INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Influence of Collisions on the Ion-Acoustic Instability in a Weakly Ionized Plasma

M. Fujiwara, M. Raether* and M. Yamada**

IPPJ-81 March 1969

* Permanent address: Coordinated Science Laboratory and Department of Physics, University of Illinois, Urbana, Ill.

** On leave from the University of Tokyo, Department of Nuclear Engineering.

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, JAPAN.

Abstract

The influence of collisions on the onset and linear stage of the ion-acoustic instability in a dc-discharge in Helium is studied experimentally. The threshold values of the drift velocity and frequency and the range of the unstable frequency spectrum are in good agreement with the values calculated from the dispersion relation based on the Vlasov equation with inclusion of ion-neutral collision. The intensity of charged particle density fluctuation has been measured over 4 orders of magnitude covering the range from thermal level to the fully developed instability.

§1. Introduction

Excitation of the ion-acoustic instability by a relative drift motion of electrons with respect to ions in a fully ionized plasma has been treated extensively in the literature(1-4). The predicted drift velocity for onset of instability and the unstable wavenumber range have been confirmed experimentally for a mercury plasma(5). The influence of ion-neutral collisions on the critical drift velocity and the unstable wavenumber range was first pointed out by Ichimaru(4) and more recently by Zaitsev *et al.*(6). The latter authors also investigated the influence of Coulomb collisions and electron neutral collisions and presented some experimental data. Collisions not only affect the onset of instability but also have a profound effect on the resulting turbulent state as suggested by Ichimaru and Nakano(7,8). For selected plasma parameters such influence has been observed experimentally by Arunasalam and Brown(9) and by Coleman(10).

In the following we present measurements of the onset condition for ion-acoustic instability in a weakly ionized He-plasma, the unstable frequency range as a function of a universal parameter and the total energy in the unstable waves as a function of drift velocity. The results are compared with theoretical predictions.

§2. Theoretical description

The influence of collisions on the instability of ion-acoustic waves has been discussed by Ichimaru and Zeitsev *et al.*. The behavior of a plasma in which electrons drift with respect to ions with a velocity v_D is characterized by the dielectric response function, $\varepsilon(k, \omega)$. In low frequency range $\varepsilon(k, \omega)$ is calculated by the expansion of plasma dispersion function with the assumption that $\omega/k \gg v_i$ and $|\,\omega/k - v_D\,| \ll v_e$.

$$\varepsilon(k,\omega) = 1 + \frac{k_e^2}{k^2} - \frac{\omega_i^2}{\omega(\omega + i/\tau_i)} + i\sqrt{\frac{\pi}{2}}\,\frac{k_e^{\,2}}{k^{\,2}}\left[\frac{\omega - kv_D}{kv_e} + \frac{T_e}{T_i}\frac{\omega}{kv_i}\exp\left(-\frac{\omega^2}{2k^2v_i^{\,2}}\right)\right] \qquad (1).$$

The distribution function of electrons and ions are assumed to be drift Maxwellian and Maxwellian with temperature T_e and T_i. k_e is the Debye wave number for electrons, ω_i the ion plasma frequency, and v_e and v_i are the thermal speeds of electrons and ions, respectively. Only ion-neutral collisions are included through the collision frequency $1/\tau_i$, and electron-neutral collisions can be neglected provided that $\frac{\nu_e}{\omega}\sqrt{\frac{m}{M}} \ll 1$. This simple expression for collision term is modified by the model of Bhatnagar$^{(11)}$ to explain the experimental results in reference 10. As shown in ref.4, this dispersion relation leads to a critical drift velocity v_c for the onset of unstable waves with wave-number k, which is given by

$$\frac{v_c}{v_e} = \left(\frac{m}{M}\right)^{\frac{1}{2}} \frac{1}{(1 + k^2/k_e^2)^{1/2}} \left\{ 1 + \left(\frac{M}{m}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{1}{2}\left[\frac{T_e/T_i}{1+k^2/k_e^2} + 3\right]\right)\right\} + \sqrt{\frac{2}{\pi}}\,\frac{1}{\omega_i\tau_i}\,\frac{(1 + k^2/k_e^2)^{3/2}}{k/k_e} \qquad (2).$$

The first term of the eq.(2) is due to electron and ion Landau damping and the second term due to ion-neutral collision damping. This "boundary curve" (curve of zero growth rate) is plotted schematically in Fig.1.

