INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

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Perpendicular Ion Cyclotron Resonance in a Two-Ion Plasma

Hiroshi Toyama and Kiyokata Matsuura

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Abstract

The rf power absorbed at the perpendicular ion cyclotron resonance of an H-D plasma is calculated using the model of an infinitely long coil, with electron density, ion temperature, and the H-D concentration ratio as the parameters.

§1. Introduction

Coil-to-plasma coupling becomes an important problem in the efficient heating of a plasma, when ion cyclotron resonance heating is adopted as the heating system[1-5].

We have examined experimentally the dependence of the absorbed rf power on the diameter of the plasma column in the density region of 10^{11} cm^{-3} to 2×10^{12} cm^{-3} using the QP machine[6]. When we set the absorbed rf power proportional to a^{γ} (a is the radius of the plasma column), the experimental value of γ is found to be less than 2 (γ is a function of the electron density).

In this paper, the absorbed rf power is calculated using the model that the plasma column is surrounded by an infinitely long coil. When the density of the plasma column is uniform, the absorbed rf power is proportional to the imaginary part of χ^2, where χ is a function of the wave number and the elements of the dielectric tensor.

$\mathrm{Im}(\chi^2)$ is calculated as a function of Ω (the ratio of applied frequency to ion cyclotron frequency) with electron density, ion temperature, and the concentration ratio of deuteron to hydrogen being the parameters. It is shown that $\mathrm{Im}(\chi^2)$ has its maximum value at the perpendicular resonance.

§2. Calculation

The absorbed rf power will be calculated using the model of a cylindrical plasma, infinitely long, and surrounded by vacuum. The radius of the plasma column is denoted by a. At the radial distance R (R > a), there is a sheet current of density j exp i(κz - ωt) in the θ direction. The rf power absorbed by the plasma cylinder per unit length, P, is given by calculating the Poynting vector at the surface of the plasma column,

$$P = 2\pi a(\vec{E} \times \vec{H})_r \; . \tag{1}$$

The electric field $\vec{E}$ in the plasma is given by

$$\nabla \times \nabla \times \vec{E} = \frac{\omega^2}{c^2} \hat{\varepsilon} \, \vec{E}, \tag{2}$$

where

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & ig & 0 \\ -ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix} \; . \tag{3}$$

The magnetic field is given by the Maxwell equation

$$-\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} \; . \tag{4}$$

Substituting the solution of equation (2) into equations (4) and (1), one obtains[6]

$$P = \pi\mu_0 j^2 \omega R^2 K_1^2(\kappa R) \int_0^a dr r \mathrm{Im}(\chi^2) I_1^2(\kappa r), \qquad (5)$$

where

$$\chi^2 = \kappa^2 + \frac{\omega^2}{c^2}\left(\varepsilon - \frac{c^2}{\omega^2}\kappa^2 - \frac{g^2}{\varepsilon - c^2\kappa^2/\omega^2}\right), \qquad (6)$$

and I_1 and K_1 are the modified Bessel functions of the first order. The components of the dielectric tensor ε and g are expressed for a Maxwellian plasma with $T_\perp = T_\parallel = T$ and zero drift velocity, as follows;

$$\varepsilon = 1 + \sum_k \frac{\Pi_k^2}{2\omega\kappa u_k}[z(\xi_+) + z(\xi_-)], \qquad (7)$$

$$g = 1 + \sum_k \frac{\Pi_k^2}{2\omega\kappa u_k}[z(\xi_+) - z(\xi_-)], \qquad (8)$$

where

$$\xi_\pm = (\omega \mp \Omega_k)/(\kappa u_k), \qquad u_k = (2T_k/m_k)^{\frac{1}{2}},$$

$$\Omega_k = e_k B_0/m_k, \qquad \Pi_k^2 = n_k e_k^2/(\varepsilon_0 m_k),$$

and the function $Z(\xi)$ is the plasma dispersion function. When the density of the plasma column is uniform, the absorbed rf power P is proportional to $\mathrm{Im}(\chi^2)$; $\mathrm{Im}(\chi^2)$ is r independent and the other factors in equation (5) are constant in the experiment. $\mathrm{Im}(\chi^2)$ is calculated as a function of $\Omega(\equiv\omega/\Omega_H)$ in the density region of 10^{10} to 10^{14} cm^{-3}, with electron density, ion temperature, and the concentration ratio of deuteron to hydrogen as the parameters, as shown in Figs. 1-5. As the electron density increases, the peak value of $\mathrm{Im}(\chi^2)$ increases and the halfwidth narrows. When electron density is higher than 7×10^{11} cm^{-3}, the peak value becomes lower. When the density is 2×10^{12} cm^{-3}, there are two peaks; one is narrow and the other is broad(see Fig. 4.). The maximum value of $\mathrm{Im}(\chi^2)$ is plotted versus electron density in Fig. 6, ion temperature and D^+ concentration being the parameters. The value of Ω, which maximizes $\mathrm{Im}(\chi^2)$ is plotted versus electron density in Fig. 7.

The experimental plasma column has a density profile such as shown in Fig. 8(a). In order to show that there exists an absorption layer which contributes to the absorbed power P, the integrand of the equation (5) is plotted vs. r in Fig. 8(b) for the case of following parameters; plasma radius: 5.7 cm, ion temperature: 4 eV, peak density: 10^{12} cm^{-3}, Ω: 0.992.

