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Harmonic Generation by Landau-Damped Ion Acoustic Waves in a Plasma*

Noriyoshi Sato†

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* This work was carried out when the author was the Guest Assistant Professor of the Institute of Plasma Physics, Nagoya University, Nagoya, JAPAN.

† Permanent address: Department of Electronic Engineering, Tohoku University, Sendai, JAPAN.

Abstract

A experiment is made on harmonics generated by two Landau-damped ion acoustic waves in a plasma. The results are in qualitative agreement with simple physical predictions.

Landau damping[1] of ion acoustic waves have been studied experimentally by a number of authors.[2,3] This paper reports preliminary measurements of harmonics generated by Landau-damped ion acoustic waves in a plasma.

The experiment is performed on the TP-C Machine (single ended Q-Machine[4]) of Nagoya University (see Fig.1).[3] The plasma, about 2 cm in diameter and 130 cm long, is produced by surface ionization in a magnetic field of 2.5 kG. The density $\sim 10^9$ cm^{-3} is uniform within a variation of 5 % along the column, where the experiment is performed. Waves are excited by applying simultaneously two sinusoidal signals of frequencies f_1 and f_2 to the exciting grid. Their voltages are kept to be smaller than 5 V peak to peak in order to avoid nonlinear damping[3] of the excited waves. The signals picked up by the receiving grid are fed to a synchronous detector (lock-in amplifier), together with the reference signals $|mf_1 \pm nf_2|$ (m, n: integers) generated by mixing f_1 and f_2. The wavelength λ and the damping rate are obtained from the detector output against the grid separation.

Two excited (primary) ion acoustic waves propagate along the column, decaying by Landau damping.[3] The phase velocities v_p and the normalized damping distances δ/λ do not depend on the wave frequencies. Harmonics of frequencies $|mf_1 \pm nf_2|$ are also observed to propagate, though their amplitude have a higher order of smallness compared with the primary waves. The phase velocities of those harmonic ion acoustic waves are equal to those of the primary waves. A typical example of them is demonstrated for the harmonics $f_2 - f_1$ in Fig.1. It is to be noted that the damping is not exponential. Similar patterns are obtained for other harmonics. If those harmonics would be generated only due to the nonlinearity of the ion sheath surrounding the grid, they would decay by the Landau damping, in the same manner as the primary waves. The most important point, however, is that their damping differs from that of the primary waves. Here are reported mainly the results on the harmonics $f_2 - f_1$, the damping of which is more interesting. Some of them were already reported.[5]

The amplitudes are plotted against the grid separation for the harmonics $f_2 - f_1$ of 85 kc/sec, with f_1 as a parameter, in Fig.2(a). Slow damping is found over a few wavelengths. Beyond there, the waves are damped more strongly. An exponential line is assumed for each curve in order to estimate the damping rate in this region. Figure 2(b) shows the damping distances δ given by those lines, normalized with the wavelengths λ, as a function of f_1. The value δ/λ is increased by decreasing f_1 and f_2 even if $f_2 - f_1$ is fixed. With $f_2 - f_1$ as a parameter, the amplitudes are plotted against the grid separation for f_2 = 195 kc/sec in Fig.3(a). The flat region of the curves over a short distance becomes distinct with the increase of $f_2 - f_1$, and there appear a growth and subsequent peak in amplitude, typically shown for $f_2 - f_1$ = 125 kc/sec. Further away from the exciter, δ/λ becomes large as $f_2 - f_1$ is increased by decreasing f_1, as shown in Fig.3(b), where are plotted the normalized values of δ/λ and v_p with those of the primary waves. For the harmonics $f_1 + f_2$ and $2f_{1,2}$, δ/λ is obtained to be larger by about 20 % than that of the primary waves, independent of the frequencies.

The same experiment is carried out by using two exciting grids G_1 and G_2 spaced ℓ apart (G_2 is located between G_1 and the receiver). In order to eliminate the harmonic generation due to the sheath nonlinearity, ℓ is kept to be about 1 mm, much longer than the sheath length. The waves of f_1 and f_2 are excited by G_1 and G_2, respectively. The observed harmonics have the same properties as in case of the one-grid excitation. This fact shows that the effect of the sheath is not important in the experiment. No difference is observed between the harmonics $|f_2 - f_1|$ for $f_2 > f_1$ and for $f_1 > f_2$. As ℓ is increased, however, they change into the echoes[6] for $f_2 > f_1$, but disappear gradually for $f_1 > f_2$.

