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NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

The Density Fluctuation

and Transport Process in a Plasma

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IPPJ-80 March 1969

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, JAPAN.

Abstract

The relation between the fluctuation and the transport process is clarified in a quiescent plasma and the diffusion coefficient, determined from the spectrum of the density fluctuation received by a single Langmuir probe, is found to be in good agreement with the coefficient derived from the correlation measurement of the density fluctuation detected by two Langmuir probes located at different positions within the plasma.

1. Introduction

In recent years the study of the relation between the fluctuations and the transport phenomena became an important problem in plasmas. The statistical fluctuations in plasmas were studied by many workers. Ichimaru[1] and Weinstock[2] gave clear formulations of the correlation of the density fluctuation and the diffusion coefficient in plasmas. According to their theory, Tsukishima *et al.* measured the diffusion coefficient from the correlation of the density fluctuations picked up by two Langmuir probes in a positive column[3]. On the other hand, the fluctuation received by a probe immersed in a quiescent plasma was studied by several workers[4]. In the frequency range lower than the electron plasma frequency, the fluctuation in the probe current consists of shot noise coming from the fluctuation of the number of particles present in the sheath.

Near the plasma frequency, the shot noise disappears because of the transit time effect, while the thermal noise becomes dominant because the probe conductance increases due to the sheath-plasma resonance.

In this report we describe the first observation of the relation between the transport phenomena and the shape of the frequency spectrum of the density fluctuation in the very low frequency range. We also present the measurement of the correlation function of the density fluctuation and compare the transport coefficient, determined from the first zero-crossing frequency of the correlation function, with the one drived from the shape of the spectrum of the fluctuation.

2. Theoretical description

In the very low frequency range ($f \sim 100$ kHz), the spectrum of the noise has $1/f$ characteristics and it is considered to be caused by the macroscopic transport phenomena present in plasmas.

In general the current fluctuation is expressed by the equations,

$$< \Delta i \Delta i > = e^2 <(v_o \Delta n + n_o \Delta v)(v_o \Delta n + n_o \Delta v)>$$

$$= e^2 [v_o^2 <\Delta n \Delta n> + 2 v_o n_o <\Delta n \Delta v> + n_o^2 <\Delta v \Delta v>] \quad (1),$$

where subscript 0 means the direct component of quantities. The first term of eq.(1) appears when an macroscopic flow exists in the medium and the second term comes from the correlation between density and velocity fluctuation, which is negligibly small in the quasi-neutral medium like a plasma. The last term is due to the dispersion of the particle velocity and consequently it is called as thermal fluctuation.

In plasmas the term $< \Delta v \Delta v >$ is dominant source of the fluctuation in high frequency region, that is, the contribution from electrons is strong near the electron plasma frequency and the one from ions is dominant near the ion plasma frequency. The term $< \Delta n \Delta n >$, however, is connected to the macroscopic transport phenomena through the flow velocity v_o and has strong relation with the fluctuation is rather low frequency range.

The treatment for $< \Delta n \Delta n >$, which is called transport noise, is done systematically by Fasset *et al.*[5]. In their theory the next formula is assumed for the spatial correlation.

$$< \Delta n(\vec{r}, t) \Delta n(\vec{r}', t) > \propto \delta(\vec{r} - \vec{r}') \quad (2).$$

Because the correlation length is the order of the distance between molecules in stable condition of liquids (If a liquid is in the critical region like as the liquid-gas phase transition[6], the correlation is maintained $1/r$ spatially), the above assumption is right for the fluctuation accompanied with the macroscopic transport phenomena.

On the other hand, in plasmas the equation (2) is modified a little by Debye-Hückel theory (or Random Phase Approximation).

$$<\Delta n(\vec{r},t)\Delta n(\vec{r}',t)> \propto \delta(\vec{r}-\vec{r}') - \frac{1}{2}\lambda_D^{-1} \exp(-\lambda_D^{-1} |\vec{r}-\vec{r}'|) \qquad (3),$$

where λ_D is the Debye length.

But the second term of equation (3) can be neglected for the transport noise problem because the scales of time and space are much larger than ω_p^{-1} and λ_D, respectively.