§3. Discussion

It is predicted that $\mathrm{Im}(\chi^2)$ is maximized when (ε_r -

$n_\parallel^2$) is equal to zero, where ε_r is the real part of ε. From equation (7), the dominant term of ε_r in the neighbourhood of the ion cyclotron resonance of H^+ is

$$\varepsilon_r \sim \frac{\Pi_H^2}{2\omega\kappa u_H} Z(\xi_+). \tag{9}$$

A rough sketch of ε_r vs. ξ_+ is shown in Fig.9. In the low density region ($n < 2 \times 10^{11}$ cm^{-3}, where the ion temperature is 4 eV and the concentration of H^+ ions is 100%), ($\varepsilon_r - n_\parallel^2$) is always negative, so that the maximum value of $Im(\chi^2)$ is low. In the higher density region, $Im(\chi^2)$ is maximized when $\varepsilon_r - n_\parallel^2 = 0$. This is the condition of the perpendicular ion cyclotron resonance mentioned by Stix[7]. We denote the value of Ω which satisfies the equation $\varepsilon_r - n_\parallel^2 = 0$ by Ω_m. Under the condition that $g_i << g_r$, where g_i and g_r are the imaginary and real parts of g, respectively, $Im(\chi^2)$ is expressed as follows, in the neighborhood of the perpendicular resonance,

$$Im(\chi^2) = \frac{\omega^2}{c^2} \varepsilon_r^2 \frac{\varepsilon_i}{(\varepsilon_r - n_\parallel^2)^2 + \varepsilon_i^2}. \tag{10}$$

The halfwidth of the resonance, $\Delta\varepsilon_r$, is given by

$$\Delta\varepsilon_r = 2\varepsilon_i \tag{11}$$

The peak value of $Im(\chi^2)$, P, is given by

$$P = \frac{\omega^2}{c^2} \frac{\varepsilon_r^2}{\varepsilon_i} . \tag{12}$$

Using the approximate relation

$$\varepsilon_r \sim \frac{\Pi_H^2}{\omega^2} \frac{\Omega^2}{1 - \Omega^2} , \tag{13}$$

the halfwidth of the resonance may be given in terms of Ω

$$\frac{\Delta\Omega}{\Omega} = (1 - \Omega^2) \frac{\varepsilon_i}{\varepsilon_r} . \tag{14}$$

As the electron density increases, Ω_m shifts from 1 (as shown in Fig. 7); ε_i becomes smaller. From equations (12) and (14), P increases and the halfwidth narrows; $\Delta\Omega/\Omega \sim 3 \times 10^{-5}$ when $n = 7 \times 10^{11}$ cm^{-3}. A too narrow peak is meaningless from the standpoint of the experiment; we have omitted this peak in the density region of $n \geq 10^{12}$ cm^{-3}. The halfwidth of the resonance $\Delta\Omega$ is also written in the following way:

$$\frac{\Delta\Omega}{\kappa u} = 2Z_i(\xi_+) \Big/ \frac{dZ_r(\xi_+)}{d\xi_+}$$

$$\sim 2\sqrt{\pi}\ \xi_+^2\ e^{-\xi_+^2} . \tag{15}$$

As the resonance condition is not so sensitive to temperature, as shown in Fig. 7, the resonance becomes broader as the ion temperature increases, from equation (15). The short wavelength of the coil also contributes to the broadening. In the higher density region ($n \geq 2 \times 10^{12}$ cm^{-3}), $(\varepsilon_r - n_{\parallel}^2)$ is positive when $\Omega \sim 0.9$. $(\varepsilon_r - n_{\parallel}^2)$ becomes zero when Ω is near 1. In this case g_i is larger than g_r, so that the value of $Im(\chi^2)$ is expressed as follows;

$$Im(\chi^2) = \frac{\omega^2}{c^2} (\varepsilon_i + \frac{g_r^2 - g_i^2}{\varepsilon_i})$$

$$\sim \frac{\omega^2}{c^2} (\varepsilon_i - \frac{g_i^2}{\varepsilon_i}).$$

The first term and the second term in the $Im(\chi^2)$ nearly cancel, so that peak value of $Im(\chi^2)$ is very small (Fig. 6). Cyclotron damping also contributes to $Im(\chi^2)$.

In conclusion, the deuteron concentration also has an effect on the resonance condition through the dispersion relation. Also, depending on the electron density there is an appropriate wavelength of the coil. That is, a short wavelength coil should be used when we want to heat dense plasma. Finally, the rf absorption curve is broadened as the ion temperature increases.

Acknowledgments

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Figure Captions

Fig. 1. $\mathrm{Im}(\chi^2)$ versus Ω. Electron density = 3×10^{11} cm^{-3}. Temperature of ions and electrons = 4 eV. $\kappa = 0.1257\ \mathrm{cm}^{-1}$. (a) D^+ 0%, H^+ 100%, (b) D^+ 30%, (c) D^+ 60%, (d) D^+ 90%.

