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Ion-Acoustic Wave Response
to an Impulse Disturbance

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Abstract

The ion acoustic wave response to an impulse disturbance is experimentally examined for two differenct exciting mechanisms. The differences and similarities are noted for the double-plasma (D-P) and plate excitation methods. The observed response agrees well with that predicted from a self-similar study of a fluid equation model which includes the effects of Poisson's equation.

I. Introduction

In this work, we describe experiments designed to investigate the linear ion acoustic wave response to an impulse disturbance. Since the duration of the impulse in these experiments was less than the ion plasma period, we can use the experimental results to obtain some information on the dispersive properties of ion acoustic waves. The experiments were performed with two different methods of excitation. In this fashion, we shall also be able to examine the differences (and/or similarities) between double plasma (D-P) and plate excitation of ion acoustic waves. The former has recently achieved success in exciting large amplitude ion acoustic wave shocks [e.g. TAYLOR, et al (1970)] while the latter has been used over a number of years to ascertain the basic properties of ion acoustic waves [e.g. JONES & ALEXEFF (1965), SHEN, et al (1970)]. Our experiments are described in Section II.*

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* We have also performed some experiments using a large single grid of the type used in many ion wave experiments. This led to the excitation of pseudowaves (free streaming ions without collective interaction). [ALEXEFF, et al (1968)].

To predict the expected response, we examine the fluid equations which describe the ion acoustic wave propagation using the "Method of Self-Similar Solution of Partial Differential Equations." [e.g. AMES (1965)(1972)] Self-similar studies have previously been made on ion acoustic waves using the assumption of charge neutrality, i. e. Poisson's equation has been neglected. In those cases it has been shown that the differential equations that define the wave propagation are similar to those of fluid dynamics which have a similarity variable $\xi = z/\tau$ where z and τ are the position and time normalized by the Debye length and the ion plasma frequency respectively, [LANDAU and LIFSHITZ (1959), GUREVICH et al (1965), KORN et al (1970), ALLEN and ANDREWS (1970), ALEXEFF et al (1971), ANDERSEN et al (1971)]. If we eliminate the assumption of charge neutrality and include Poisson's equation, we find that the self-similar variable is changed to $\zeta = \frac{z - \tau}{(z)^{1/3}}$. This derivation is valid at least within the domain where the ion acoustic waves can be described by a Kortweg-deVries equation [WASHIMI and TANIUTI (1966)]. This calculation is presented in Section III. Section IV is the conclusion.

II. Experiments

The experiments were performed in nearly the same type of quiescent Argon plasma which had typical numbers of $n_e \sim 10^9$ cm^3, $T_e \sim 2$e.v., and $T_i \lesssim 1/10$ e.v. The experimental setup for the double-plasma (D-P) excitation experiment is shown in Fig 1 and for the plate excitation experiment in Fig 2.

For the D-P excitation, an exciting voltage ϕ_{ex} of magnitude $\phi_{ex} < \frac{K_T}{e}e$ need only be applied between the two chambers to launch an easily detectable ion acoustic wave. Larger values led to nonlinear effects. An ion perturbation from the driver chamber is quickly transported to the target chamber. The flow of neutralizing electrons from the driver to the target is severely limited by the negatively biased screen separating the two chambers without any limitation on ion motion (See Fig 1). We therefore expect the excitation of ion waves to be very efficient. For the case of the plate excitation however, the exciting voltage has been found to require a value $\phi_{ex} >> \frac{K_T}{e}e$. A recent computer simulation and accompanying experimental verification revealed that most of the large exciting voltage ϕ_{ex} appeared across a thin sheath surrounding the plate which was formed on the time scale of electron motion [WIDNER, et al (1970)]. The resulting evolution of this sheath as the ions started to move led to the excitation of the ion acoustic wave.

