INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Stochastic Model of Electron Cyclotron Heating in a Magnetic Mirror

Takaichi KAWAMURA, Hiromu MOMOTA*, Chusei NAMBA**
and
Yoshinosuke TERASHIMA

IPPJ-100 AUGUST 1970

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, JAPAN.

* Permanent Address: Department of Electrical Engineering, Kyoto University, Kyoto.

** Permanent Address: Department of Electronics, Nagoya University, Nagoya.

Abstract

The mechanism of heating of electron plasmas by high-power microwaves in a magnetic mirror is investigated theoretically, in reference to the experiments in a device named TP-M. A microwave mode propagating perpendicular to the magnetic field is assumed to be responsible for heating, as there exist higher harmonic resonances, the main features of the experiments. The heating mechanism can be interpreted in terms of a stochastic process, a random walk in velocity space.

It is numerically confirmed that the localized resonance zones of finite width exist and electrons are effectively heated only in the resonance zones, and that the presence of a randomization process of the relative phase relation between the wave and the electron gyration is essential to heating. On the basis of the numerical analysis the heating process is analytically treated. Electrons are supposed to pass the resonance zones repeatedly in the course of oscillatory motions between turning points, and the relative phase relation is assumed to be random at each passage through the resonance zone. The heating rate is calculated and is found to be in agreement with the experimental value. The causes of phase randomization and the observed staturation of electron temperature are also discussed.

§ 1. Introduction

From experiments on hot-electron plasmas produced in magnetic mirrors by high-power microwaves we obtain much informations about heating mechanism of electrons. An interesting experiment was carried out in the device named TP-M [1∿3]. The characteristic features of this experiment are as follows. The discharge is produced by a microwave pulse at 6.4 GHz with 20 msec duration and 5 kW power. The microwave power is introduced radially through the wave guide ports in the cavity wall surrounding a magnetic mirror. The plasma consists of hot electrons, cold electrons and ions. The density is about $10^9 \sim 10^{10}$ hot electrons/cm^3 and $10^{10} \sim 10^{12}$ cold electrons/cm^3. The average energy is about 150 keV for the hot electrons and about 10 eV for the cold electrons. The device seems to have the fundamental resonance zone, where the microwave frequency is equal to the local electron cyclotron frequency, as well as the second harmonic one (see Fig.(1)). The hot electrons have a shell-structure and are situated along the magnetic lines of force passing through the second harmonic resonance zone. This fact indicates the possibility that the harmonic resonance makes an important contribution to heating [3]. The purpose of this paper is to reveal the heating mechanism of the hot electrons in the magnetic mirror on the basis of a stochastic model.

Previous theoretical works [4 ∿ 9] treated the heating process by microwaves which propagate along the mirror field. Here we are concerned with the heating by microwaves propagating perpendicular to the magnetic lines of force, since higher harmonic resonances appear only when a microwave has the perpendicular component of the wave vector.

In this paper it is demonstrated that the electrons are accelerated or decelerated only in the resonance zones of certain width, and that the heating mechanism is interpreted in terms of a stochastic process.

Namely, the change in an electron velocity component perpendicular to the magnetic lines of force is assumed to be stochastic, because it depends on the difference between the phases of the electric field and the electron gyration, and because the phase should be modulated randomly by fluctuations in the cold-electron density.

We first work out a numerical study in order to examine the validity of the model, of which the partial results are given in the reference [10]. The computation is carried out on a simplified magnetic mirror. The essential features of heating mechanism come from the fact that the heating electric field has a component perpendicular to the magnetic field besides perpendicular propagation, so that for simplicity we assume a longitudinal wave for it. The results of numerical study are described in §2. Secondly, on the basis of this numerical study we treat the problems analytically to some extent in §3. The heating rate is calculated and is found to be consistent with the experimental result.

In the subsequent section the discussions are given for the causes of phase randomization. The saturation of the electron mean energy is also discussed.

§ 2. Numerical Computations

We here numerically track the electron orbits in the following electric and magnetic fields:

$$\vec{E} = \{ E_0 \sin(kx - \omega t + \psi),\ 0,\ 0 \}, \tag{1}$$

$$\vec{B} = \{ -\frac{x}{2}\frac{\partial B_z}{\partial z},\ -\frac{y}{2}\frac{\partial B_z}{\partial z},\ B_M - (B_M - B_m)[(\frac{z}{L})^2 - 1]^2 \}, \tag{2}$$

where the z - coordinate is taken along the magnetic line of force, k and ω are the wave number and the frequency of the electric field, respectively. And B_M and B_m are the field strengths at the end of the magnetic mirror, $z = \pm L$, and at the midplane, $z=0$, respectively. These quantities, except the amplitude of the electric field, E_0, are assigned to the appropriate values refered to the TP-M experiment. The value of E_0 is taken to be somewhat large to make computed results distinct.

§ 2.1. Single Particle Motion

In order to assure the presence of the resonance zones we track the orbits of those electrons, which start at the midplane with the identical initial condition except the value of ψ. The assignment of the various values of ψ corresponds to that of the various initial phase differences between the phases of the electric field and the electron gyration. In Fig.2 we show the computed results of the total energy and the magnetic moment of the electrons. One sees that the changes in total energy depend on the values of ψ, or on the initial phase differences, and also sees that the changes are remarkable only in the vicinity of the fundamental and the second harmonic resonance points. The result confirms the existence of the localized resonance zones of finite width, only in which electrons effectively interchange their energies with the electric field.

In Fig.2 we see that the magnetic moment changes similarly as the total energy changes. Analyzing the computed results, we also find that for the change in total energy $\Delta W_{\parallel} \doteq \Delta W_{\perp} + \Delta W_{\parallel} \simeq \Delta W_{\perp}$ ($\Delta W_{\perp} = \Delta(mV_{\perp}^2/2)$, $\Delta W_{\parallel} = \Delta(mV_{\parallel}^2/2)$) and the parallel velocity $V_{\parallel}$ does not change appreciably in the resonance zone.