Fig. 2. $\mathrm{Im}(\chi^2)$ versus Ω. Electron density = 5×10^{11} cm^{-3}. (a) D^+ 0%, H^+ 100%, (b) D^+ 30%, (c) D^+ 60%, (d) D^+ 90%.

Fig. 3. $\mathrm{Im}(\chi^2)$ versus Ω. Electron density = 10^{12} cm^{-3}. (a) D^+ 0%, H^+ 100%, (b) D^+ 30%, (c) D^+ 60%, (d) D^+ 90%.

Fig. 4. $\mathrm{Im}(\chi^2)$ versus Ω. Electron density = 2×10^{12} cm^{-3}. (a) D^+ 0%, H^+ 100%, (b) D^+ 30%, (c) D^+ 60%, (d) D^+ 90%.

Fig. 5. $\mathrm{Im}(\chi^2)$ versus Ω. Electron density = 7×10^{12} cm^{-3}. Temperature of ions and electrons = 7 eV. (a) D^+ 0%, H^+ 100%, (b) D^+ 30%, (c) D^+ 60%, (d) D^+ 90%.

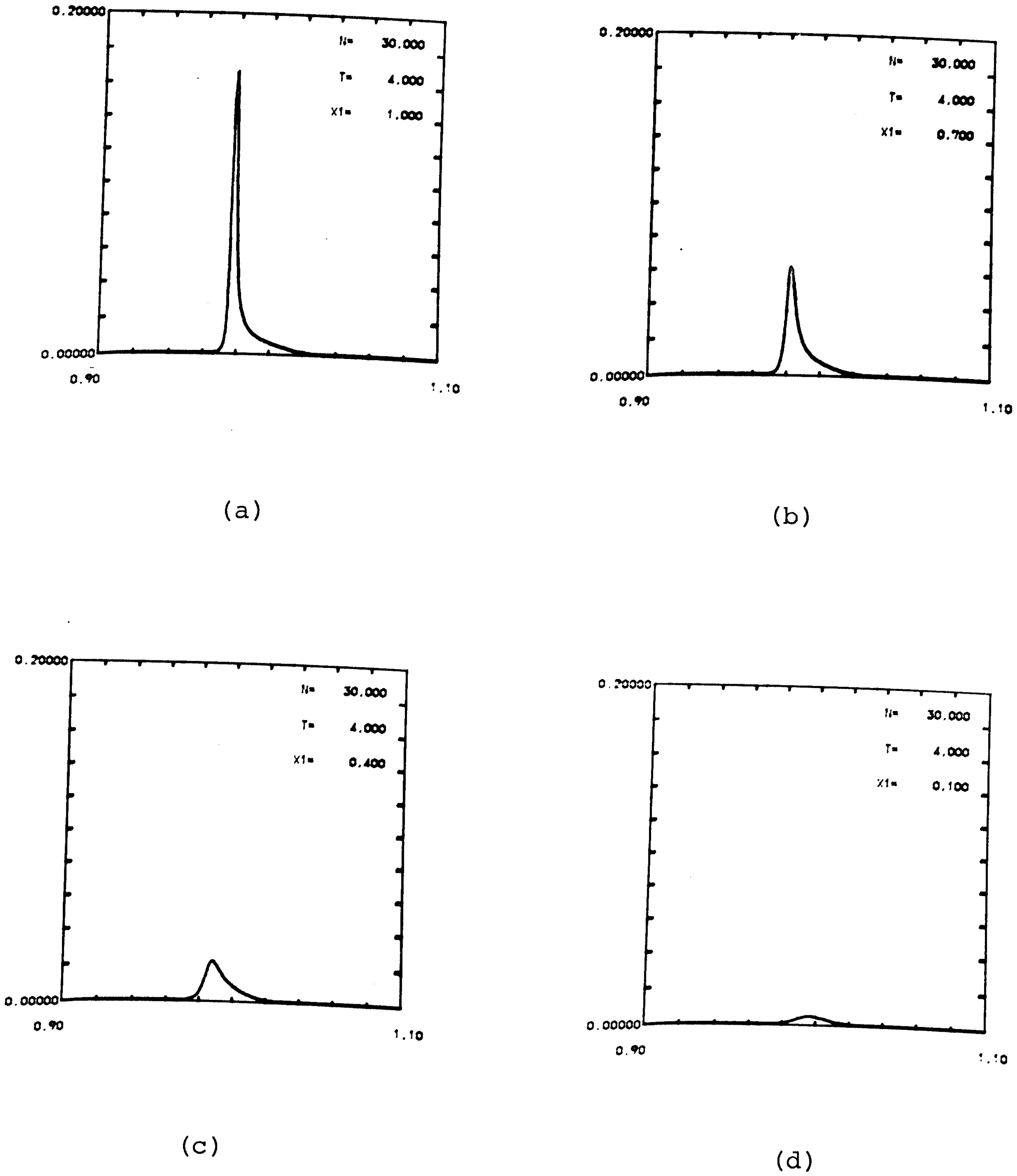
Fig. 6. $\mathrm{Im}(\chi^2)$ versus electron density. (1) Temperature = 4 eV. H^+ 100%. (2) Temperature = 7 eV. H^+ 100%. (3) Temperature = 4 eV. H^+ 70%, D^+ 30%.