For small mass ratio m/M and high electron temperature the first term in eq.(2) is negligibly small and we have in a good approximation

$$\frac{v_c}{v_e} = \sqrt{\frac{2}{\pi}}\,\frac{1}{\omega_i\tau_i}\,\frac{(1 + k^2/k_e^2)^{3/2}}{k/k_e} \qquad (3).$$

For a spontaneously excited instability we are interested in the wavenumber that will first becomes unstable when the drift velocity is increased beyond the critical value. We obtain this from the condition, $dv_c/dk = 0$, as

$$k_{\text{opt.}} = k_e/\sqrt{2}\ .$$

With this value for k the critical drift velocity for onset of instability becomes

$$\left(\frac{v_c}{v_e}\right)_{\text{min.}} = \frac{2}{\sqrt{\pi}}\left(\frac{3}{2}\right)^{3/2}\frac{1}{\omega_i\tau_i} \qquad (4).$$

The frequency at which oscillations first appear follows from the dispersion relation for the real part of ω

$$\omega^2 = \frac{\omega_i{}^2}{1 + k_e^2/k^2} \quad \text{as} \quad \omega_{\text{opt.}} = \frac{\omega_i}{\sqrt{3}} \qquad (5).$$

It is convenient to express eq.(3) in terms of ω rather than k.

From eq.(3) and (5) we find the boundary curve between the stable and unstable regions as

$$\sqrt{\frac{\pi}{2}}\,\frac{v_c}{v_e}\,\omega_i\tau_i = \frac{1}{(\omega/\omega_i)[1 - (\omega/\omega_i)^2]} \qquad (6).$$

The growth rate for the unstable waves is found from eq.(1) as

$$\frac{\gamma}{\omega} = -\frac{1}{2\omega\tau_i} + \frac{1}{2}\sqrt{\frac{\pi}{2}}\,\frac{kv_D-\omega}{kv_e}\,\frac{\omega^2}{k^2}\,\frac{k_e^2}{\omega_i^2} \qquad (7).$$

The Landau damping term has been neglected in this expression. Eq.(7) can be written more conveniently in terms of the critical drift velocity v_c. In the long wavelength regime we find

$$\gamma = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,\omega\,\frac{v_D - v_c}{v_e} \qquad (8).$$

Here we have used the relation

$$\frac{\omega}{k} = \frac{\omega_i}{k_e} = S \qquad \text{for} \qquad k \ll k_e\,,$$

s is the ion-acoustic velocity. In particular, we have close to threshold

$$\gamma = \frac{1}{2}\sqrt{\frac{\pi}{6}}\,\omega_i\,\frac{v_D - v_c}{v_e} \qquad (9).$$

As described in the next section, v_e is much larger than v_D in our experiment ($v_D \lesssim 0.1\ v_e$). We could realize the transition region between stable and unstable state predicted by the linear theory. But it was impossible to make a condition for a stationary turbulent state, $v_D = s(1 + M\tau_e/m\tau_i)$, which is a equivalent condition, $v_D \simeq v_e$, for He-plasma(7, 8).

§3. Experimental results and discussions

The experiments were carried out in the positive column of hot cathode discharge(12) in Helium in the pressure range 0.065 - 0.21 Torr. Two discharge tubes were used, 7 cm in diameter 75 cm long and 10 cm in diameter, 100 cm long. In order to get quiescent discharge conditions, two grids whose shapes are similar to the equipotential surface of the positive column, are located near the anode and cathode. Langmuir probes were located along the discharge tubes for the measurement of the plasma parameters and the oscillations spectrum. The electron density was varied from 10^9 /cc to 5×10^{10} /cc by changing the discharge current between 0 to 2 amps. The electron temperatures were 4 - 7 eV and the energy distribution function was Druyvesteyn one. But according to the probe measurement the lack of high energy electron begins from 20 eV. The energy range of electrons which interact with the ion acoustic wave is very low compared with 20 eV. So we can apply the theory described in the previous section which is developed by the assumption of drift Maxwellian distribution for electrons. The parameter $\omega_i\tau_i$ covered the range 7.5 - 31. For the measurements of the oscillations spectrum the ac-component of the ion saturation current of a plane probe, which was negatively biased to collect ions only, were amplified and displayed on a Panoramic Spectrum Analyzer covering the range 0 - 15 MHz. The discharge tubes were completely shielded with Al-foil to avoid sprious pick-up of oscillations and external feedback of oscillations originating in the tubes. The indirectly heated oxide coated cathode was run at reduced temperature, a measure that proved very successful in suppressing low frequency cathode oscillations. No moving striations were present in the covered pressure and current range. The neutral gas pressure was monitored by a Pirani gauge that was calibrated against an oil manometer.

