let the distance from P to the boundary between B an C, a_0, be less than $a/2$, then a particle with a momentum greater than some critical one drifts toward right ($x > 0$) gyrating along the strip and one with a less momentum toward left gyrating along the boundary between B and C (c.f. Fig. 1), provided the following conditions are satisfied:

$$k(\xi - 1) - (\eta - 1) > 0 , \tag{1}$$

$$P_2 > (a - a_0)(eH_B/c) > P_1 > a_0(eH_B/c) \tag{2}$$

where P_1 and P_2 are the magnitudes of the initial momentums, ξ and η are the ratios of field strength

$$\xi = H_B/H_A , \qquad \eta = H_B/H_C$$

and k is defined by

$$k = [\{R_B^2 - (a-a_0)^2\}/(R_B^2 - a_0^2)]^{\frac{1}{2}}$$

with the cyclotron radius $R_i \equiv \dfrac{cP_2}{eH_i}$ (i = A, B, C). The inequality (1) is the condition for the particle of the greater momentum P_2 to drift over the regions A, B and C as is required (c.f. Fig. 1) and is obtained easily by noting that the particle must first return to the right of the point P. The condition (2) is the consequence of the inequalities

$$r_C < r_B ,$$

$$a - a_0 > r_B > a_0 ,$$

in which r_i is the cyclotron radius $\frac{cP_1}{eH_i}$ ($i = BC$) and by means of which the particle with the momentum P_1 drifts leftwards along the boundary. This may be regarded as the conditions for the initial momentums for a given value of H_B whilst the condition (1) determines the admissible range of H_A and H_C for the values of P_2 and H_B. (c.f. Fig. 2). If the two particles have the same mass, the result implies that the small difference in the initial velocities leads to the large deviation in the motion on later times. This seems to suggest that a non-adiabatic change of magnetic field would play an important role in the dissipation in phase space.

In the non-relativistic case result obtained so far may, of course, be applied to the motion of the particles with the same initial velocity but of different masses. The Figure 1 shows the trajectries for a proton and a deuteron with the energy 300 e V. The values of the other parameters are specified as follows:

$a = 4.20$ cm

$a_0 = 1.00$ cm

$k = 0.979$

$\xi = 6.91$

$\eta = 2.52$

$H_A = 144.7$ gauss

$H_B = 1000$ gauss

$H_C = 396.7$ gauss

The distance of the closest approach to the strip is equal to 2.9 cm for the deuteron whilst 0.7 cm for the proton. The authors are greatly indebted to Dr. Y. Kato for his useful advices.

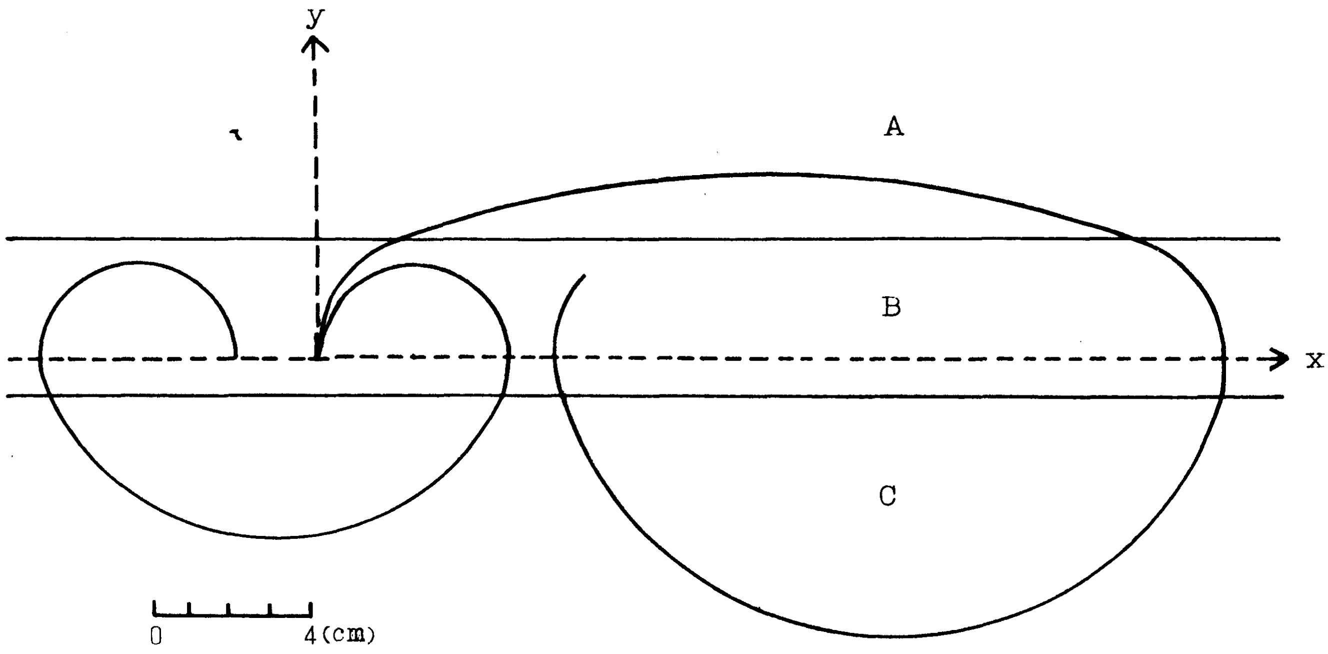

Fig. 1. The trajectries for a deuteron and a proton with the same energy.

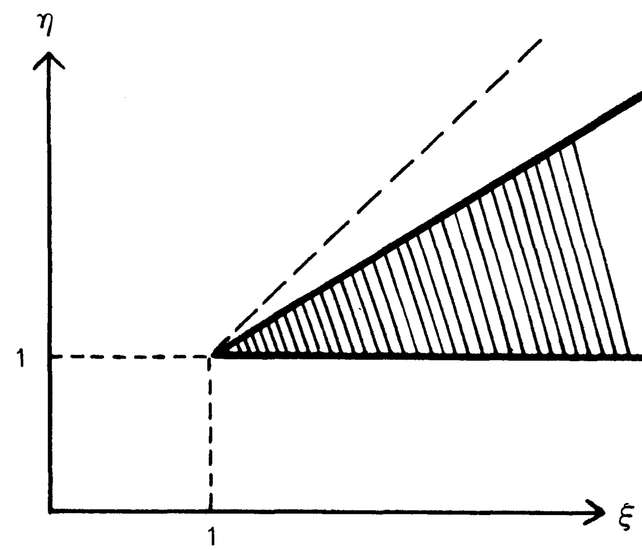

Fig. 2. The admissible range of ξ and η for a given value of k is shaded.