Fig. 7. Ω value corresponding to the maximum of $\mathrm{Im}(\chi^2)$ versus electron density. (1) Temperature = 7 eV. H^+ 100%. (2) Temperature = 4 eV. H^+ 100%. (3) Temperature = 4 eV, H^+ 70%, D^+ 30%.

Fig. 8. Profile of electron density and $r\mathrm{Im}(\chi^2)I_1^2(\kappa r)$.

Plasma radius = 5.7 cm. Temperature = 4 eV. Ω = 0.992.

Fig. 9. Variation of ε_r versus ξ_+ for increasing density.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X1= 1.000
0.00000
0.90
1.10
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N= 30.000
T= 4.000
X1= 0.700
0.00000
0.90
1.10
(b)
0.20000
N= 30.000
T= 4.000
X1= 0.400
0.00000
0.90
1.10
(c)
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N= 30.000
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X1= 0.100
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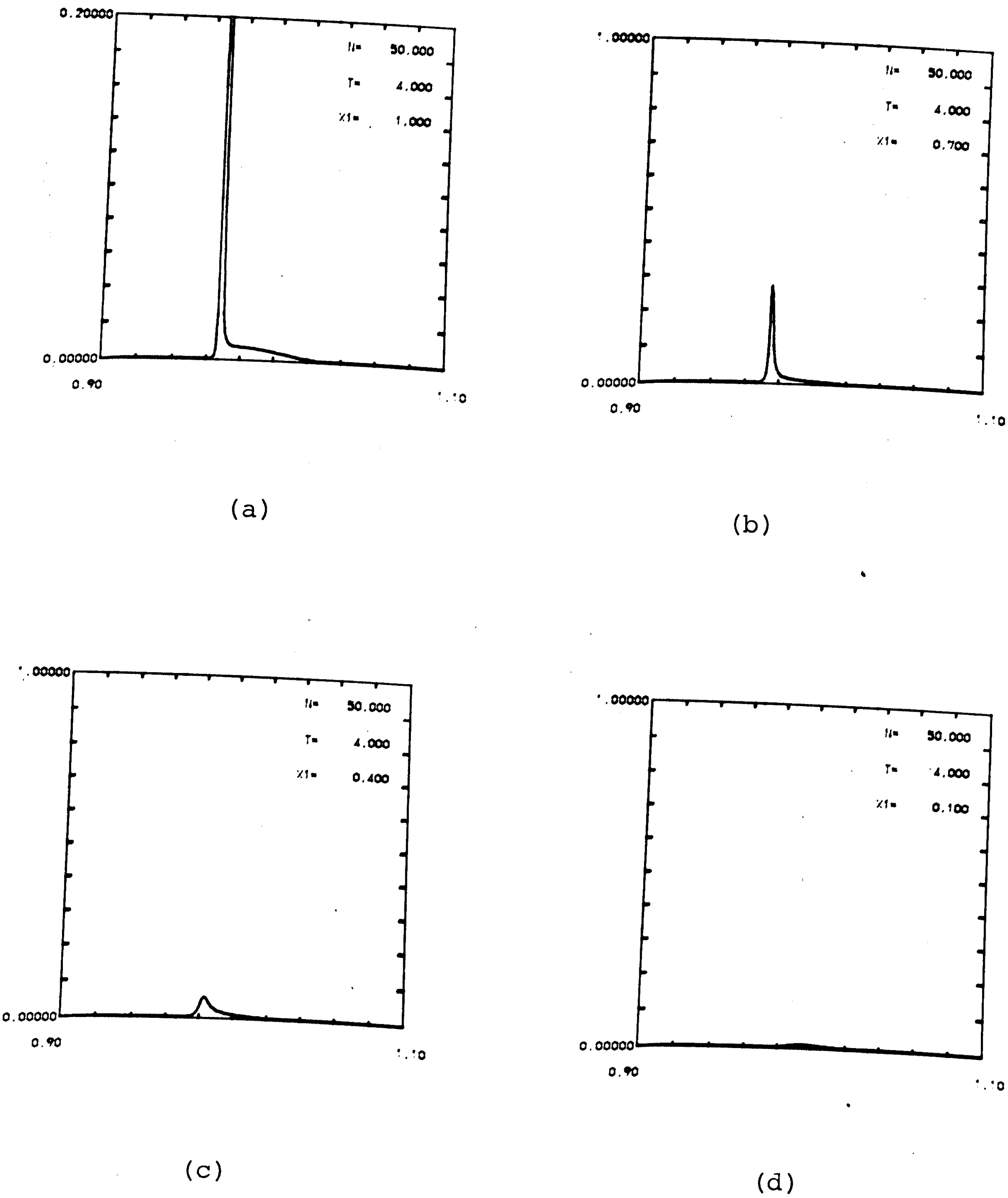
0.90
1.10
(d)