At a given pressure no oscillations are detected below a certain threshold current. (This statement required some qualification that will be given below). Above this critical current a narrow continuous spectrum of random oscillations appears which grows in amplitude as the current increases and shifts to higher frequencies in accordance with the increase in plasma density and consequently ion plasma frequency with increasing current. The phase measurement of this oscillation was impossible because the wavelength is too short ($k \sim k_D$). But we observed the fluctuation by using two types of plane probes (5 mm in diameter). One of them was located so that its surface was perpendicular to the axis of the discharge tube and the other one was placed so that the surface was parallel to the axis of the discharge tube. The signal level observed by the latter probe was about -40 db lower than the signal of the former probe. This fact means that the wavelength of this oscillation is very short and if we observe by the latter probe, the phase of the oscillation is averaged out and the signal level becomes very low.

It should be noted that increasing the current does not increase the drift velocity of the electrons, which is constant to a good approximation in the current range 0 - 2 amps. Increasing the plasma density, however, reduces the critical drift velocity necessary for onset of the ion wave instability (eq.(4)).

In Fig.2, the frequency and the critical drift velocity are compared with their theoretical values as a function of pressure. The drift velocity was inferred from the current and the measured electron density. Assuming that the electron density has a Bessel function profile, the two quantities are connected via the relation,

$$I = e(n_e)_{\mathrm{max.}} v_D \int_0^r J_0(\alpha_0 \frac{r}{R}) r dr$$

here I is the total discharge current, $(n_e)_{\mathrm{max.}}$ the density at the center, J_0 the 0th Bessel function, R the radius of discharge tube and α_0 the first root of J_0. The value of the drift velocity calculated from the above equation was compared with the experimental value given as the function of E/p and both value were found to be nearly equal. The ion-neutral collision frequency $\nu_i = e/M\mu_i$ was determined from the experimental value of the mobility of He^+ in He, $\mu_i = 10$ cm^2/Vs at 300° K. The ion temperature is very difficult to measure and must be assessed in a different way. The best agreement between experiment and theory is obtained by assuming $T_i = 300°$ K, although one would expect the ion temperature to be more like 500 - 600° K$^{(13)}$.

In Fig.3, the upper and lower limit of the frequency spectrum are compared with the theoretical boundary curve of eq.(6). The frequency spectrum of the fluctuation has only one peak near $\omega_{\mathrm{opt.}}$ when the plasma state is near the bottom of the boundary curve. In this region the plasma is in the linear stage and the mode-mode coupling is very weak.

But as the unstable parameter $\sqrt{\frac{\pi}{2}}\, \omega_i \tau_i \frac{v_D}{v_e}$ becomes large, the spectrum has two peaks as indicated schematically in the figure, the one at high frequency being much smaller in intensity. This high frequency maximum has its origin in the dispersion relation for ion acoustic waves, which has a cut off at the ion plasma frequency but no cut off for wave numbers. Consequently the oscillation spectrum in the region $k > k_e$ is concentrated in a narrow frequency region around ω_i. The main error source is the measurement of the absolute value of n_e which is certainly not better than 20 %.

With respect to the determination of $(V_c)_{min.}$ a practical problem arises. With sufficiently sensitive receivers oscillations are already detected for $v_D < (v_c)_{min.}$. These are the critical fluctuation discussed in detail by Ichimaru$^{(4)}$. For $v_D << (v_c)_{min.}$ only thermal noise with a broad spectrum is present. As v_D approaches $(v_c)_{min.}$, the fluctuation in the neighborhood of $\omega_{opt.}$ are greatly enhanced and the spectrum narrows. As $(v_c)_{min.}$ is exceeded the spectrum broadens again in accordance with eq.(6). Fig.4 shows an example of the spectral width as a function of current. The current at minimum spectral width is taken as corresponding to the onset of instability. Measurements of the intensity of the charge density fluctuations were carried out in both tubes. The range of the parameter $\sqrt{\frac{\pi}{2}}\, \omega_i \tau_i \frac{v_D}{v_e}$ that could be covered differed for both tubes due to the different geometry. The 10 cm diameter tube was more suitable for covering the range below threshold, the 7 cm diameter tube for the range above threshold. The data joined smoothly in the overlap region.

Fig.5 shows a plot of $|n(\omega)|^2$ at the frequency where the main peak of the oscillations spectrum occured as a function of $\alpha = \sqrt{\frac{\pi}{2}}\, \omega_i \tau_i \frac{v_D}{v_e}$.