We have carried out the computations for $k = 2 \sim 200$ cm^{-1}. Fig.2 -

(a), (b) and (c) are some examples corresponding to the various values of wave number, k. The widths of the resonance zones are estimated from the numerical results and found to be almost independent of k, while the absolute values of ΔW depend on it. Therefore one needs not solve the dispersion equation in order to determine the resonance width of the mode of interest.

§ 2.2. Statistical Treatment

Here we present numerical calculations of heating process of an electron plasma based on the statistical method. We have numerically tracked the orbits of test electrons with the same procedure as in the above. Here trackings are continued over a number of turnings between the mirrors. The phase angle, ψ, is treated as a random variable and is assigned to a random number at each turning point. So that the test electron randomly walks in velocity space and one step of random walk is the velocity change after a travelling between two subsequent turning points.

The numerically computed values of $< V_{\perp}^2 >$ are shown in Fig.3, where the abscissa is the number of turnings, N. The average is taken over a group of test electrons. The values of $< V_{\perp}^2 >$ fluctuate for small N, however, they less fluctuate as N increases and are in agreement with the analytical prediction of the random walk theory as will be shown in §3. The distribution of $V_{\perp}$ should become broad as N increases. The distribution of $V_{\perp}$ obtained for $N = 20$ is shown in Fig.4, which is almost Maxwellian. The solid curve indicates $f(V_{\perp}) = C \cdot P(V_{\perp}, N{=}20)$, where $P(V_{\perp}, N)$, is the Gaussian distribution achieved as a result of random walk, given by Eq.(23) below, and C is a normalization factor. This indicates that the velocity distribution becomes Maxwellian after a number of turnings as a result of phase randomization, and that the heating results.

§ 3. Analytical Treatments

We treat the heating process analytically on the basis of the results of the numerical study. First, the equation of motion of an electron in the inhomogenous magnetic field is solved under some approximations, and the expressions for the changes in velocity of the electron in the resonance zones are obtained. Secondly, the heating process is treated as a stochastic process, and the expression of heating rate is obtained.

§ 3.1. Solution of Single-Particle Motion

In the vicinity of the resonance point we are interested only in the change of the electron velocity perpendicular to the magnetic lines of force, because the change of the parallel component of the electron velocity is little and so it is assumed constant. The motion of an electron on the plane perpendicular to the magnetic field is governed by

$$\dot{v}_{\perp} - i\Omega v_{\perp} - i\,\frac{1}{2}\,\Omega' v_{\parallel}\xi = -\frac{e}{m}E_{\perp}, \tag{3}$$

where the dot means the differentiation by time, the prime does the one by the coordinate z along the line of force, and the complex velocity $v_{\perp} = v_x + iv_y$ and the complex position $\xi = x + iy$ are introduced. $\Omega = eB_z(z)/mc$ is the local electron cyclotron frequency and $E_{\perp}$ is the assumed electric field.

Let the electron pass the resonance point z_r at $t = 0$, then $\Omega(z)$ in Eq.(3) is approximated by $\Omega \simeq \Omega_r + \Omega_r' v_{\parallel} t$, substituting $z = z_r + v_{\parallel} t$.

The subscript r means the value at $z = z_r$. The electron is assumed to stay in the resonance zone longer than $1/\Omega_r$.

Then, under the condition that $2\Omega_r'/\Omega_r''v_{\parallel} >> |t| >> 1/\Omega_r$, Eq.(3) is reduced to

$$\dot{v}_\perp - i\Omega_r v_\perp - i\Omega_r' v_\parallel t v_\perp = -\frac{e}{m} E_\perp \quad . \tag{4}$$

The solution of this equation becomes

$$v_\perp(t) = e^{i\theta(t)}\{v_\perp(t_0)e^{-i\theta(t_0)} - \frac{e}{m}\int_{t_0}^{t} dt' E_\perp(x(t'),t')e^{-i\theta(t')}\}, \tag{5}$$

with

$$\theta(t) = \Omega_r t + \frac{1}{2}\Omega_r' v_\parallel t^2 \quad . \tag{6}$$

Introducing a new variable $\tilde{v}_\perp(t) \equiv v_\perp(t)e^{-i\theta(t)}$, we have

$$\tilde{v}_\perp(t) = \tilde{v}_\perp(t_0) + A(t, t_0), \tag{7}$$

where

$$A(t, t_0) = -\frac{e}{m}\int_{t_0}^{t} dt' E_\perp(x(t'),t')e^{-i\theta(t')}. \tag{8}$$

When we put $A = |A|e^{i\theta_A}$ and $\tilde{v}_\perp(t_0) = V_\perp(t_0)e^{i\theta_0}$, the change in energy for one passage through the resonance zone becomes

$$\Delta W \simeq mV_\perp(t_0)|A|\cos(\Theta_0 - \theta_A) \quad , \tag{9}$$

where $V_\perp(t_0) \gg |A|$ is assumed. This formula expresses the behaviour of the change in electron energy near the resonance, as is shown in Fig.2.

We substitute the assumed electric field given in Eq.(1) into Eq.(8). Within accuracy of the present approximation, x is replaced by the value of the unperturbed orbit:

$$x \doteqdot \rho_0 \sin(\Omega_r t + \phi_0), \qquad \rho_0 = V_\perp(0)/\Omega_r . \tag{10}$$

Then we have

$$A(t, t_0) = -\frac{e}{m} E_0 \sum_{\nu=-\infty}^{\infty} f_\nu(t, t_0) J_\nu(k\rho_0) , \tag{11}$$

where

$$f_\nu(t, t_0) = \frac{1}{2i} \{ e^{i\Phi_\nu} \int_{t_0}^{t} dt' \exp i\, (\Lambda_{\nu-1} t' - \frac{1}{2} \Omega_r' v_{\shortparallel} t'^2)$$
$$- e^{-i\Phi_\nu} \int_{t_0}^{t} dt' \exp i\, (-\Lambda_{\nu+1} t' - \frac{1}{2} \Omega_r' v_{\shortparallel} t'^2) \}, \tag{12}$$

$$\Lambda_\nu = \nu\Omega_r - \omega , \qquad \Phi_\nu = \nu\phi_0 + \psi \tag{13}$$

and $J_\nu(z)$ is the ν-th order Bessel function of the first kind.