In the experiments described here, a narrow voltage impulse was applied to the exciter (electron plasma period $\ll$ impulse duration $<$ ion plasma period; D-P pulse amplitude $\sim$ 0.5 v, plate pulse amplitude $\sim$ 60 v). The ion acoustic wave was detected by monitoring the electron saturation current of a Langmuir probe. In the case of the plate excitation, a coherent detector was also used. Typical results obtained at different probe positions are shown in Fig 3a for the D-P excitation and in Fig 4 for the plate excitation.*

Due to the short distance of probe travel in these experiments, it was not possible to clearly isolate whether the damping was exponential or algebraic. Collisions would suggest the former while dispersion and/or Landau damping would suggest the latter. We note, however, that the increase of the period of the trailing oscillation for the D-P excitation increases proportionally with $(\text{distance})^{1/3}$. See Fig 3b.

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* Similar results could be obtained in the D-P chamber if a plate were used to excite the ion waves instead of the normal D-P excitation.

III. Analysis of the Ion Acoustic Wave Response

Starting from the continuity equation and the equation of motion for cold ions, a Boltzmann relation for electrons and Poisson's equation, it is possible to derive a linearized wave equation for $u^{(1)}$, $n_e^{(1)}$, and $n_i^{(1)}$ which are the normalized velocity, electron density, and ion density perturbations respectively. The derivation and normalizations follow the work of Washimi and Taniuti [WASHIMI and TANIUTI (1966)] and yields a linearized Korteweg-deVries equation, say for $u^{(1)}$.

$$\frac{\partial u^{(1)}}{\partial \eta} + \frac{1}{2}\frac{\partial^3 u^{(1)}}{\partial \xi^3} = 0 \tag{1}$$

In eq(1), η and ξ are defined from a scale transformation

$$\xi \sim \varepsilon^{1/2}(z - \tau)$$

$$\eta \sim \varepsilon^{3/2} z$$

where ε is a bookkeeping parameter, z and τ are the normalized position and time respectively.

We solve eq(1) using the "Method of Self-Similar Solution of Partial Differential Equations" [e.g. AMES (1965) (1972)]. This technique allows one to repeatedly transform a partial differential equation with n variables to one of (n - 1) or (n - 2) or... 1 variable in a methodical manner.

Hopefully the final equation can be easily solved. The price that one must pay in using this technique is that the correct transformation may be hard to find or the boundary conditions may be difficult to transform properly. Eq(1) falls into the class of equations which can be treated using this technique. The nonlinear Korteweg-deVries equation is also in this treatable class.

Using the terminology of Ames, [AMES (1965), (1972)], we search for the invariants of a transformation using a finite dimensional group. We choose the group

$$\eta = a^{\alpha}\bar{\eta}$$

$$\xi = a^{\beta}\bar{\xi} \tag{2}$$

$$u^{(1)} = a^{\gamma}\bar{u}^{(1)}$$

where a, α, β, and γ are constants. After substituting the group defined by eq.(2) into eq.(1), we obtain

$$a^{\gamma - \alpha}\frac{\partial \bar{u}^{(1)}}{\partial \bar{\eta}} + a^{\gamma - 3\beta}\frac{1}{2}\frac{\partial^3 \bar{u}^{(1)}}{\partial \bar{\xi}^3} = 0 \tag{3}$$

The transformation is invariant if $\alpha = 3\beta$. The group invariants are defined as

$$\zeta \equiv \frac{\xi}{\eta^{\beta/\alpha}} \qquad \text{and} \qquad f \equiv \frac{u^{(1)}}{\eta^{\gamma/\alpha}} \tag{4}$$

The first invariant becomes $\zeta = \frac{\xi}{\eta^{1/3}} = \frac{z - \tau}{z^{1/3}}$ which demonstrates the modification to the self-similar variable that Poisson's equation makes in the plasma equations compared with the ordinary equations of fluid dynamics. A self-similar variable of this type was noted also in a calculation of waves in a magnetized cold plasma. [BEREZIN and KARPMAN (1964)].

After substituting eq. (4) into eq. (1), we obtain the ordinary differential equation

$$\frac{1}{2} \frac{d^3 f}{d\zeta^3} - \frac{\zeta}{3} \frac{df}{d\zeta} + \frac{\gamma}{\alpha} f = 0 \tag{5}$$

At this point, we shall examine the boundary and initial conditions from the physics of the problem and will find that this will allow us to determine the remaining constant γ/α. For the nonlinear Korteweg-deVries equation, we find $\gamma/\alpha = -2/3$ which is specified by the invariance properties.