If the system is stationary, the time shift is permissible for the density correlation,

$$<\Delta n(r,t)\Delta n(r',t')> = <\Delta n(r,\tau)\Delta n(r',0)> \qquad (4).$$

And the density fluctuation at certain position, which has an extent of the order λ_D (the fluctuation is picked up by a Langmuir probe surrounded by the sheath whose depth is the order λ_D), is expressed as the sum of the fluctuation transported from other points during the time τ,

$$\Delta n(\vec{r},\tau) = \int g(\vec{r},\vec{r}',\tau)\Delta n(\vec{r}',0)\,d\vec{r}' \qquad (5),$$

where $g(\vec{r}, \vec{r}', \tau)$ is the Green function of the transport equation, that is, the response at the point $(\vec{r},\tau)$ when the single disturbance is applied at the points $(\vec{r}', 0)$. The correlation function is

$$< \Delta n(\vec{r},\tau)\Delta n(r'0) > = \int g(\vec{r},\vec{r}'',\tau) < \Delta n(\vec{r}'',0)\Delta n(\vec{r}',0) > d\vec{r}'' \qquad (6).$$

By the use of the Wiener-Khintchine's theorem the spectral density of the density fluctuation is written as follows;

$$S(\vec{r},\vec{r}';\omega) \doteq 2 \int_{\infty}^{\infty} e^{i\omega\tau} d\tau \int g(\vec{r},\vec{r}'',\tau) <\Delta n(\vec{r}'',0)\Delta n(\vec{r}',0)> d\vec{r}'' \qquad (7).$$

In the same way the spectral power density of the fluctuation in the total number of particles in the volume V is calculated by the assumption before described ($<\Delta n(r)\Delta n(r')> \propto \delta(r-r')$). The final result after some algebra is

$$< \Delta n^2 >_\omega = \frac{4<\Delta n^2>}{V} Re \int_v \int_v d\vec{r} d\vec{r}' G(\vec{r},\vec{r}';\omega)$$

$$\text{and} \quad G(\vec{r},\vec{r}';\omega) = \int_0^\infty e^{-i\omega\tau} g(\vec{r},\vec{r}';\tau) d\tau \qquad (8),$$

where ΔN is the fluctuation of the total number N of particles contained in the volume V and $G(\vec{r}, \vec{r}', \omega)$ is the Green function of the transport equation, in other words $G(\vec{r}, \vec{r}', \omega)$ is the inverse Fourier transform in the $\vec{k}$ space of the dynamic form factor $S(\vec{k}, \omega)$ derived by Ichimaru. The frequency spectrum of the fluctuation was calculated for various cases by Fasset *et al.*(5). When the transport due to the one dimensional diffusion is taken into account, the frequency spectrum of the fluctuation is given by:

$$< \Delta n^2 >_\omega = \frac{<\Delta N^2>}{D\theta^2} R^2 \{ 1 - e^{-\theta}(\cos\theta + \sin\theta) \} \text{ and } \theta = R\sqrt{\frac{\omega}{2D}} \qquad (9),$$

where D is the diffusion coefficient and R is the typical length of the volume where the fluctuation is studied. The spectrum has a $f^{-1/2}$ dependence in the lower frequency range and a $f^{-3/2}$ dependence in the higher frequency range. The critical value f_c, where the frequency dependence of the density fluctuation changes from $f^{-1/2}$ to $f^{-3/2}$, is $(1/\pi)DR^{-2}$.

As another example we consider shot noise. In this case the transport equation is pure drift equation,

$$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} = 0 \qquad (10),$$

$$< \Delta N^2 >_\omega = \frac{4<\Delta N^2>}{\omega^2} v(1- \cos \omega\tau_o), \ \tau_o = \frac{L}{v} \qquad (11),$$

and $$< \Delta i^2 > = e^2 v^2 <\Delta N^2>_\omega = 2eI \frac{\sin^2(\omega\tau_o/2)}{(\omega\tau_o/2)^2} \qquad (12),$$

where we use the relation $I = eN/\tau_o$. When we consider the fluctuation in the probe current, the particles come into the probe by various transport processes in actual case. In usual positive columns, electrons or ions are carried by ambipolar diffusion to the boundary of the sheath around the probe and in the sheath the particles are transported by electric field. The depth of the sheath is the order of the Debye length and transit time going through the sheath is the order of ω_p^{-1}. So the shot noise has continuous spectrum up to f_p. On the other hand when we consider about the transport by ambipolar diffusion, the transit time is the order of $R(D_a \nabla n/n)^{-1}$(here R is the radius of a positive column.), which is very large compared with ω_p^{-1}.