As stated before the fluctuation was picked up by a negatively biased probe, therefore the fluctuation means ion density fluctuation. The oscillations start from thermal noise and first experience an enhancement below the critical drift velocity as α is increased. This is the region of critical fluctuations discussed in detail by Ichimaru(4). Above threshold they grow approximately exponentially with α until the growth rate levels off when the intensity has increased by about a factor of 10^4 above thermal level. It appears that non-linear coupling limits the growth when α reaches a value of 4 - 5, because at the same time a distortion of the spectrum sets in which shifts oscillations to lower frequencies. This is in qualitative agreement with the considerations of ref.7, but it must be realized that the theory of a stationary turbulent state is not without modification applicable to the present experimental situation. The reason for this is, that the growth rate is not that of the hydrodynamic theory but that of the kinetic theory. Also the lower cut-off in wavenumber $k_{\text{min.}} = \frac{1}{2s\tau_i}$ as assumed in the hydrodynamic theory of the stationary turbulent state is not realized in the present experiment.

§4. Conclusion

The onset and the linear stage of the ion acoustic instability in a He-plasma has been studied under conditions where ion-neutral collisions are important. The results lead to the conclusion that the onset condition and the excited frequency or wavenumber range is correctly described by the dispersion relation for the low frequency range as derived from kinetic theory. The excited wave number is different from previous experiments(9,10), and our results agree better with the theory.

The main uncertainty and a possible systematic error lies in the value of the ion temperature which was assumed to be 300° K. The results, however, do not agree with Zaitsev's condition $\omega_i\tau_i > \sqrt{M/m}$ for the excitation of ion acoustic waves in a dc-discharge$^{(6)}$. In fact we find that ion-acoustic waves are excited for values of $\omega_i\tau_i$ which are a factor 10 lower than those predicted from Zaitsev's criterium. The measurements of $|n(\omega)|^2$ are in quantitative agreement with theoretical expectations but a quantitative comparison with i.e. the theory of stationary turbulent states is not possible at the present time.

Acknowledgements

For one of us, M.R., this work was made possible through a travel grant by the US-Japan Cooperative Science Program of the National Science Foundation. M.R. and M.Y. are much indebted to Prof. K. Takayama for the kind hospitality extended to us during our stay at the Institute of Plasma Physics.

References

1. G. V. Gordeev, Zh. Eksp. Teor. Fiz. 27 (1954) 19.
2. E. A. Jackson, Phys. Fluids 3 (1960) 786.
3. I. B. Bernstein and R. M. Kulsrud, Phys. Fluids 3 (1960) 937.
4. S. Ichimaru, Ann. of Phys. 20 (1962) 78.
5. H. Tanaka, A. Hirose and M. Koganei, Phys. Rev. 161 (1967) 94.
6. A. A. Zaitsev, B. Milish, A. A. Rukhadze and B. N. Shilkin Sov. Phys. Rech. Phys. 12 (1968) 1175.
7. S. Ichimaru and T. Nakano, Phys. Rev. 165 (1968) 231.
8. S. Ichimaru and T. Nakano, Phys. Rev. 165 (1968) 251.
9. V. Arunasalam and S. C. Brown, Phys. Rev. 140A (1965) 471.
10. J. T. Coleman, J. Appl. Phys. 38 (1967) 2655.
11. P. L. Bhatnagar, E. P. Gross and M. Krook, Phys. Rev. 94 (1954) 511.
12. H. Ikezi, K. Takayama and S. Miyake, Proc. Intern. Conf. on Ionization Phenomena in Gases, Wien, Austria, Aug.- Sept. 1967, p.553.
13. E. C. Beaty and P. L. Patterson, Phys. Rev. 137 (1965) 346.