From an elementary physical argument, the phenomena can be interpreted qualitatively. For simplicity, the harmonics $f_2 - f_1$ is treated below. It is reasonable to provide for nonlinear media that the perturbation, $\omega_3 = \omega_2 - \omega_1$ $(=2\pi(f_2 - f_1))$, is produced at every position by two travelling waves $a_1(x,t)=A_1e^{-\beta_1 x}\exp[j(\alpha_1 x-\omega_1 t)]$ and $a_2(x,t)=A_2e^{-\beta_2 x}\exp[-j(\alpha_2 x-\omega_2 t)]$. Suppose that the same kind of wave is generated by this perturbation at $t=t'$ and $x=x'$, and propagates in the direction of the primary waves. This wave should be expressed by $A_3e^{-\beta_3(x-x')}\exp[-j\{\alpha_3(x-x')-\omega_3(t-t')\}]$, where $A_3 \propto a_1(x',t') \times a_2(x',t')$. Since the harmonic waves are generated at all positions between the exciter and the receiver, the signal received $a_3(x,t)$ is the integral of them over x' from zero to x. Thus, we get

$$a_3(x,t)\propto \int_0^x dx' a_1(x',t')a_2(x',t')d^{-\beta_3(x-x')}\exp[-j\{\alpha_3(x-x')-\omega_3(t-t')\}]. \quad (1)$$

For the Landau-damped ion acoustic waves $(\alpha,\beta \propto \omega)$, $\alpha_3 = \alpha_2 - \alpha_1$ and $\beta_3 = \beta_2 - \beta_1$. Then, Eq.(1) is rewritten as

$$a_3(x,t)\propto A_1A_2e^{-(\beta_2-\beta_1)x}(1-e^{-2\beta_1 x})\exp[-j\{(\alpha_2-\alpha_1)x-(\omega_2-\omega_1)t\}]. \quad (2)$$

The wave $a_3(x,t)$ decays as $e^{-(\beta_2-\beta_1)x}(1-e^{-2\beta_1 x})$, which increases up to its maximum at $x_0 = (2\beta_1)^{-1}\ln(\beta_1+\beta_2)/(\beta_2-\beta_1)$, beyond where it decreases monotonically.

The curve of $e^{-(\beta_2-\beta_1)x}(1-e^{-2\beta_1 x})$ against x is similar to the experimental curves shown in Figs.2(a) and 3(a). Roughly speaking, the values x_0 (for example, $x_0 \sim 1.4$ cm for $f_1 = 70$ kc/sec and $f_2 = 195$ kc/sec) do not contradict with the experimental results. After the amplitude maximum, the factor $(1-e^{-2\beta_1 x})$ makes the damping smaller as f_1 $(\beta \propto f)$ is decreased even if $f_2 - f_1$ is fixed. The result is in reasonable agreement with the experimental data for δ/λ, though they are obtainted by assuming an exponential line for the damping curve as shown in Figs.2 and 3. For the harmonics $f_1 + f_2$, the damping $xe^{-(\beta_1+\beta_2)x}$ is obtained ($\beta_1=\beta_2$ for $2f_{1,2}$) by the same method. Thus, the damping of $f_1 + f_2$ and $2f_{1,2}$ is small compared with the primary waves, and their damping rates are independent of the frequencies. The result well explains the experimental data. For the Landau-damped ion acoustic waves, the amplitudes are larger for the lower frequencies at some fixed positions, and so the harmonic generation for $f_2 - f_1$ becomes strong with the decreases of f_1 and f_2, in contrast with the case of $f_1 + f_2$, because $f_2 - f_1$ can be kept to be constant for the decreases of both f_1 and f_2. Details of the phenomena could be clarified by a theoretical description basing on the Vlasov equation. Analysis along this line has been made by Katayama and Nishikawa[7] whose results are similar to the experimental results. We must also take account of the harmonic generation due to the sheath, which displaces the position x_0 and changes the damping rate, though its effect is weak. Even for the two-grid excitation, we may not neglect the transient sheath reported[8] recently to be much longer than the dc sheath.