The frequency range of the spectrum which is characterized by the transport process of ambipolar diffusion is very low. The present report is on the problem of the electron free diffusion treated by the same way, which is described in the next section.

Experimentally the correlation function of the density fluctuation $G(\vec{r}, \vec{r}', \omega)$ itself is measured by picking up the density fluctuation at different points $\vec{r}$, $\vec{r}'$ in the plasma. The diffusion coefficient can be evaluated by measuring the first zero-crossing frequency of $G(\vec{r}, \vec{r}', \omega)$.

3. Experimental results and Discussions

In the present experiment an anode glow mode plasma is used[(7)]. The structure of the discharge tube filled with neon gas and the block diagram of the instrumentation are shown in Fig.1. The noise is picked up by a cylindrical probe (1 mm in diameter and 2 mm in length) located longitudinally at 1 cm from the cathode. The electron temperature and density are 0.2 $\sim$ 0.3 eV and $10^9 \sim 10^{10}$/cc. respectively. The density distribution is proportional to r^{-2} and the static radial electric field is 0.05 $\sim$ 0.1 V/cm. In this plasma the electrons are supplied by the thermal emission from the cathode while the ions are produced by ionization near the anode. The electrons emitted from the cathode are carried by diffusion and by drift due to the electric field. The radial transport velocity of the electrons is expressed as follows:

$$v = D\frac{1}{n}\frac{\partial n}{\partial r} + \mu E_r \quad \text{and} \quad D = \frac{v_{th}\lambda}{3} \qquad (13),$$

where D, μ, E_r, V_{th} and λ are the diffusion coefficient, the mobility, the electric field, the thermal velocity of the electrons and the mean free path, respectively. For example, when the gas pressure is 1 Torr, the diffusion velocity, $D\ \nabla n/n = 2D/r$ is about 3×10^6 cm/sec at a distance $r = 1$ cm from the center and the drift velocity due to the electric field is about 5×10^5 cm/sec; the transport of the electrons in the radial direction is then due mainly to diffusion rather than to drift. The density fluctuation is measured as a function in the probe current.

The power of the noise current $< i^2 >_f$ is measured by comparison with the saturated diode used as the standard noise generator producing the full shot noise $< i^2 >_f = 2eI_n$ corresponding to the anode current I_n. The noise in the probe is expressed as the anode current I_n of the standard diode having the same level noise as in the probe current. The theoretically predicted error, ε, of the measurement of the fluctuation is determined by the bandwidth Δf of the receiver and by the time constant τ of the detector, $\varepsilon = (\tau \Delta f)^{-1/2}$.
In our experiment, ε is less than 5%.

The relation between the probe current I_p and the equivalent noise current I_n is shown in Fig.1. When ionization does not occur in the sheath, I_n is proportional to I_p; this fact indicates that the density fluctuation $< \Delta N^2 >$ is equal to N(the fluctuation is purely random and follows the Poisson process). When the electrons begin to ionize the neutral atoms in the sheath, the noise level I_n is proportional to $I_p{}^2$ and the fluctuation is considered to be flicker type. When ionization does not occur in the sheath, the frequency spectrum of the fluctuation is shown in Fig.2. The frequency spectrum of the noise is proportional to $f^{-1/2}$ in the lower frequency and to $f^{-3/2}$ in the higher frequency region. This dependence is well explained by equation (2) and this fact indicates that the electrons are transported by diffusion. The diffusion coefficient of the electrons can be determined from the critical frequency f_c by putting in equation (2) R = 1 cm, because the electrons are transported to the probe at the distance of 1 cm from the cathode.

It is, however, expected that the spectrum becomes flat in the lower frequency range when the effect of the boudary conditions is strong. The effect of the boundary conditions is explained in connection with the correlation length.

$$|\Delta n(\vec{k},\omega)|^2 = \int\int \phi(\vec{s},\tau)e^{i\omega\tau}e^{-i\vec{k}.\vec{s}}d\tau d\vec{s} \qquad (14),$$

where $\phi(\vec{s},\tau) = < \Delta n(\vec{r},t)\Delta n(\vec{r}-\vec{s},t-\tau) >$