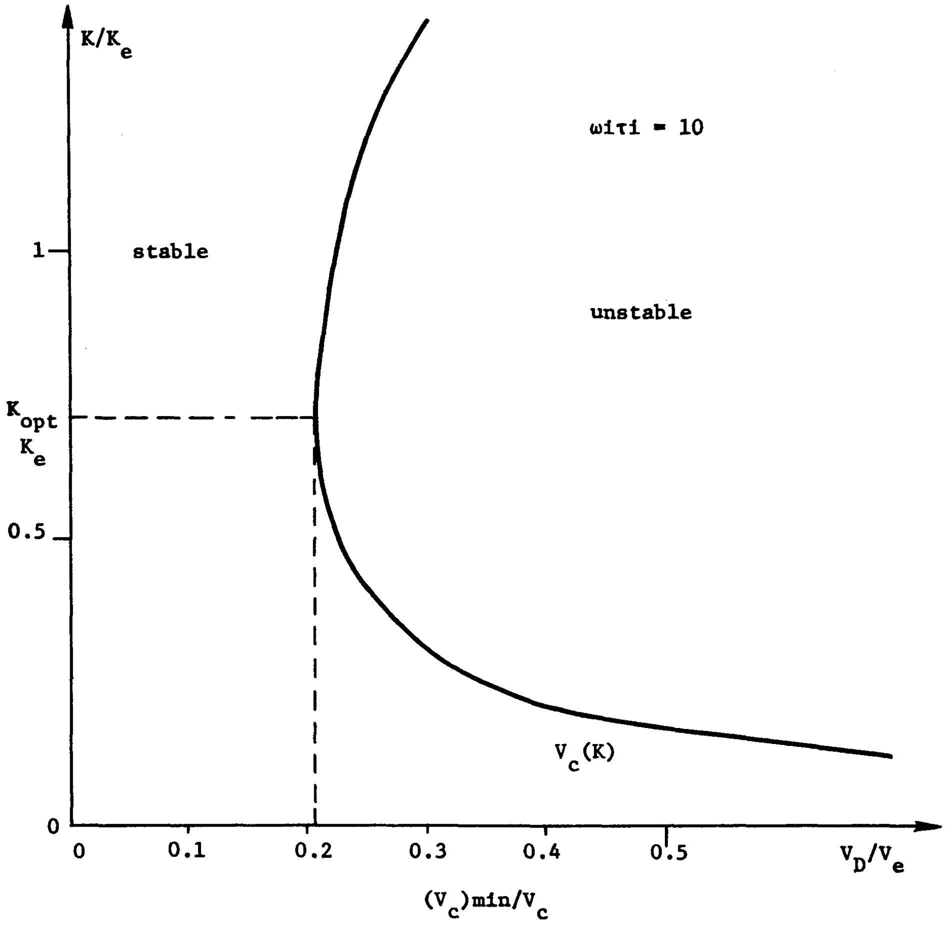

Fig.1 Theoretical curve of the boundary between stable and unstable regime. This curve is calculated by neglecting Landau damping of ions and electrons.