The boundary conditions which are germane to the experiment are

$$\begin{aligned} &u^{(1)}(z = 0,\ \tau = 0^+) = 0 && \text{(a)} \\ &u^{(1)}(z \neq \infty,\ \tau = \infty) = 0 && \text{(b)} \\ &u^{(1)}(z = \infty,\ \tau \neq \infty) = 0 && \text{(c)} \end{aligned} \tag{6}$$

$$u^{(1)}(z = \tau,\ \tau = 1) = 1 \qquad \text{(d)}$$

These four boundary conditions transform to

$$u^{(1)}(\zeta = -\infty) = 0 \qquad \text{(a)} \quad \text{(The first two consolidate)}$$

$$u^{(1)}(\zeta = +\infty) = 0 \qquad \text{(b)} \qquad (7)$$

$$u^{(1)}(\zeta = 0) = 1 \qquad \text{(c)}$$

Using the ansatz that $\gamma/\alpha = -1/3$,* we write eq.(5) as

$$\frac{d^3 f}{d[(\frac{2}{3})^{1/3}\zeta]^3} - \frac{d\{[(\frac{2}{3})^{1/3}\zeta]\, f[(\frac{2}{3})^{1/3}\zeta]\}}{d[(\frac{2}{3})^{1/3}\zeta]} = 0$$

which has a solution

$$f = Ai[(\tfrac{2}{3})^{1/3}\zeta]$$

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* By using the ansatz that $\gamma/\alpha = 0$, we find that we solve the piston problem as outlined in Washimi and Taniuti [WASHIMI and TANIUTI (1966)]

$$u_1 = \int Ai\ [(\tfrac{2}{3})^{1/3}\tilde{\zeta}]\,d\tilde{\zeta}$$

or

$$u^{(1)} = z^{-1/3} \mathrm{Ai} \left[\frac{z - \tau}{z^{1/3}} \left(\frac{2}{3}\right)^{1/3}\right] \tag{8}$$

This solution can also be obtained using Fourier-Laplace transform techniques and the dispersion relation for ion acoustic waves. [See e.g. DEMULIERE et al (1971)] It predicts that the signal should appear as an Airy Function whose amplitude decreases as $z^{-1/3}$ and whose period of oscillation increases as $z^{1/3}$ with increasing distance z.

IV. Conclusion

We note that eq. (8) predicts the experimentally observed properties for the ion wave excited by D-P excitation. A decay in the amplitude of the pulse for the plate excitation was observed although no higher frequency trailing oscillations were found. The difficulty in exciting higher frequency ion acoustic waves (frequency $\gtrsim \frac{1}{10}$ ion plasma frequency) with a plate has been previously noted [LONNGREN et al (1967)]. We believe that this explains why the higher frequency trailing oscillations were not observed. Therefore in an experiment designed to measure the dispersion of ion acoustic waves as $f \to f_{pi}$, a plate could not be used [JOYCE et al (1969)] while the D-P excitation easily produced the expected dispersion curve [IKEZI et al (1972)].

Finally, all of the experimental results presented here were obtained under linear conditions. By this, we mean that the same results could be found for $\phi_{ex} > 0$ and $\phi_{ex} < 0$. As the exciting voltage of the D-P plasma was increased to $\phi_{ex} \sim \frac{K_T}{e}e$, a difference in results appeared. The rarefaction wave excited by $\phi_{ex} < 0$ was not altered. However, the compression wave excited by $\phi_{ex} > 0$ rapidly developed into soliton propagation [IKEZI, TAYLOR and BAKER (1970)]. Such an a symmetry was not observed for the plate excitation.