Fig. 1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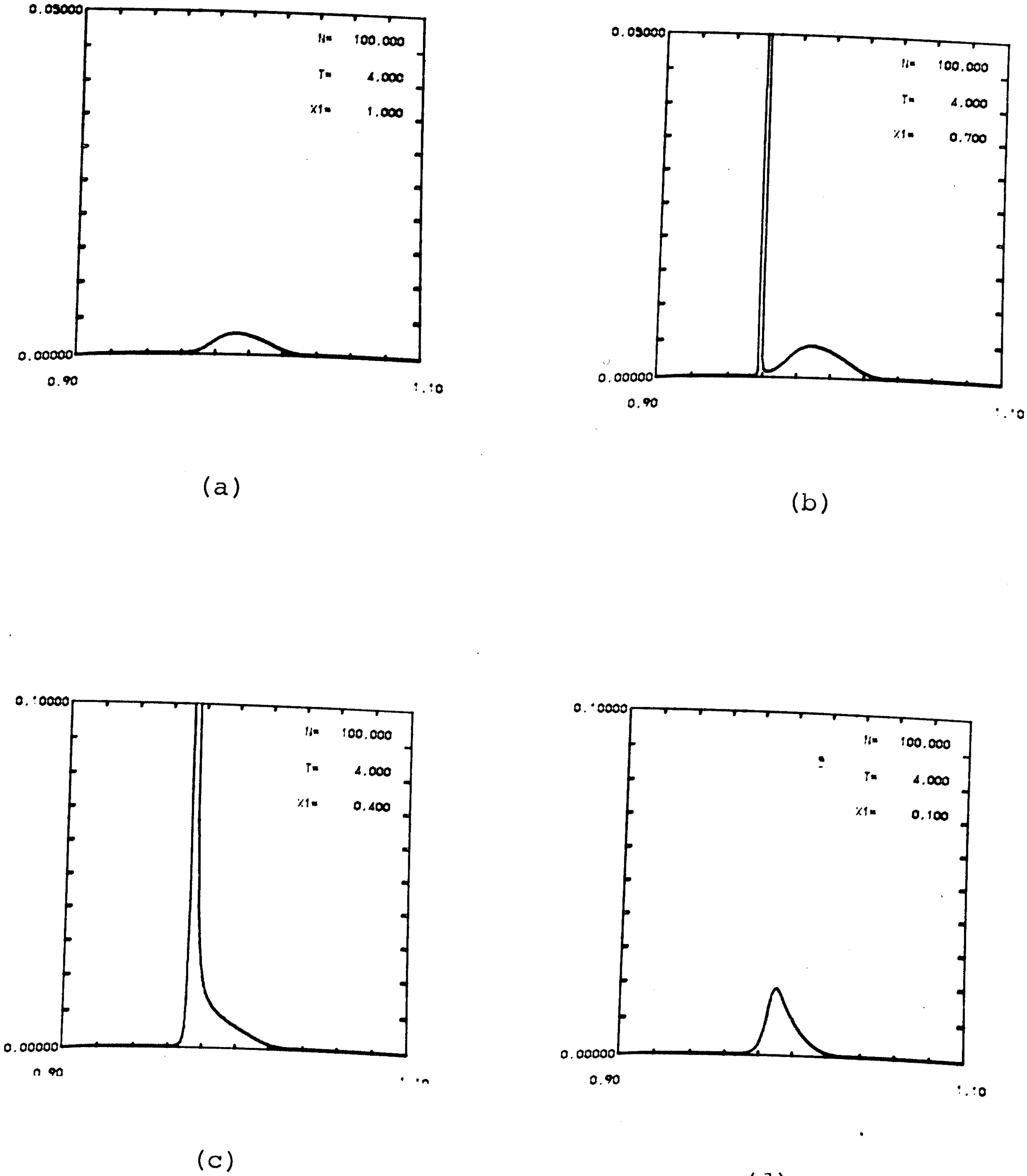
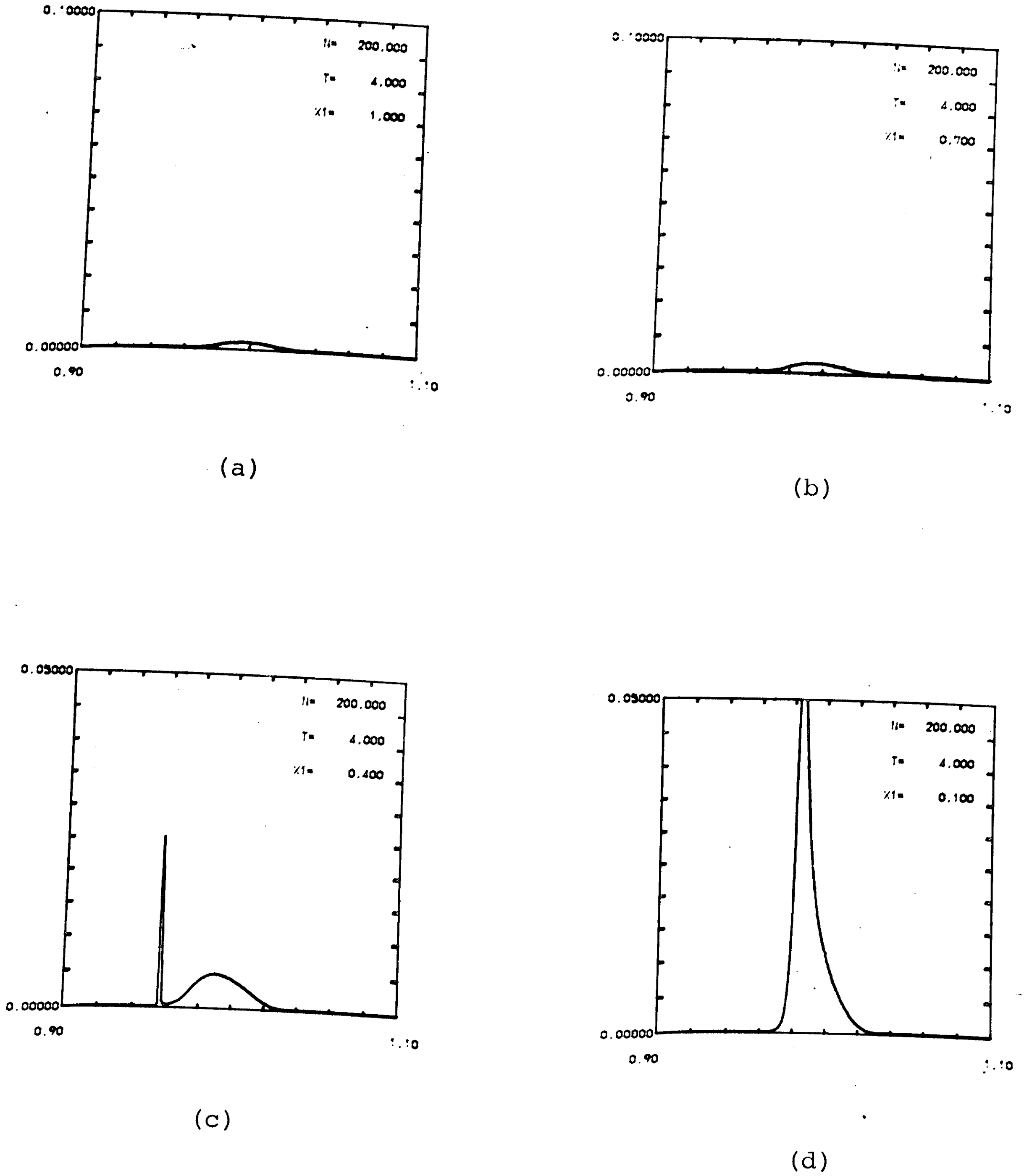
N= 50.000
T= 4.000
X1= 1.000
0.00000
0.90
(a)
1.00000
N= 50.000
T= 4.000
X1= 0.700
0.00000
0.90
(b)
1.00000
N= 50.000
T= 4.000
X1= 0.400
0.00000
0.90
(c)
1.00000
N= 50.000
T= 4.000
X1= 0.100
0.00000
0.90
(d)