In Eq.(11) the terms with $\Lambda_{\nu\pm1} = 0$ dominate over other terms. The leading terms include the integral estimated as

$$\int_{t_0}^{t} dt' \exp(-i \frac{1}{2} \Omega_r' v_{\shortparallel} t'^2) \doteqdot \int_{-\infty}^{+\infty} dt' \exp(-i \frac{1}{2} \Omega_r' v_{\shortparallel} t'^2) = t_r e^{-i\frac{\pi}{4}} ,$$

where

$$t_r = \left(\frac{2\pi}{\Omega_r' v_{\parallel}} \right)^{1/2} \tag{14}$$

is considered to be the effective resonance time. Consequently the effective width of the resonance zone along the magnetic lines of force is given by

$$\Delta z_r = v_{\parallel} t_r = \left(\frac{2\pi v_{\parallel}}{\Omega_r'} \right)^{1/2} . \tag{15}$$

Essentially the same result is given by Kuckes [6]. With use of these relations A in Eq.(11) for the fundamental resonance $(\omega = \pm\Omega_r)$ is approximated to be

$$A \cong \pm \frac{eE_0}{2mi} t_r J_0(k\rho_0) e^{\mp i\Phi_0 - i\frac{\pi}{4}} \tag{16}$$

and for the second harmonic resonance $(\omega = \pm 2\Omega_r)$

$$A \cong \pm \frac{eE_0}{2mi} t_r J_1(k\rho_0) e^{\mp i\Phi_{\pm 1} - i\frac{\pi}{4}} . \tag{17}$$

Finally the general form of A in the n-th harmonic resonance is approximately given by

$$A \cong \frac{eE_0}{2m} t_r J_{n-1}(k\rho_0) \cdot e^{i\chi_n} \tag{18}$$

and χ_n depends on the phases of the field oscillation and the electron gyration.

Strictly speaking, χ_n becomes a function of time in an inhomogeneous magnetic field and the exact expression of χ_n is very complicated. However, if any stochastic phenomenon like collisions or fluctuations takes part in the motion of the electron, χ_n becomes a random variable and the acceleration process becomes stochastic. Namely the process of heating electrons essentially becomes a random walk in two-dimensional velocity space owing to Eq.(7) and Eq.(18) with the random phase χ_n .

§ 3.2. Stochastic Process

The change of the electron velocity perpendicular to the magnetic field, $\vec{v}_\perp$*, is governed by the probability law if any random process takes part in its motion. Suppose that the phase randomization takes place at intervals of time and $\vec{u}_i$ is the change of $\vec{v}_\perp$ between two successive occurences of randomization, we have $(\vec{v}_\perp)_N = \vec{v}_{\perp 0} + \vec{u}_1 + \vec{u}_2 + \cdots + \vec{u}_N$ where $\vec{v}_{\perp 0}$ is the initial value of $\vec{v}_\perp$. If the transition from the $(N-1)$-th process to the N-th process is a Markoffian, the probability $P(\vec{v}_\perp, N)$ with which the electron has $\vec{v}_\perp$ in the N-th process, can be defined and obeys

$$P(\vec{v}_\perp, N+1) = \int P(\vec{v}_\perp - \vec{u}, N) g(\vec{v}_\perp - \vec{u}, \vec{u}) d\vec{u} , \tag{19}$$

where $g(\vec{v}_\perp, \vec{u})d\vec{u}$ is the transition probability of the change of $\vec{v}_\perp$. From this we obtain the Fokker-Planck equation, which is for $|\vec{v}_\perp| >> |\vec{u}|$ and $N >> 1$,

$$\frac{\partial}{\partial N} P(\vec{v}_\perp, N) = \frac{1}{4} \frac{\partial}{\partial \vec{v}_\perp^2} \{ << \vec{u}^2 >> P(\vec{v}_\perp, N) \}, \tag{20}$$

---

* The velocity is treated not as a complex variable but as a vector in this subsection.

where

$$<< \vec{u}^2 >> = \int \vec{u}^2 g(\vec{v}_\perp, \vec{u}) d\vec{u} \ , \qquad \int g(\vec{v}_\perp, \vec{u}) d\vec{u} = 1 \ . \tag{21}$$

Here we have assumed $<< u_x^2 >> = << u_y^2 >> = \frac{1}{2} << \vec{u}^2 >>$ and $<< \vec{u} >> = 0$ for simplicity. Examples of the g-function are given in the Appendix.