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Figure Captions

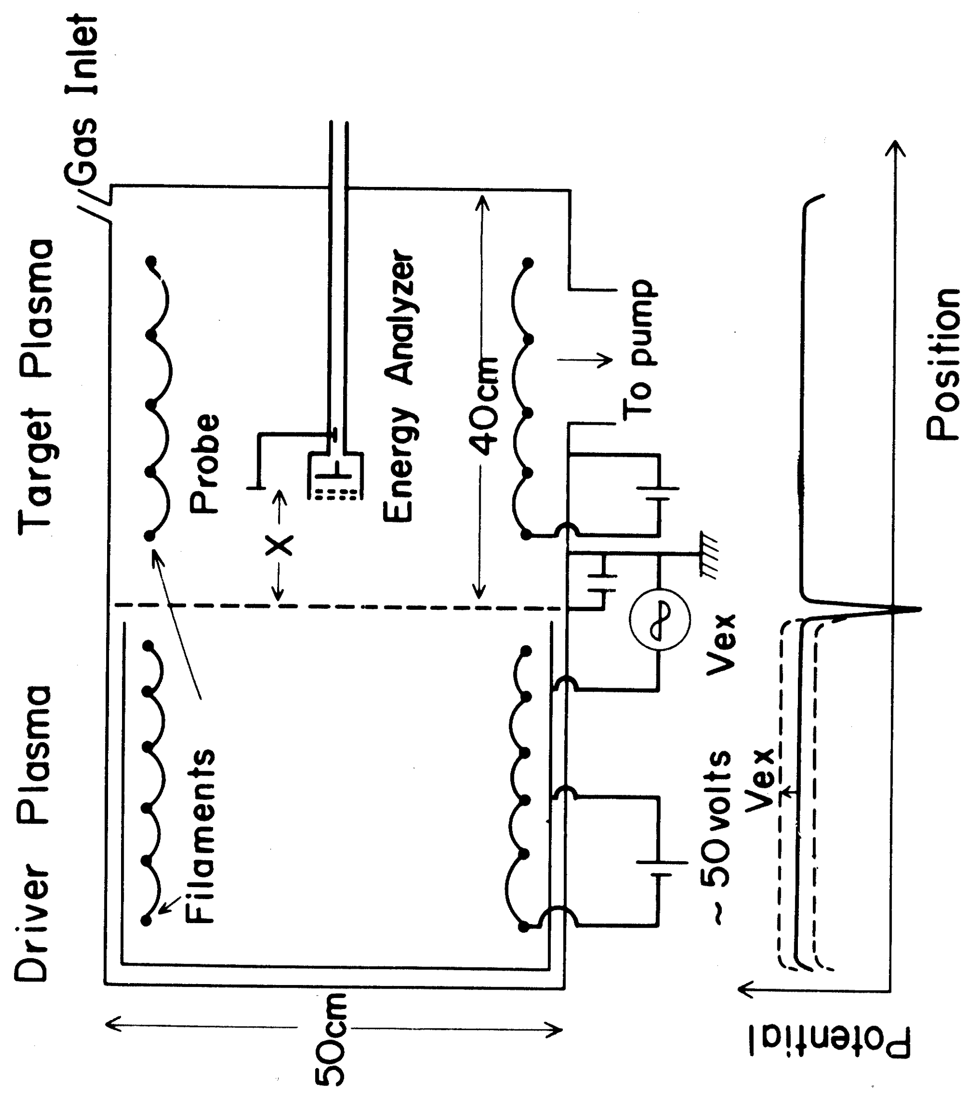
Fig. 1. Experimental setup for the double-plasma (D-P) excitation experiment. The potential profile is shown at the bottom.

Fig. 2. Experimental setup for the plate excitation experiment. The use of an additional insulated probe and a difference amplifier eliminated the large direct-coupled signal that had been previously observed in experiments of this type.

Fig. 3. (a) Typical detected ion wave signals at various distances from the D-P exciting grid. (in cm, indicated by the numbers). The top trace is the applied signal.

(b) Period of the trailing oscillations as a function of position.

Fig. 4. Typical detected ion wave signals at various distances from the exciting plate (a) $x = 3.3$ cm, $\Delta T = 20 \mu sec/div$, output of coherent detector used a "smoothing filter". (b) $x = 3.3$ cm, $\Delta T = 10 \mu sec/div$, no "smoothing filter" (c) $x = 7.0$ cm, $\Delta T = 10 \mu sec/div$, no "smoothing filter".