Fig. 2



0.03000
N= 100.000
T= 4.000
X1= 1.000
0.00000
0.90
(a)
0.03000
N= 100.000
T= 4.000
X1= 0.700
0.00000
0.90
(b)
N= 100.000
T= 4.000
X1= 0.400
0.00000
(c)
N= 100.000
T= 4.000
X1= 0.100
0.00000
0.90
(d)

Fig. 3



0.10000
0.00000
0.90
N= 200.000
T= 4.000
X1= 1.000
X1= 0.700
0.05000
X1= 0.400
X1= 0.100
(a)
(b)
(c)
(d)

Fig. 4

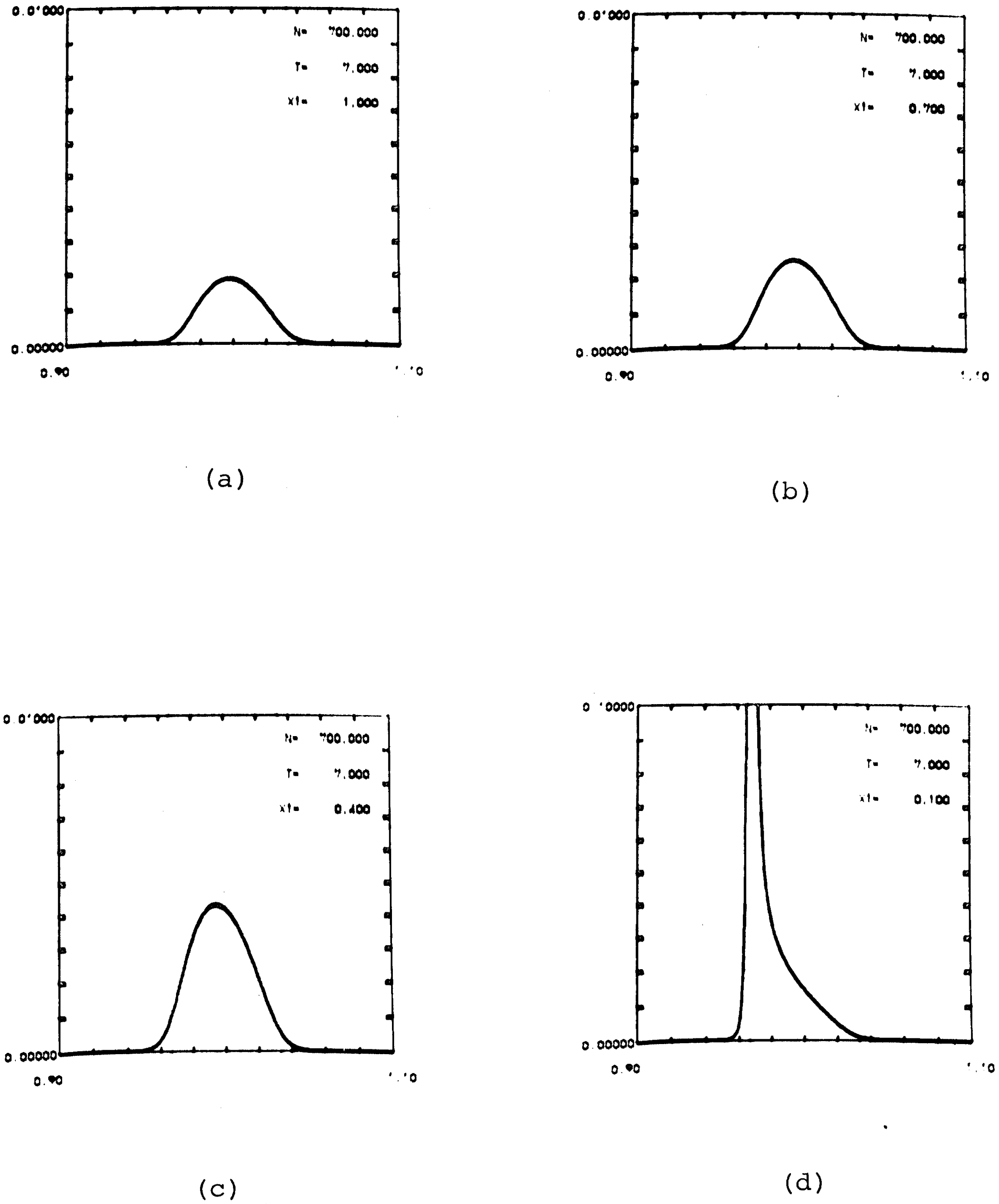

N= 700.000
T= 7.000
xi= 1.000
0.00000
0.90
(a)
N= 700.000
T= 7.000
xi= 0.700
0.00000
0.90
(b)
N= 700.000
T= 7.000
xi= 0.400
0.00000
0.90
(c)
N= 700.000
T= 7.000
xi= 0.100
0.00000
0.90
(d)

Fig. 5

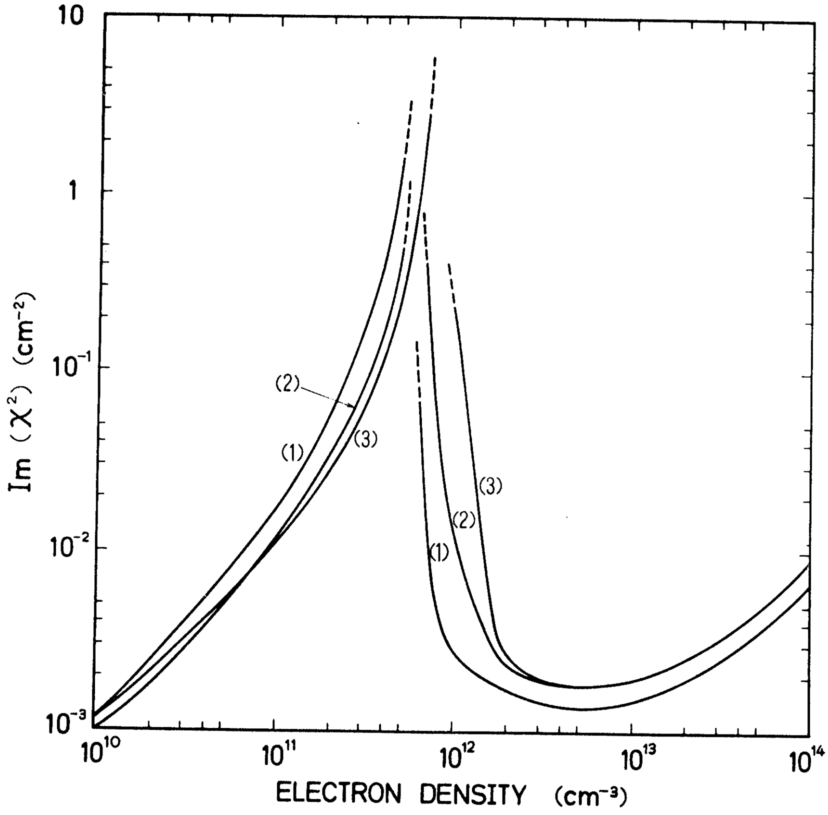

10
1
10^{-1}
10^{-2}
10^{-3}
Im (χ^2) (cm^{-2})
(1)
(2)
(3)
(1)
(2)
(3)
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
ELECTRON DENSITY (cm^{-3})