When $\vec{u}$ is sufficiently small as compared with $\vec{v}_\perp$, the dependence of $\vec{v}_\perp$ in $<< \vec{u}^2 >>$ will be weak and $<< \vec{u}^2 >>$ is taken to be constant in Eq.(20), say $<< \vec{u}^2 >> = 2a^2$. If the probability distribution at $N = 0$ is the delta function, $P(\vec{v}_\perp, 0) = \delta(\vec{v}_\perp - \vec{v}_{\perp 0})$, the solution of Eq.(20) in this case is

$$P(\vec{v}_\perp, N) = \frac{1}{2\pi a^2 N} \exp \{ - (\vec{v}_\perp - \vec{v}_{\perp 0})^2 / 2a^2 N \}. \tag{22}$$

The probability that an electron has a speed $V_\perp = |\vec{v}_\perp|$ is calculated as

$$P(V_\perp, N) \cong (2\pi a^2 N)^{-1/2} (V_\perp / V_{\perp 0})^{1/2} \exp\{-(V_\perp - V_{\perp 0})^2 / 2a^2 N\}, \tag{23}$$

where $V_{\perp 0}$ is the initial value of $V_\perp$. From this we know that the Gaussian distribution is achieved as a result of randomization. When we consider a group of electrons which have same speeds but different gyration phases at the initial moment, we conclude that their distribution function becomes a Maxwellian after a number of passages of resonances, and the heating results. The heating rate is given by

$$\frac{dT_\perp}{dt} = \frac{m}{2} \frac{d}{dt} < \vec{v}_\perp^2 > = ma^2 \frac{dN}{dt} \ , \tag{24}$$

where

$$< \vec{v}_\perp^2 > = \int \vec{v}_\perp^2 P\ (\vec{v}_\perp,\ N) d\vec{v}_\perp = \vec{v}_{\perp 0}^2 + 2a^2 N, \tag{25}$$

and dN/dt is the frequency of occurence of randomization.

If a phase randomization process certainly occurs between two successive passages of the resonance zones, the phase χ_n in Eq.(18) becomes a random variable, and $<< \vec{u}^2 >>$ at the n-th harmonic resonance is obtained by averaging the square of A in Eq.(18) over χ_n as

$$<< \vec{u}^2 >> = \left(\frac{eE_0}{2m} \right)^2 t_r^2 J_{n-1}{}^2 \left(\frac{k}{\Omega_n} V_\perp \right) , \tag{26}$$

where $V_\perp = |\vec{v}_\perp|$ and $\Omega_n = \omega/n$. The g-function defined in Eq.(19) is now $g(\vec{v}_\perp,\ \vec{u}) = (2\pi |A(V_\perp)|)^{-1} \delta(|\vec{u}| - |A(V_\perp)|)$.

In order to obtain the heating rate including the effects of the higher harmonic resonances, one must solve Eq.(20) with Eq.(26). However we treat briefly this problem as follows. We can reasonably expect that the probability $P(\vec{v}_\perp,\ N)$ tends to a Gaussian for $N >> 1$ and $|\vec{v}_\perp| >> |\vec{v}_{\perp 0}|$. That is,

$$P(\vec{v}_\perp,\ N) = \frac{m}{2\pi T_\perp} \exp\left(- \frac{m}{2T_\perp} \vec{v}_\perp^2 \right) , \tag{27}$$

where

$$T_\perp = \frac{m}{2} \int \vec{v}_\perp^2 P(\vec{v}_\perp,\ N) d\vec{v}_\perp \ . \tag{28}$$

Then the heating rate is obtained as

$$\frac{dT_\perp}{dt} = \frac{dN}{dt}\frac{m}{2}\int << \vec{u}^2 >> P(\vec{v}_\perp, N)d\vec{v}_\perp . \tag{29}$$

Substituting Eqs.(26) and (27) into Eq.(29) we have

$$\frac{dT_\perp}{dt} = \frac{dN}{dt}\frac{m}{2}\left(\frac{eE_0}{2m}\right)^2 t_r^2 \, I_{n-1}(\lambda)e^{-\lambda} , \tag{30}$$

where

$$\lambda = \frac{T_\perp}{m}\frac{k^2}{\Omega_n^2} \tag{31}$$

and $I_n(\lambda)$ is the modified Bessel function of the n-th order. Usually $\lambda >> 1$, then we finally obtain the heating rate as

$$\frac{dT_\perp}{dt} \cong \alpha \frac{1}{\sqrt{T_\perp}} \tag{32}$$

with

$$\alpha = \frac{dN}{dt}\left(\frac{m}{2}\right)^{3/2}\left(\frac{eE_0}{2m}\right)^2 t_r^2 \frac{\Omega_n}{k}$$

$$\cong 3.7 \times 10^7 \cdot E_0^2 k^{-1} \cdot \Omega_n/\tilde{L}\Omega_n' , \tag{33}$$

where t_r is already given in Eq.(14) and $T_\perp$ is in units of keV, E_0 in *esu* and k in cm^{-1}. We have assumed $dN/dt \cong v_\parallel/\tilde{L}$ where $\tilde{L}$ is the distance which an electron travels between two successive passages of the resonance zones. $\Omega_n/\tilde{L}\Omega_n'$ is a number with the same order of a mirror ratio. For the TP-M experiment we may assign these value as: $\Omega_n/\tilde{L}\Omega_n' = 3$,

$E_0 = 0.1$ *esu* (30 V/cm) and $k = 10\ \mathrm{cm}^{-1}$, then we have $\alpha = 1.1 \times 10^5 (\mathrm{keV})^{3/2}$ sec^{-1}, and $dT_\perp/dt = 11$ MeV/sec at $T_\perp = 100$ keV. This is in agreement with the experimental value. The time dependence of the heating rate is obtained as $dT_\perp/dt = 2.0 \times 10^3 t^{-1/3}$ keV/sec.

§ 4. Discussions

In the foregoing sections it is shown that the heating of electrons by high-power microwaves in the magnetic mirror can be interpreted in terms of the stochastic process. However it remains to examine the posible causes of the phase randomization which is essential to the stochastic process. Another problem is to explain the observed saturation of the heating process. We will discuss these problems in the following.

In view of the experimental conditions, we have two possible mechanism for phase randomization. One is the collisions of hot electrons, and the other is the fluctuations in density of the medium. The collisions of hot electrons are possibly with contaminated neutral atoms and with cold electrons. However the mean free paths for both processes are found to be very long and collisions are not effective for phase randomization.