For k which satisfies the condition $kR_o << 1$, $|\Delta n(k,\omega)|^2$ is constant (here R_o is the dimension of the system), that is, the spectrum becomes flat in low k region. For the diffusion mode the dispersion relation is roughly expressed

$$\omega^2 + k^4D^2 = 0 \qquad (15).$$

Consequently in low frequency region the spectrum has a flat characteristics due to the finiteness of the system. Even in our experiment the spectrum will have a flat part in the much lower frequency region. We also measured the first zero-crossing frequency, f_o, at which the correlation function G(r, r', ω) goes through zero value. This correlation function was measured by placing the Langmuir probes at different positions in the radial direction (r = 0.5 cm and r'= 1.5 cm). The measuring circuit is shown in Fig.4, and the results of measurement for $G(r, r', \omega)$ is shown in Fig.3. The lock in amplifier was used as the synchronous detector and it was impossible to get the information about the amplitude $< i_A i_B >$, so the phase relation only was observed. Zero-crossing frequency was not changed so much by changing the probe bias, because the Debye length is much shorter than the distance between two probes. In Fig.4, the critical frequency f_c, determined from the noise measurement, and the zero-crossing frequency of $G(r, r', \omega)$, f_o, are plotted as a function of the gas pressure. The solid line indicates the calculated values of f_c ($f_c = D/\pi R^2$) assuming that the electrons are transported by diffusion. In the anode glow mode plasma one should keep in mind the fact that the electrons and ions do not move by ambipolar diffusion happens only when the net electric current is equal to zero, as in the case of the radial transport in positive columns. The radial direction in the anode glow mode plasma, in our experiment, corresponds to the axial direction in positive columns. The fluctuation is composed of electron and ion density fluctuations, but in the case of data in Fig.1 and Fig.2, the electron density fluctuation is prevalent, because the larger part of the probe current is constituted by electron current. As it is shown in Fig.4, it is experimentally confirmed that the diffusion coefficient of the electrons in a quiescent plasma derived from the noise spectrum, agrees very well with the one deduced from the correlation measurement of the fluctuation. We are now doing the same experiment for the fluctuation associated with the transport process in the radial direction in positive columns and we have observed that the measured ambipolar diffusion coefficient is in good agreement with the predictions of the theory.

The authors wish to express their appreciation to Dr. H. Ikezi and Dr. T. Tsukishima for many valuable discussions.

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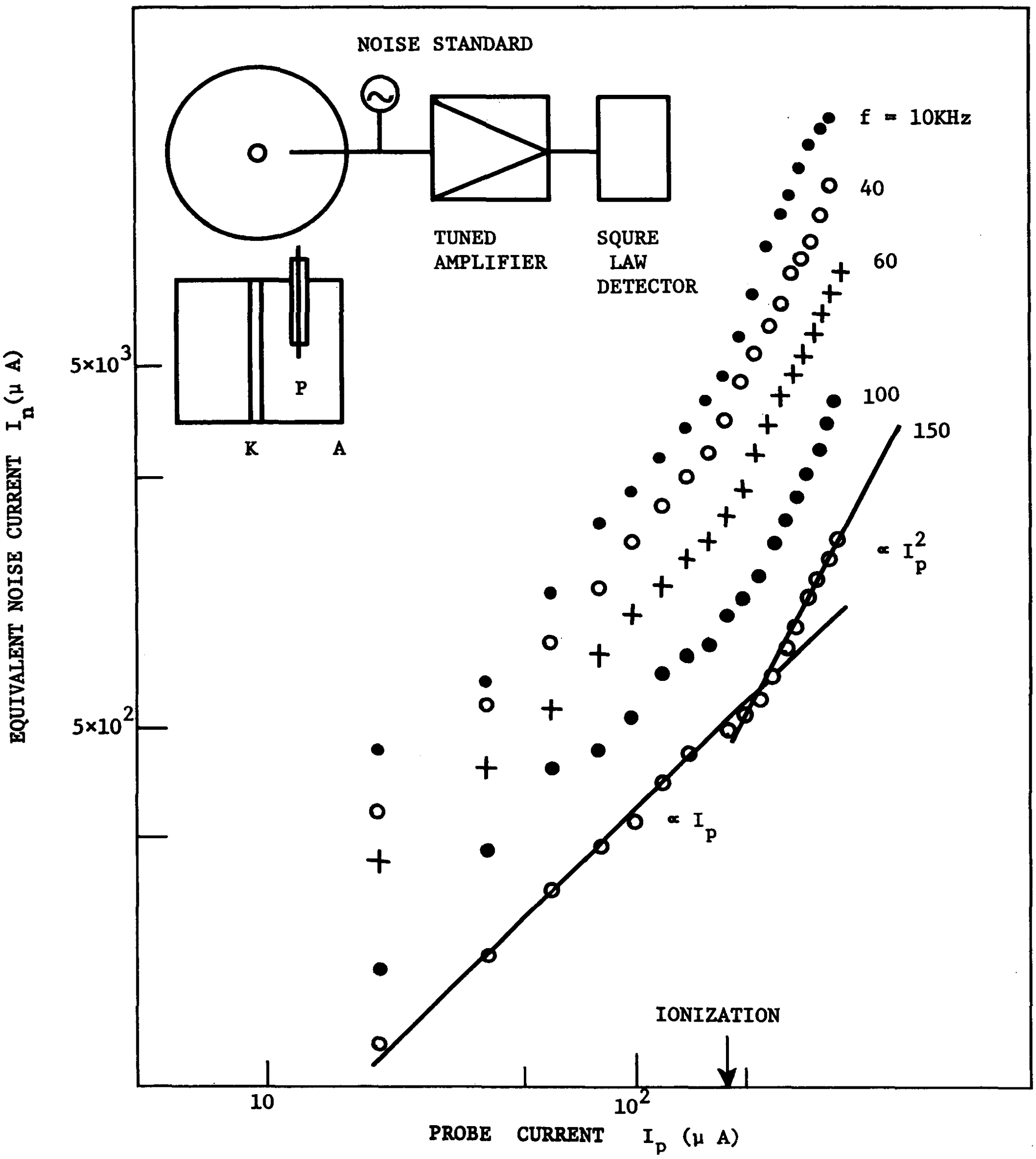