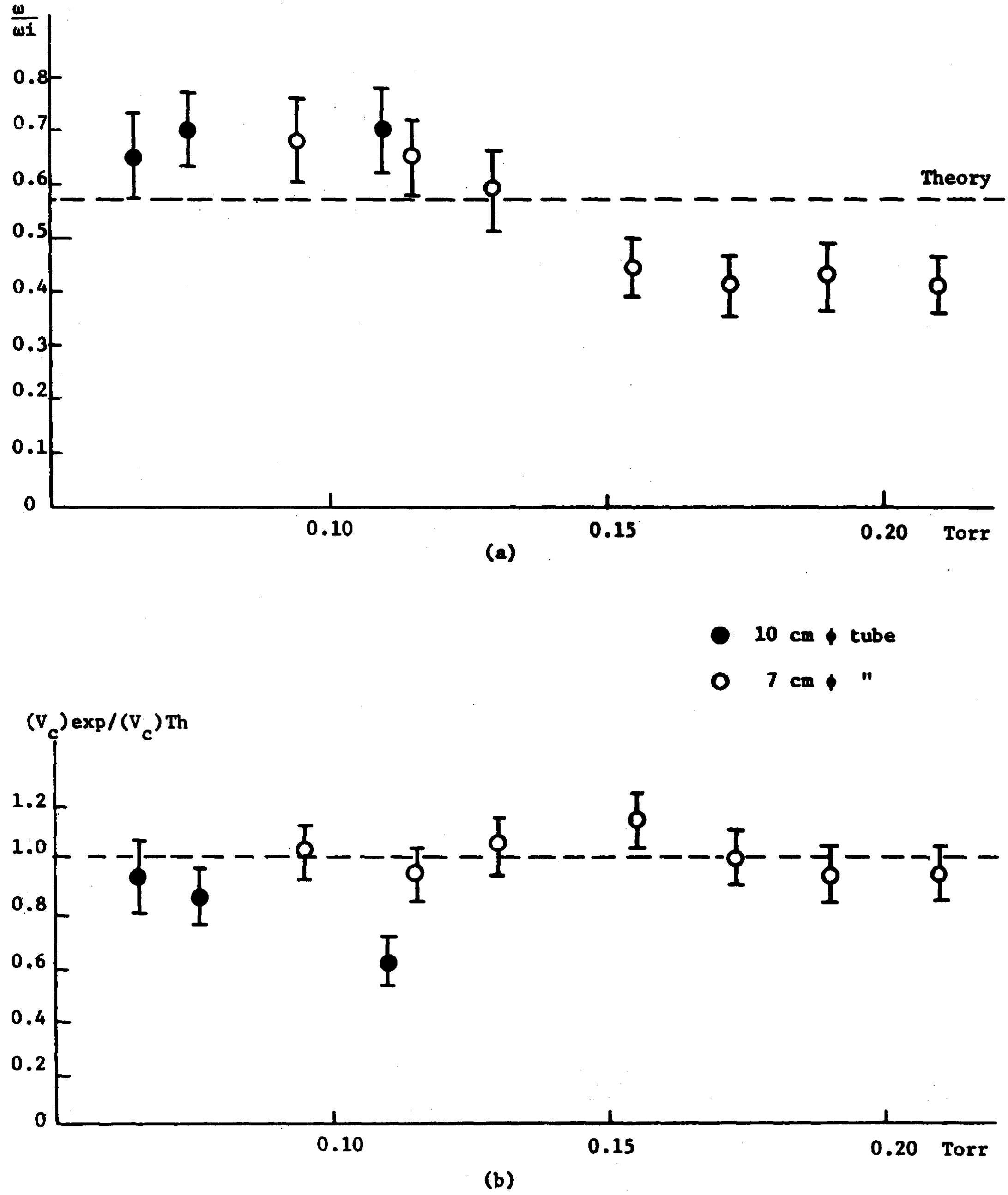

Fig.2 Critical drift velocity and frequency at the onset of instability.