Driver Plasma
Target Plasma
Gas Inlet
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X
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Fig. 1

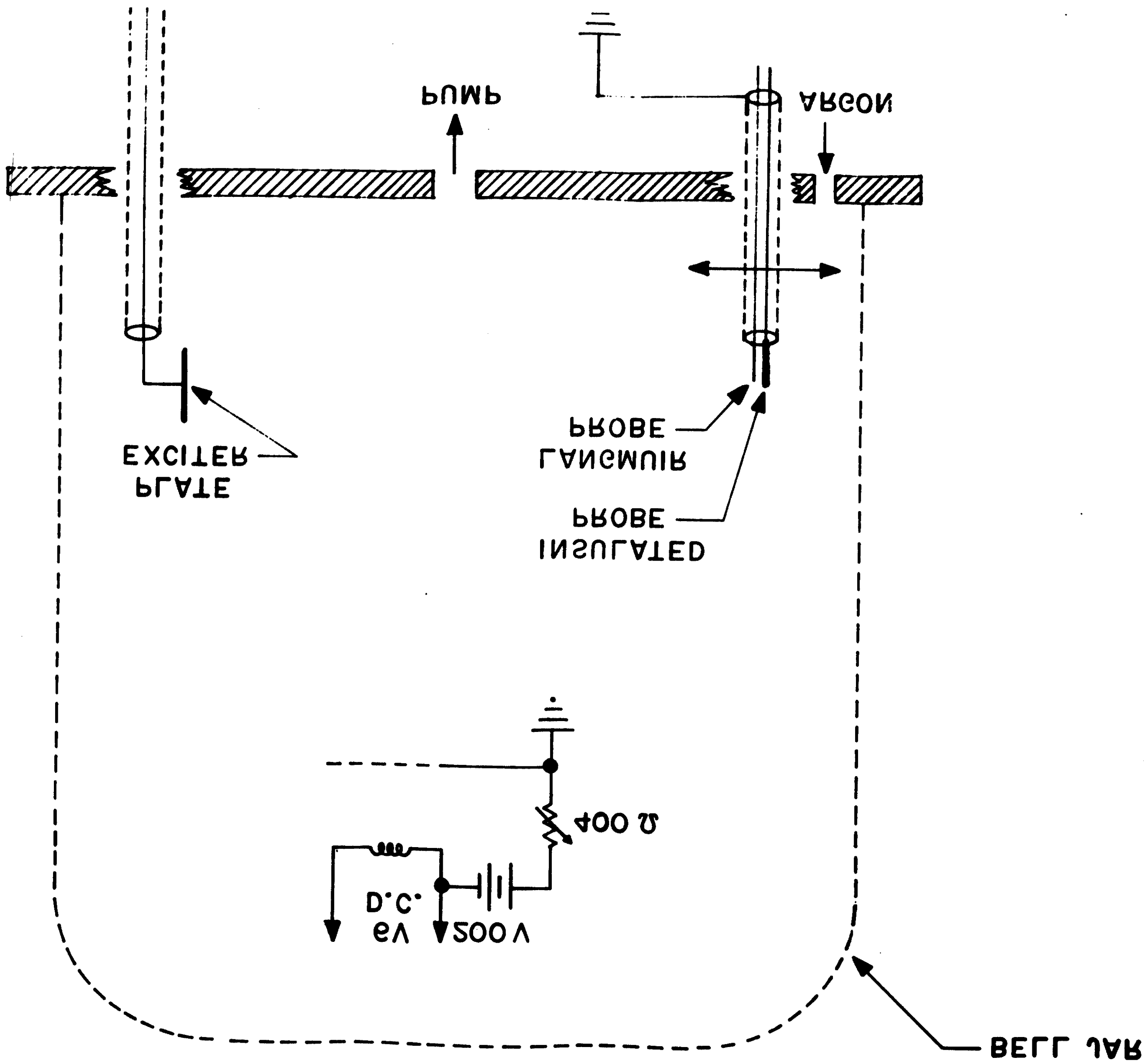
PUMP
ARGON
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Fig. 2