The phenomena should exist also in a collision-dominated plasma. For ion acoustic waves damped by ion-atom collisions, the damping of both $f_2 - f_1$ and $f_1 + f_2$ is given by $e^{-\beta x}(1-e^{-\beta x})$, because the damping factor β does not depend on the frequency.[9]

I would like to thank Professor K. Takayama and Professor Y. Hatta for their encouragements. I am grateful to Dr. H. Ikezi and Dr. K. Nishikawa for their helpful comments. Finally it is a pleasure that some similar results have been obtained by my collegue, T. Ohnuma[10] of Tohoku University, using a weakly ionized plasma.

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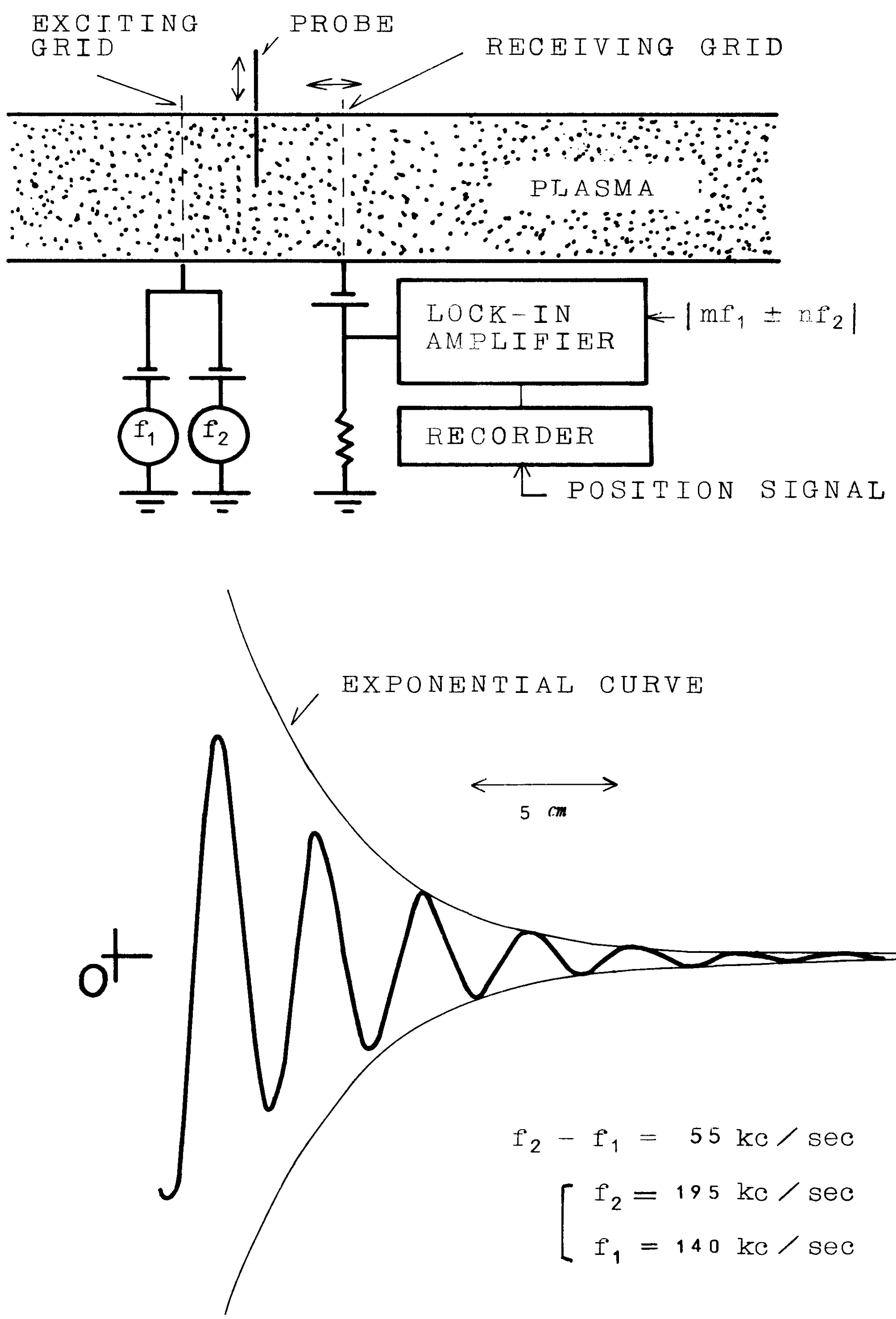
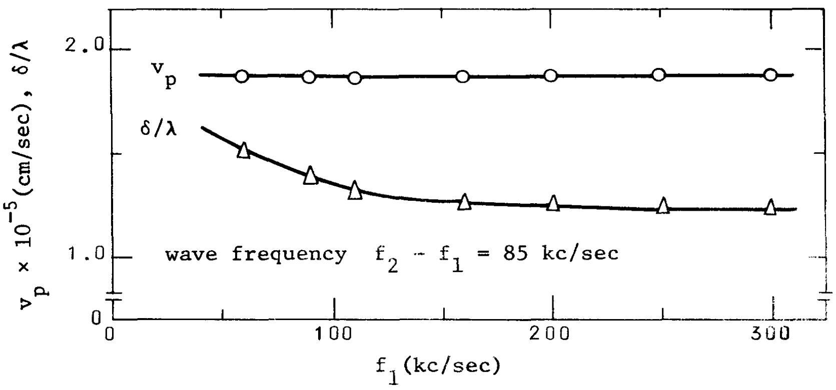