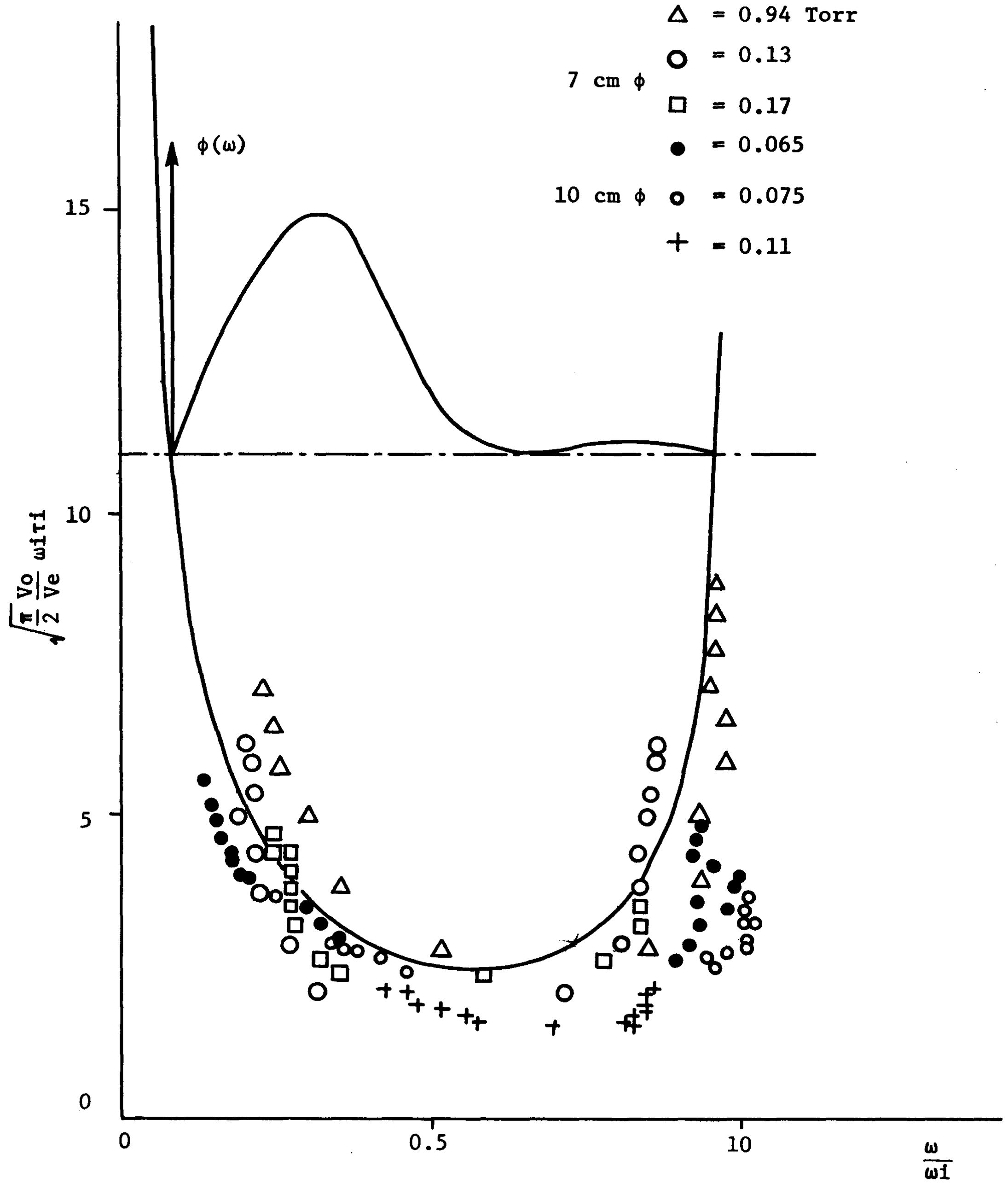

Fig.3 The width of the oscillation spectrum during transition from stable to unstable state.

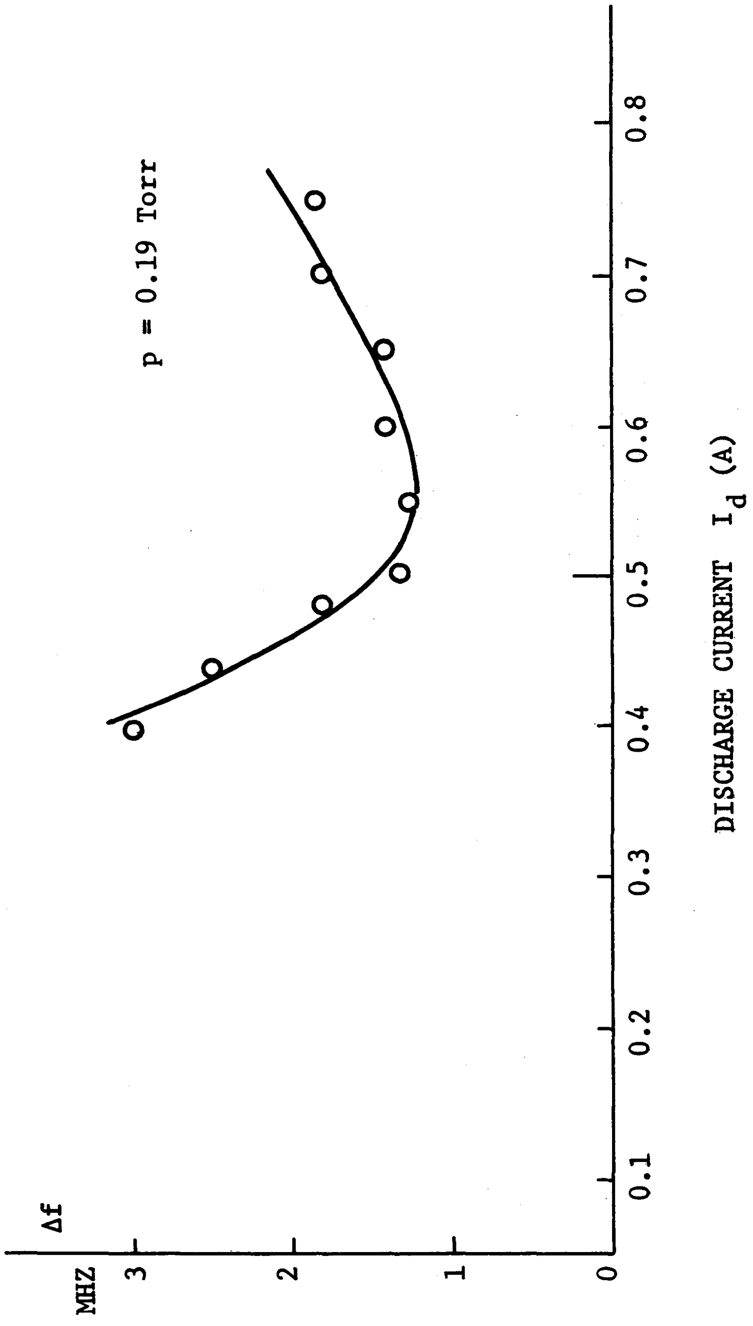

Fig.4 The upper and lower frequency limit normalized by the ion plasma frequency. The plasma state is expressed by the universal parameter $\sqrt{\frac{\pi}{2}}\,\omega_i\tau_i\,\frac{v_D}{v_e}$ and the solid curve is the boundary curve calculated in ω space by using the dispersion relation of ion acoustic wave for Fig.1.