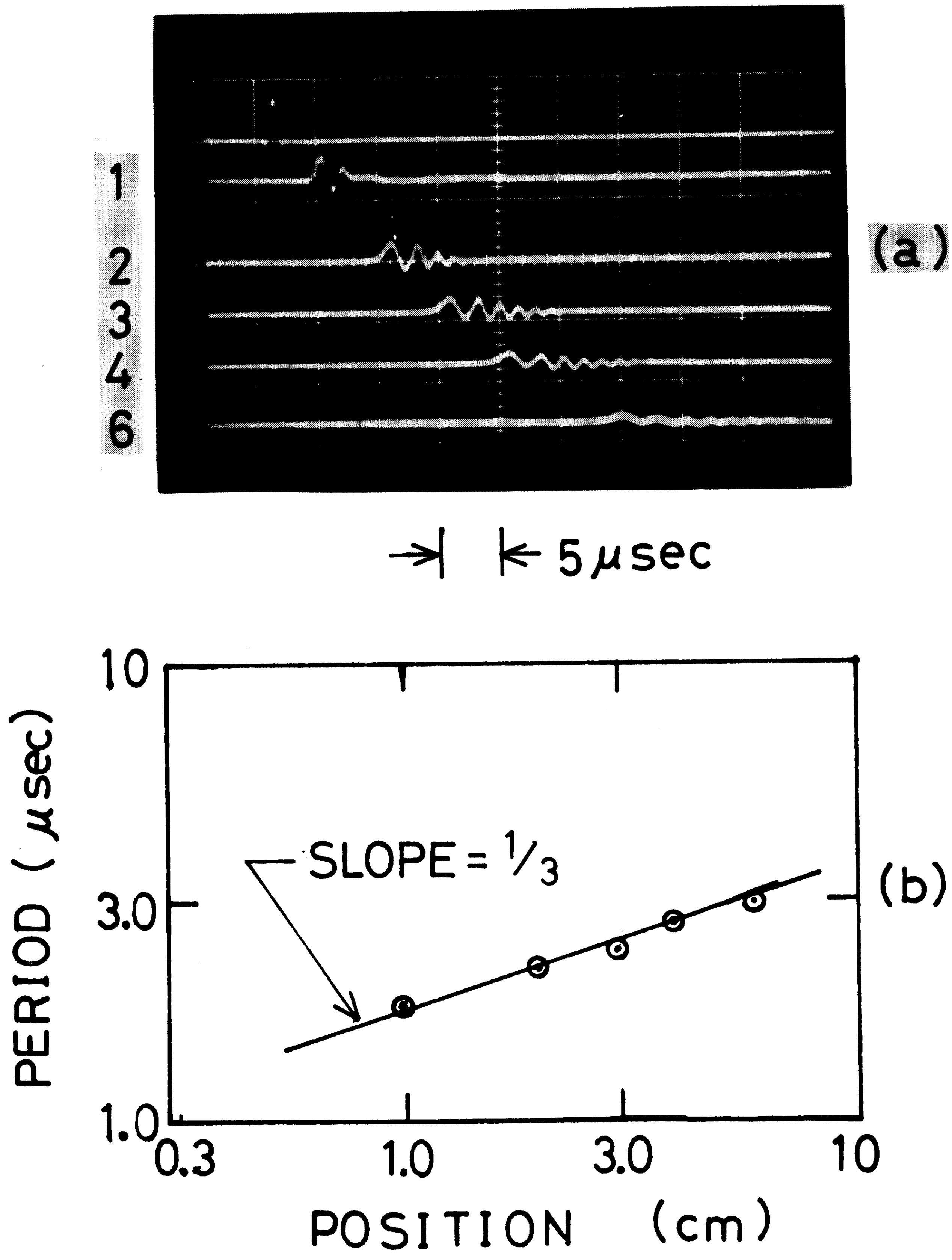
1
2
3
4
6
(a)
5 μsec
10
3.0
1.0
PERIOD (μsec)
SLOPE = 1/3
(b)
0.3
1.0
3.0
10
POSITION (cm)