Fig.1 The block diagram of the instrumentation; the structure of the discharge tube and the I_p - I_n characteristics.

A -- cylindrical anode (50 mm in diameter and 50 mm in length);

P -- cylindrical probe (1 mm in diameter and 3 mm in length);

K -- oxide cathode.

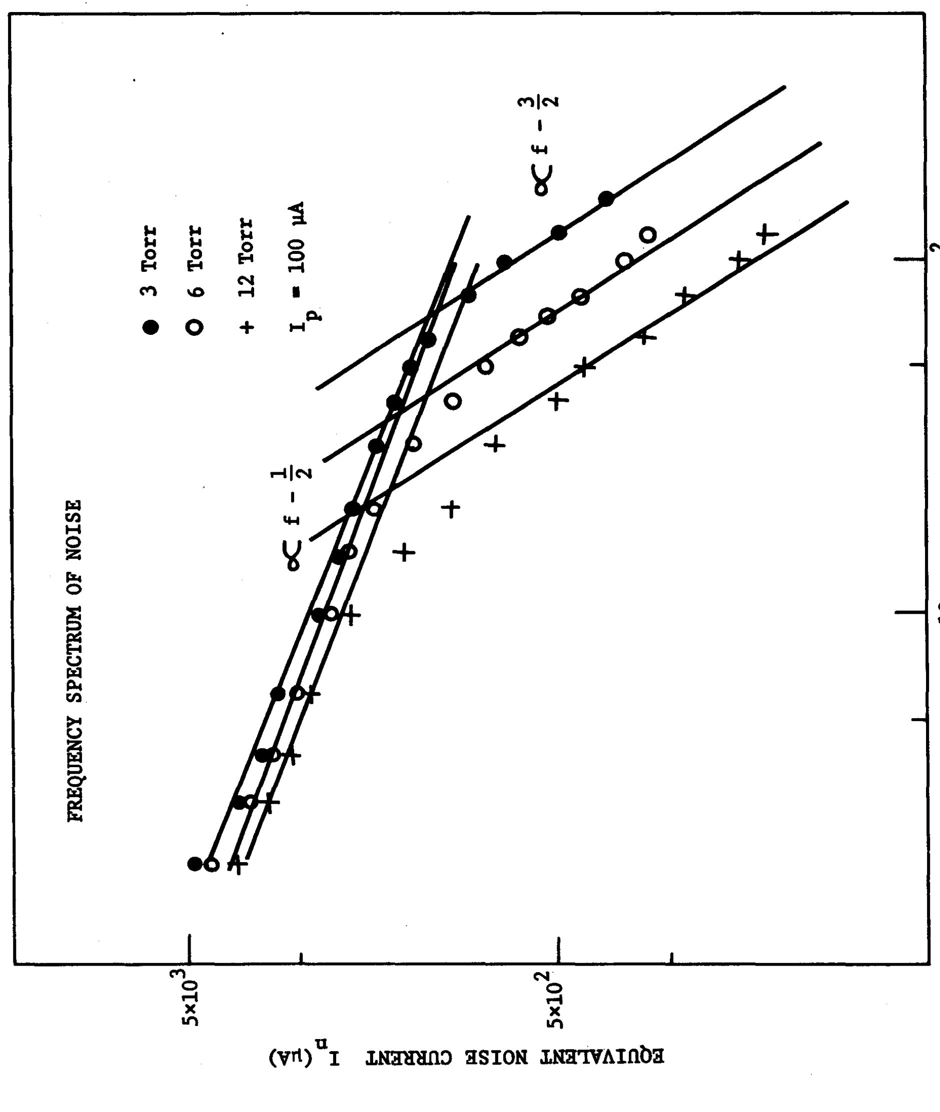

Fig.2 The frequency spectrum of the density fluctuation. In the low frequency range $I_n \propto f^{-1/2}$ and in the