Fig.1. Upper: Schematic diagram of experimental setup.
Lower: A typical example of lock-in amplifier output against grid separation, demonstrating propagation and damping of harmonics $f_2 - f_1$ for f_1 = 140 kc/sec and f_2 = 195 kc/sec. Solid lines show exponential damping.



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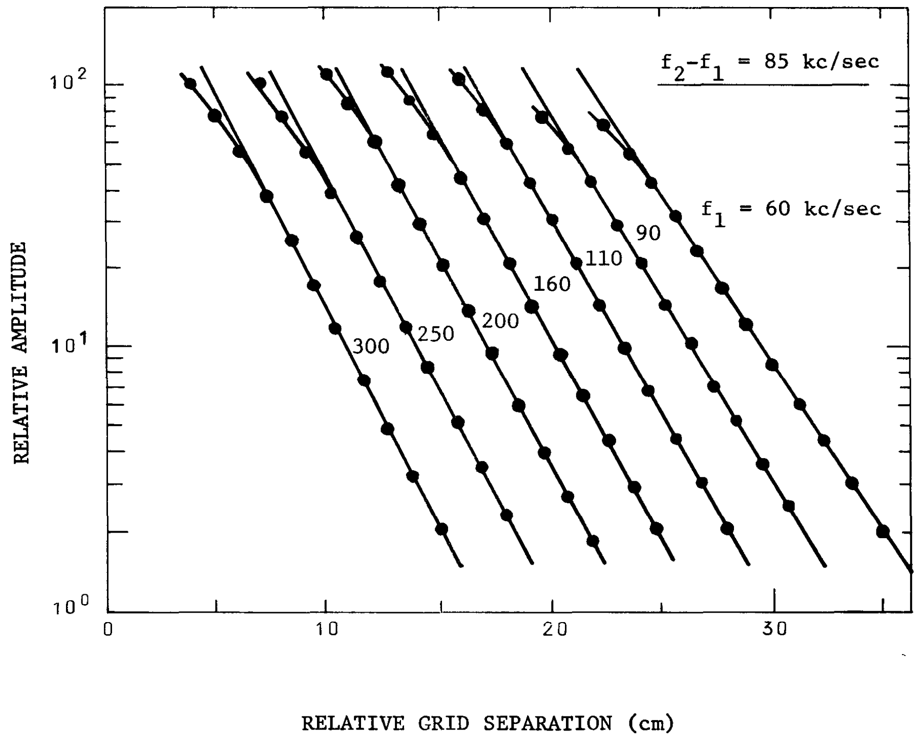
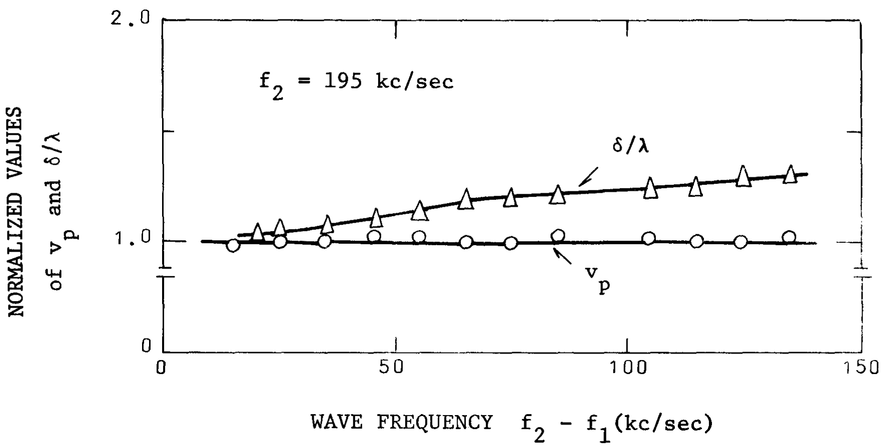