Fig. 3

(a)

(b)

(c)

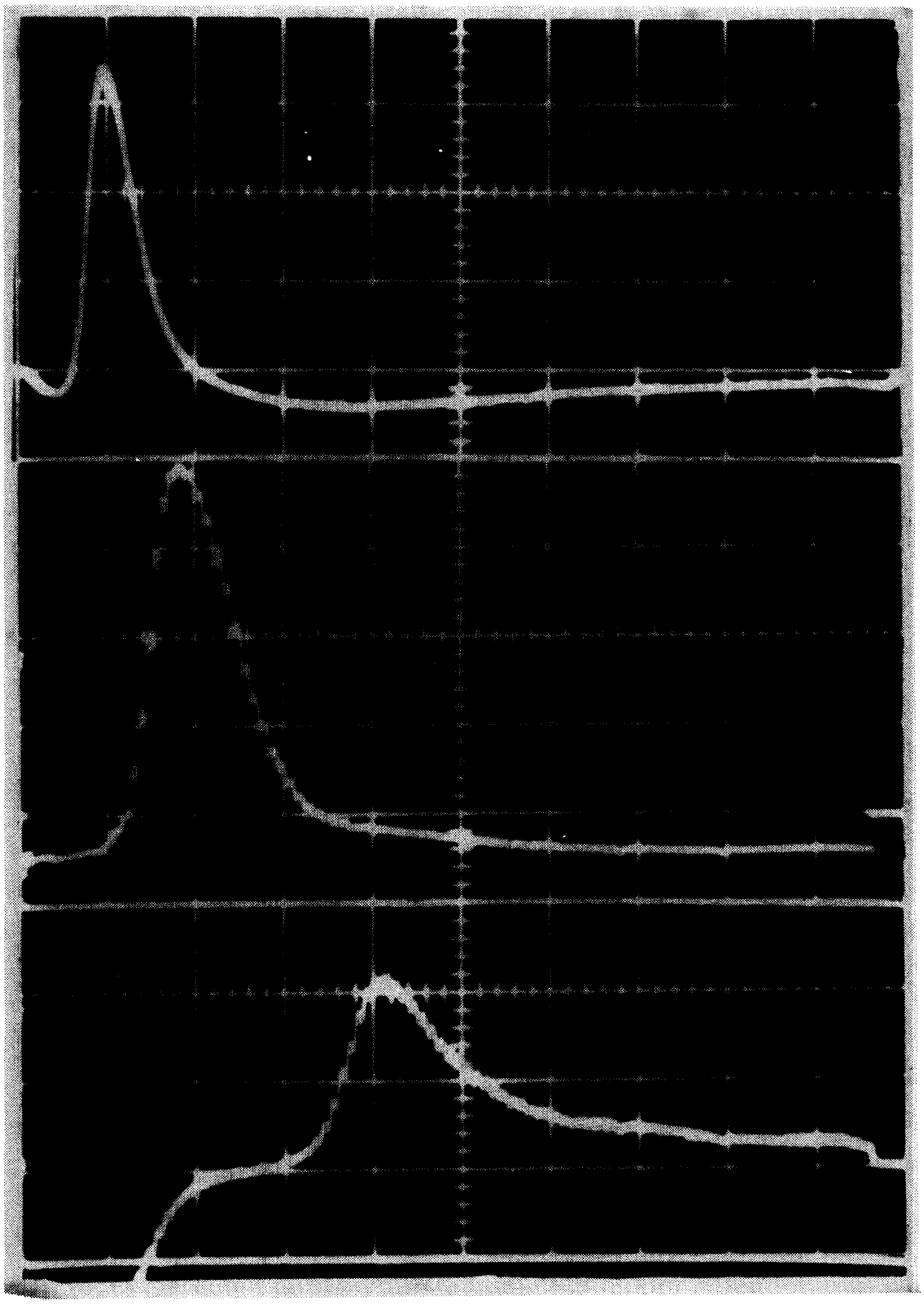

Fig. 4