higher frequency range $I_n \propto f^{-3/2}$. The frequency dependence $f^{-3/2}$ is the universal characteristics for the transport by diffusion.

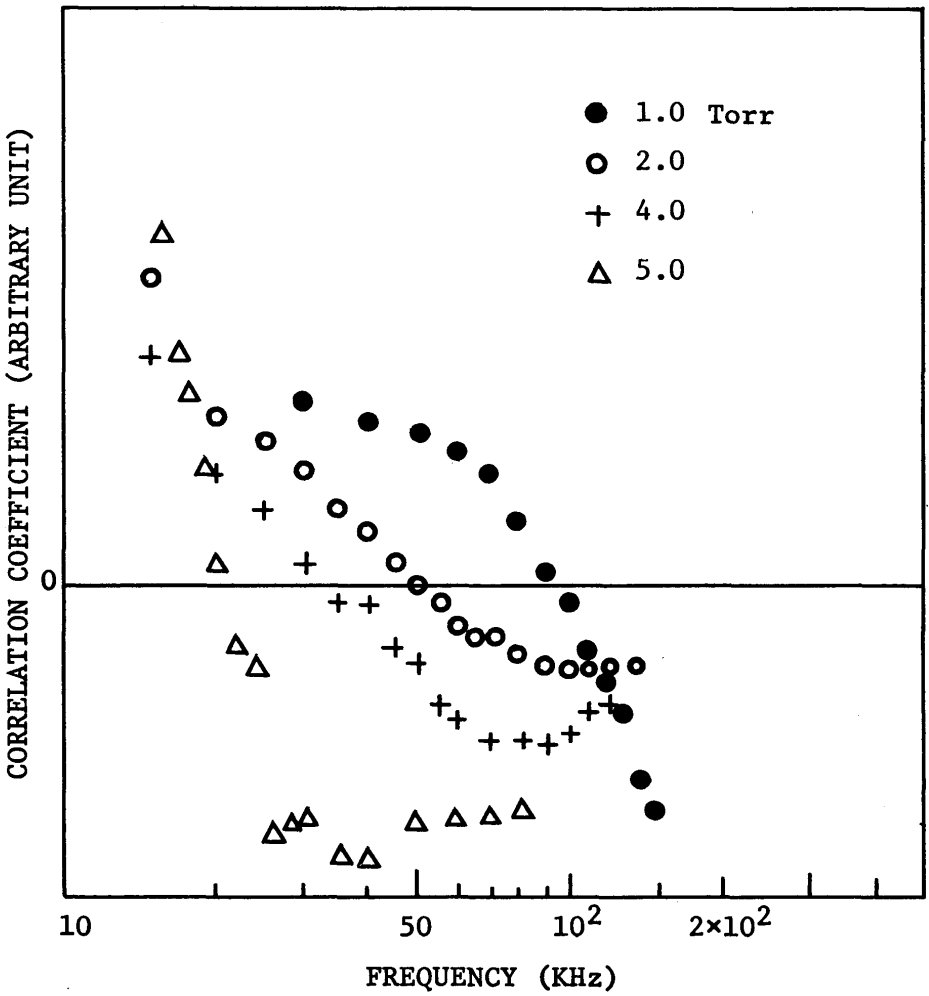

Fig.3 The results of the measurement for $< i_A i_B >$ or $G(r, r', \omega)$. As the increase of the gas pressure, zero-crossing frequency becomes lower. The signal was obtained by the probes which were biased to the space potential.