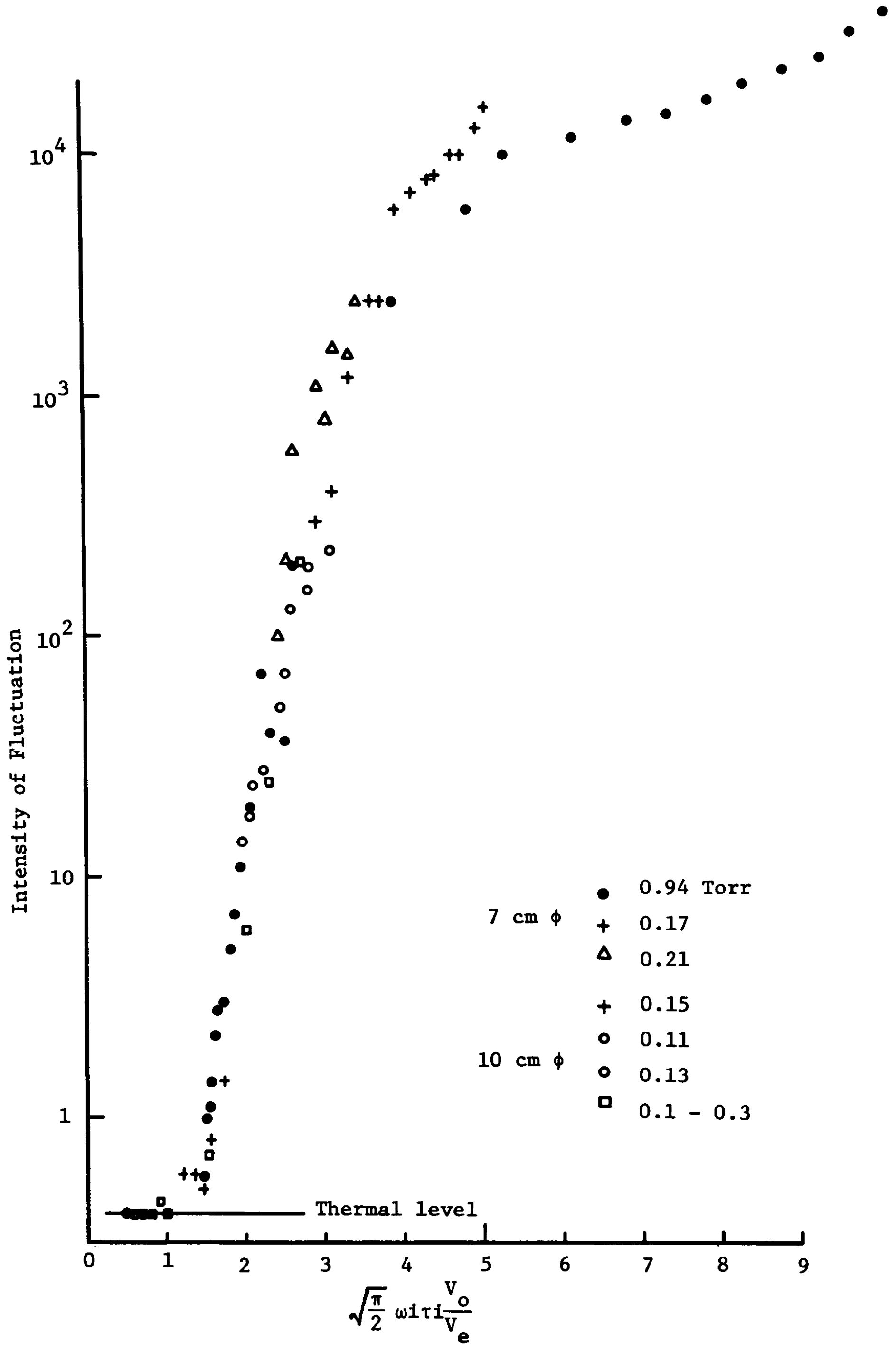

Fig.5 The enhancement of the density fluctuation at the frequency where the spectrum has a peak.