Another possible cause of the phase randomization is due to the fluctuations in density of the cold plasma which supports the wave. The variation of the wave number, k, and that of the refractive index, N, are related with the variation of the electron number density, n, in the cold plasma as $\Delta k/k = \Delta N/N = \kappa \cdot \Delta n/n$, where κ is determined by the dispersion relation of the wave, and in practice $|\kappa|$ is of the order of unity. The density fluctuation is supposed to be somewhat larger than the thermal level, say, a few percent. Consequently we may take $\kappa^2 < (\Delta n/n)^2 > \sim 10^{-3}$, where < > means the ensemble average.

After the electron travels by the length L along the magnetic lines of force, the mean square value of change of the wave phase will be estimated as

$$< (\delta\phi)^2 > \simeq < \left(\frac{\Delta k}{k} \right)^2 > L\omega/v_{\parallel} ,$$

where $v_{\parallel}$ is the mean parallel velocity of hot electrons. For $<(\delta\phi)^2>^{1/2} \sim 1$ radian, the phase seen from electron will be randomized. Therefore we can define the effective length of phase randomization as

$$L_e \simeq \ell \{ \kappa^2 < \left(\frac{\Delta n}{n} \right)^2 > \}^{-1} ,$$

where $\ell = v_{\parallel}\omega^{-1}$ and is of the order of the pitch length of the electron gyration.

The effect of the fluctuating electric field parallel to the magnetic field is also expected. Due to the scatterings of the wave by the density fluctuations, we expect its mean value as $\hat{E}_{\parallel} \simeq E_{\perp} < (\Delta N/N)^2 >^{1/2}$, where $E_{\perp}$ is the amplitude of the wave. For $E_{\perp} \sim 30$ V/cm(0.1 *esu*) we have $\hat{E}_{\parallel} \sim 1$ V/cm. If the correlation time of the fluctuating field is of the order of ω^{-1}, then, the average value of the change in gyration pitch over the length L along field lines is estimated to be

$$\delta z \simeq \int_0^L \frac{dz}{v_{\parallel}} \frac{e}{m} \hat{E}_{\parallel}\omega^{-1} (z\omega/v_{\parallel})^{1/2}$$

$$\simeq \frac{e}{m} \hat{E}_{\parallel}\omega^{-1/2} (L/v_{\parallel})^{3/2}$$

For $\delta z \simeq \ell$, the phase difference between the hot electron and the wave will be randomized. The effective length of the phase randomization in this case is

$$L_e' \simeq \ell \{ mv_{\parallel}^2 / e\hat{E}_{\parallel}\ell \}^{2/3} .$$

For $(1/2)mv_{\parallel}^2 = 10$ keV and $\omega = 2\pi \times 6.4$ GHz, we have $\ell \approx 10^{-1}$cm and $L_e \simeq L_e' \simeq 10^2$cm. Comparing these values with the length between mirrors of the device ($\simeq$ 40 cm), we know that the relative phase relation between the electron and the wave will be effectively randomized in the course of the electron oscillatory motions between turning points.

The heating rate obtained above does not show saturation by itself, although it decreases as the temperature increases. The heating rate is reduced more or less by cooling. However, the obserbed temperature saturation cannot be explained only by radiation loss.

We may attribute the temperature saturation to the spatial shift of the electron cyclotron resonance zones in the magnetic mirror field, due to the relativistic effect. Although the formulae given so far hold only for non-relativistic regium, we discuss qualitatively the relativistic effect. The value of the magnetic field for the n-th resonance point is $B(z) = \gamma(m_0c/e)(\omega/n)$, where m_0 is the rest mass of the electron and $\gamma = (1 - v_{\perp}^2/c^2)^{-1/2}$. As γ becomes large, the resonance zones shift toward the place of stronger magnetic field, that is, toward the ends of the mirror. The value of γ corresponding to the maximum field strength inside the cavity (4×10^3G) is 1.75 for n=1. Therefore, the heating rate will be much reduced at 300 $\sim$ 400 keV. Also, the maximum value of the electron energies will be limited by the finiteness of the device because of large gyration radii, and by finite lifetimes in the mirror trap.

§ 5. Conclusions

We summarize our conclusions as follows.

i) By numerical computation, the presence of the localized heating zones, the resonance zones, is confirmed, in which the electrons are accelerated or decelerated resonantly. Outside the resonance zones the electrons behave almost adiabatically.

ii) The widths of the resonance zones are almost independent of the wave number of the heating field, while the acceleration efficiency of the electrons depends on it.

iii) The heating mechanism can be interpreted in terms of the stochastic process with the phase randomization which is possibly caused by fluctuations of the cold-electron density.

iv) Based on this stochastic model, the expression for the heating rate including the effect of the higher harmonic resonances is obtained. By reasonable assignment of the physical parameters, the value of the heating rate is found to be in agreement with the experimental value.

v) The observed saturation of the hot-electron temperature is attributed to relativistic effect rather than to cooling. As the energies of the electrons increase to relativistic regium, their resonance zones will shift to the ends of the magnetic mirror and finally disappear.

Acknowledgements

The authors wish to thank Dr. H. Ikegami for informing them the experimental results and for his invaluable discussions. The authors are also greatful to Professor K. Takayama for his helpful advices and to Professor K. Husimi for his continual encouragement.

Appendix

We present some examples of the transition probability distributions $g(\vec{u})$ (g-function). First, in the case of one step with a constant length A and a random direction angle χ, the g-function of $u_x = A \cos \chi$, becomes

$$\tilde{g}(u_x) = (\pi A)^{-1}\{1 - (u_x/A)^2\}^{-1/2}, \tag{A1}$$

The g-function of 2-dimentional vector $\vec{u}$ is, in the polar coordinates,

$$g(\vec{u})d\vec{u} = g(u)\ u\ du\ d\chi \tag{A2}$$

with

$$g(u) = (2\pi A)^{-1}\delta(u - A). \tag{A3}$$

The g-function of this form is of the most fundamental one.