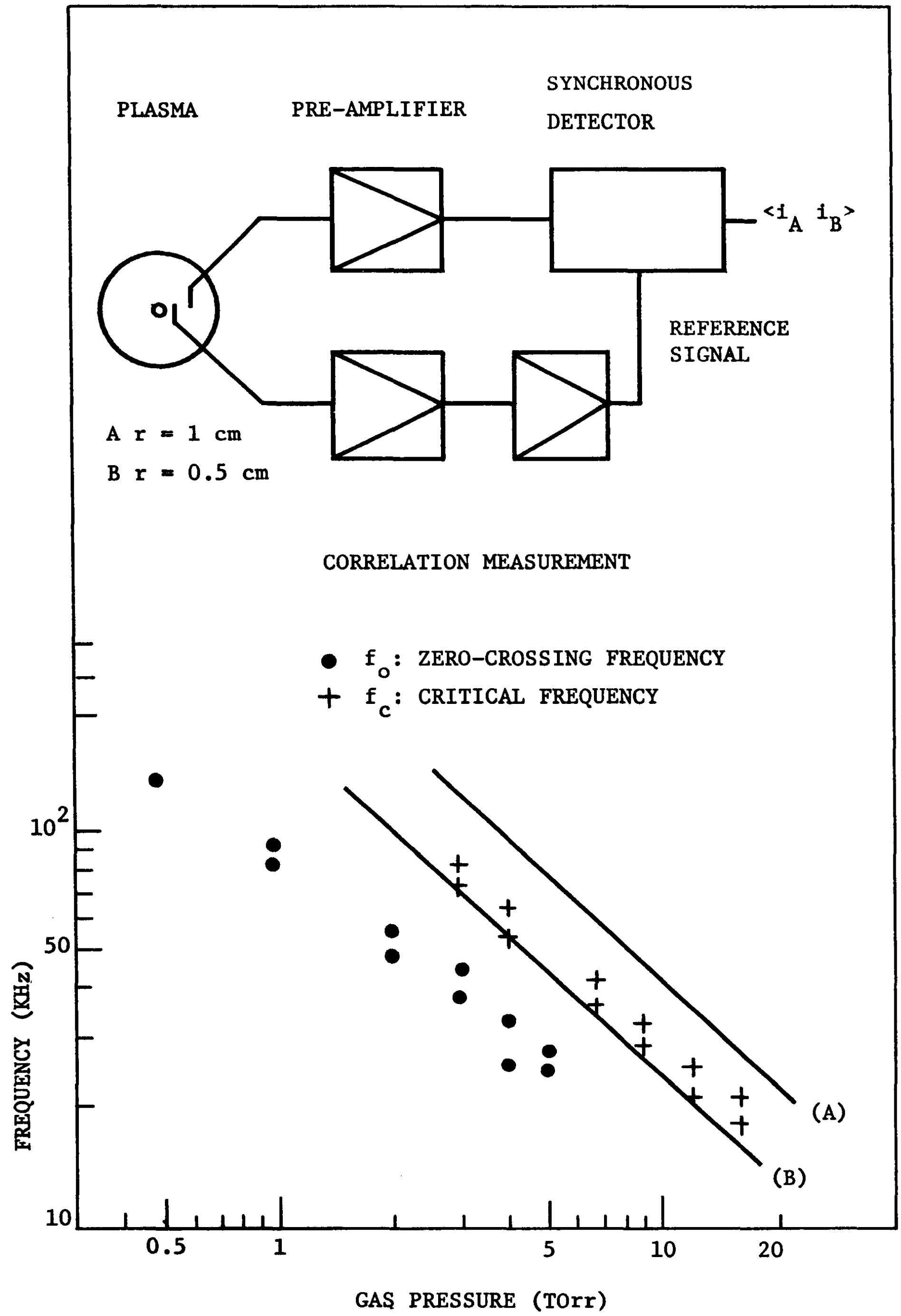

Fig.4 The block diagram for the measurement of the correlation function, and the pressure dependence of f_c and f_o. The solid line (a) is calculated from the relation, $f_c = (1/\pi)D_e r^{-2}$, with the assumption $D_e = V_{th}\lambda/3$, and the curve (b) is calculated by using Einstein relation $D\tau = r^2\alpha^2$ in cylindrical dimension (α is the first root of the Bessel function J_o).