(a)

Fig.2. (a): Relative wave amplitude of harmonics $f_2 - f_1$ as a function of grid separation, with f_1 as a parameter, for f_2 = 85 kc/sec. For the curve of f_1 = 300 kc/sec, the exciting position is at zero point, but other curves are displaced 3 cm one after another on the abscissa. (b): Normalized damping distance δ/λ and phase velocity v_p of harmonics $f_2 - f_1$ as a function of f_1 for f_2 - 85 kc/sec. The damping distances is obtained by assuming an exponential line for the damping curve as shown in (a).



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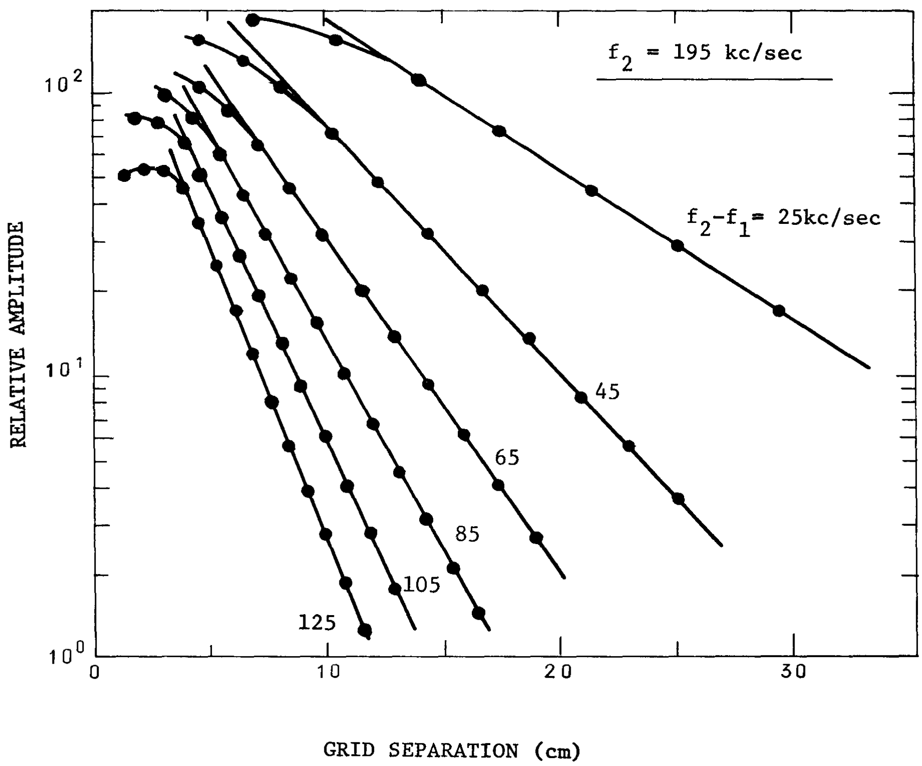

(a)

Fig.3. (a): Relative wave amplitude of harmonics $f_2 - f_1$ as a function of grid separation, with $f_2 - f_1$ as a parameter, for f_2 = 195 kc/sec. (b): Normalized values of δ/λ and v_p of harmonics $f_2 - f_1$ with those of primary waves as a function of $f_2 - f_1$ for f_2 = 195 kc/sec. The damping distance is also obtained by assuming an exponential line for the damping curve as shown in (b).