Secondly, we consider the case of $u_x = A(\cos\chi_1 + \cos\chi_2)$. If χ_1 and χ_2 are independent random angles, the probability distribution of u_x is

$$\tilde{g}^{(2)}(u_x) = \int_{-A}^{A} \tilde{g}(u_x - w)\tilde{g}(w)dw$$

$$= \frac{2}{\pi^2}\ (1 + z)^{-1}K(k), \tag{A4}$$

$$k^2 = (1 - z)^2/(1 + z)^2 : \ z = |u_x|/2A,$$

where $K(k)$ is the complete elliptic integral of the first kind. However this is not the genuine g-function, because the randomization is already applied through the above convolution procedure. When the angles χ_1 and χ_2 are mutually involved in, it is very difficult to obtain the expression of the g-function.

References

[1] Ikegami, H., Ikezi, H., Hosokawa, M., Tanaka, S., and Takayama, K., Phys. Rev. Letters 14 (1967) 778

[2] Ikegami, H., Ikezi, H., Kawamura, T., Momota, H., Takayama, K., and Terashima, Y., Third Conference on Plasma Physics and Controlled Nuclear Fusion Research (Proc. Conf. Novosibirsk 1968), IAEA, Vienna Vol.2 (1969) 423

[3] Ikegami, H., Tanaka, S., Takayama, K., Ikezi, H., Kodama, T., Hosokawa, M., Ohnishi, H., and Shiina, S., 2ème Colloque International sur les Interactions entre les Champs Oscillants et les Plasmas. Saclay, Vol.1 (1968) 429

[4] Seidl, M., Plasma Physics 6 (1964) 597

[5] Piliya, A. D., and Frenkel', V. Ya. Soviet Physics - Technical Physics 9 (1965) 1356

[6] Kuckes, A. F., Plasma Physics 10 (1968) 367

[7] Grawe, H., Plasma Physics 11 (1969) 151

[8] Lichtenberg, A. J., Schwartz, M. J., and Tuma, D. T., Plasma Physics 11 (1969) 101

[9] Canobbio, E., Nuclear Fusion 9 (1969) 27

[10] Namba, C., and Kawamura, T., Physics Letters 31A (1970) 555

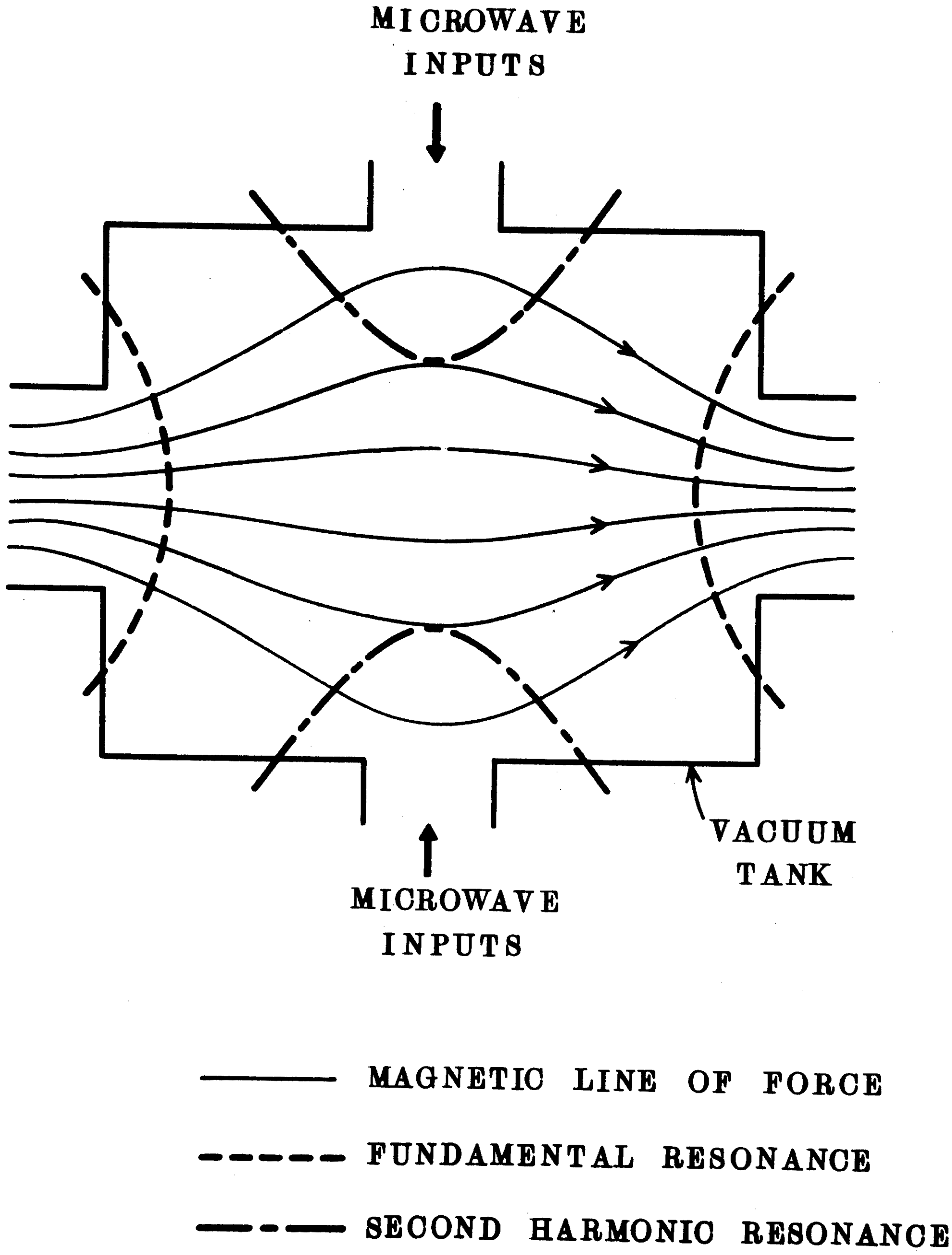

Fig.1. The magnetic mirror field of the TP-M device. The hyperbola-like lines vertically crossing the flux lines indicate the fundamental electron cyclotron resonance zones. The other hyperbola-like lines at the top and the bottom of the cavity denote the second harmonic electron cyclotron resonance zones.

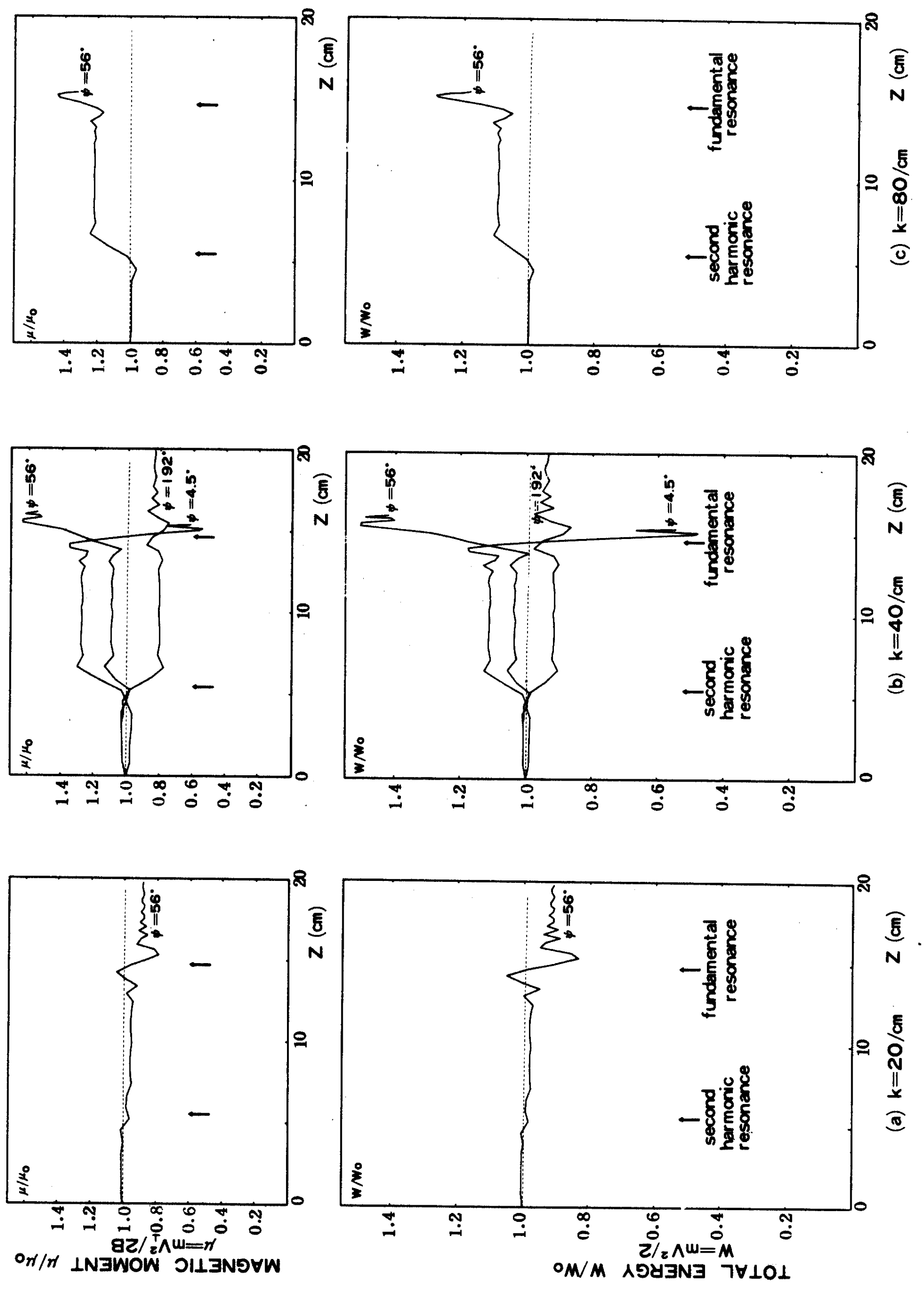

Fig.2. The computed results of the total energy and the magnetic moment of the test electrons. The solid lines indicate the numerical solutions of the equation of motion in the external electric and magnetic fields with the values of parameters: W_0 = 1.0 keV, $V_{\perp 0}/V_{\parallel 0}$ = 0.708, E_0 = 300V/cm and $\omega = 6.34 \cdot 10^{10}$/sec. The broken lines correspond to the cases without the external electric field.

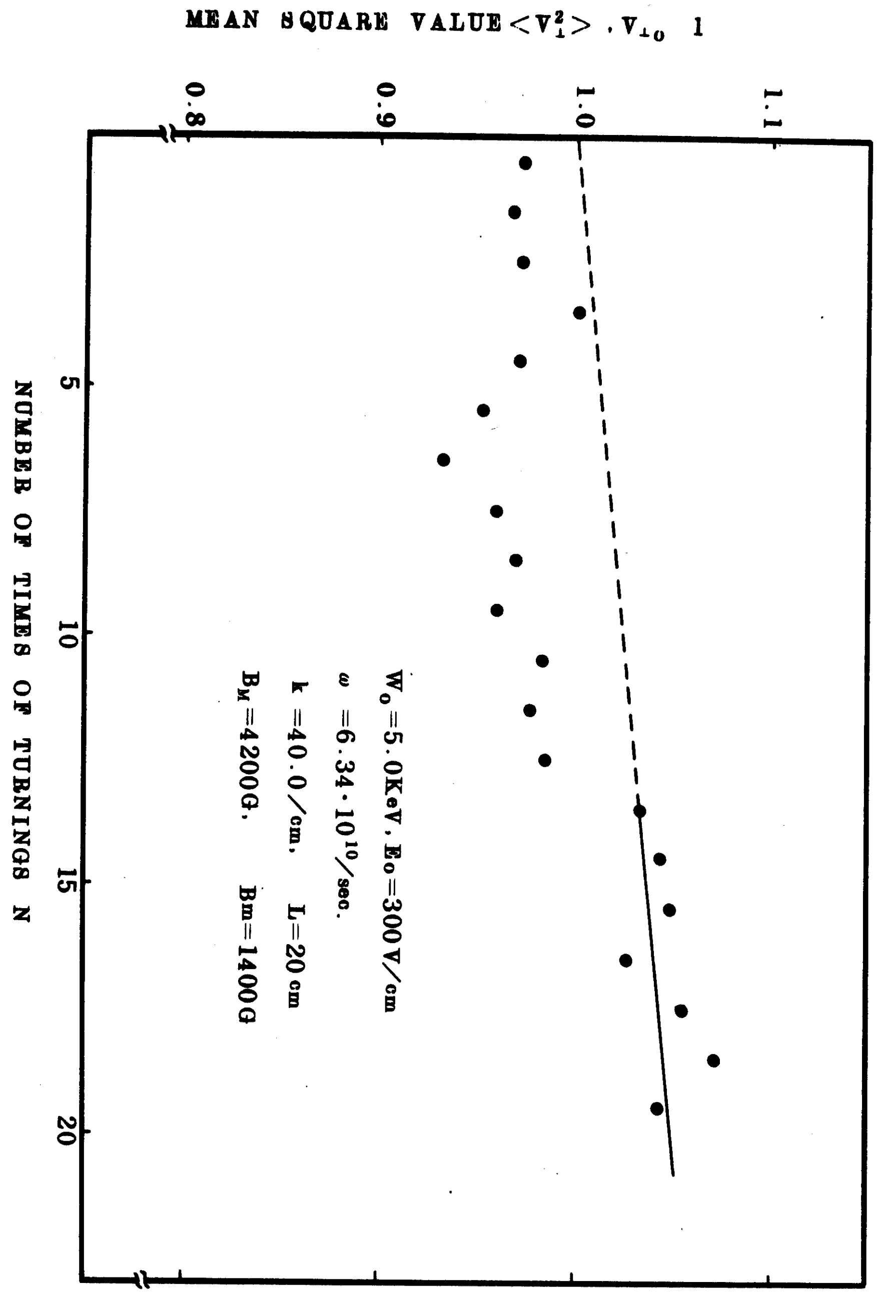

Fig.3. The computed results of $< V_{\perp}^2 >$. The solid line indicates $< V_{\perp}^2 >$, given by Eq.(25) with α^2 obtained from numerical computation.

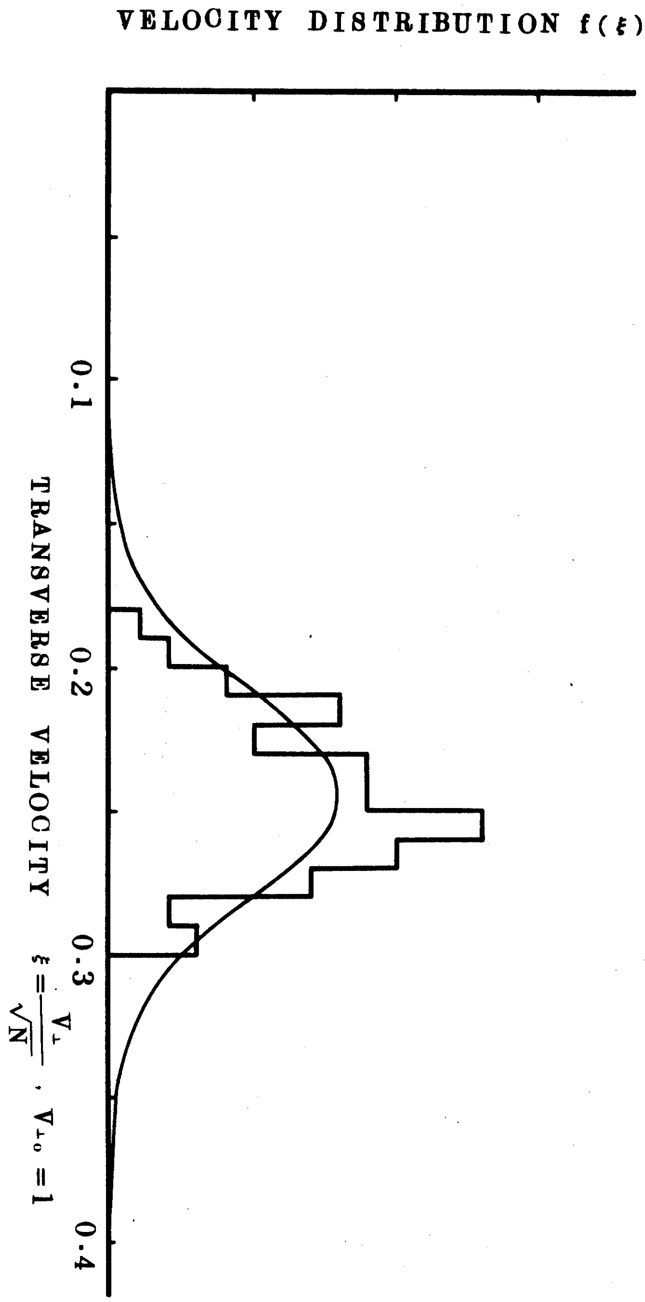

Fig.4. A distribution of $V_\perp$ for N = 20. The solid curve indicates $f(V_\perp) = C \cdot P(V_\perp, N)$, where $P(V_\perp, N)$ is the Gaussian distribution given by Eq.(23) and C is a normalization factor.