Fig. 6

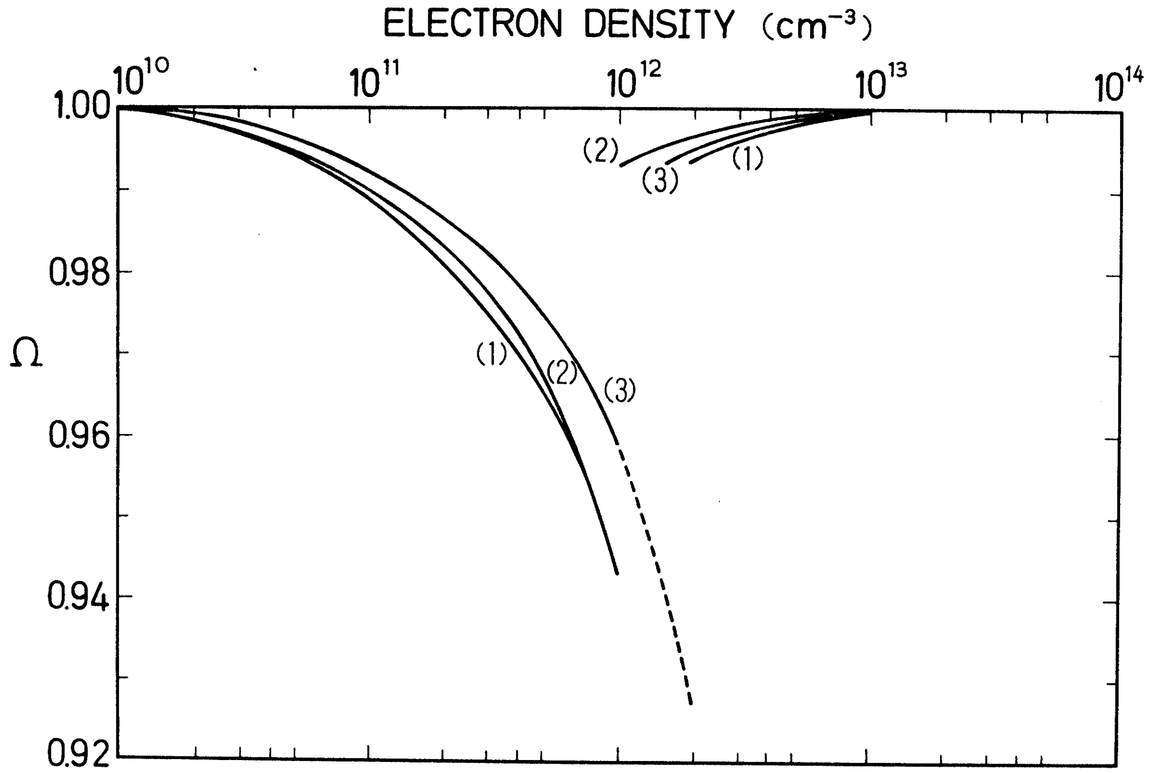
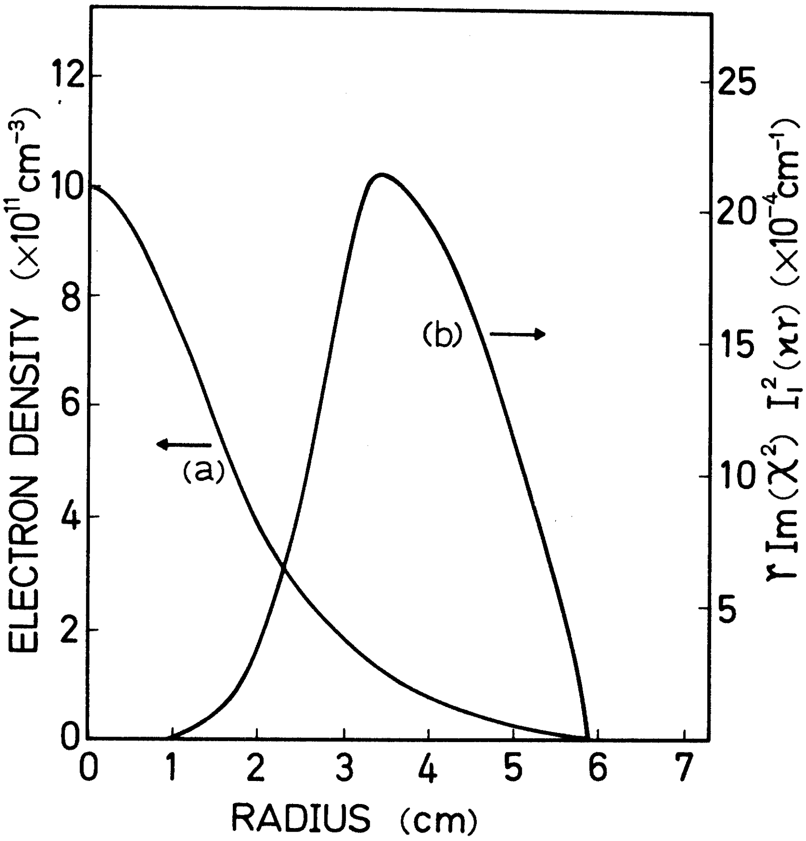

ELECTRON DENSITY (cm⁻³)
10¹⁰
10¹¹
10¹²
10¹³
10¹⁴
1.00
0.98
0.96
0.94
0.92
Ω
(1)
(2)
(3)
(2)
(3)
(1)

Fig. 7



ELECTRON DENSITY (×10^11 cm^-3)
r Im(χ²) I₁²(κr) (×10^-4 cm^-1)
RADIUS (cm)
(a)
(b)
0
2
4
6
8
10
12
5
10
15
20
25
1
3
7

Fig. 8

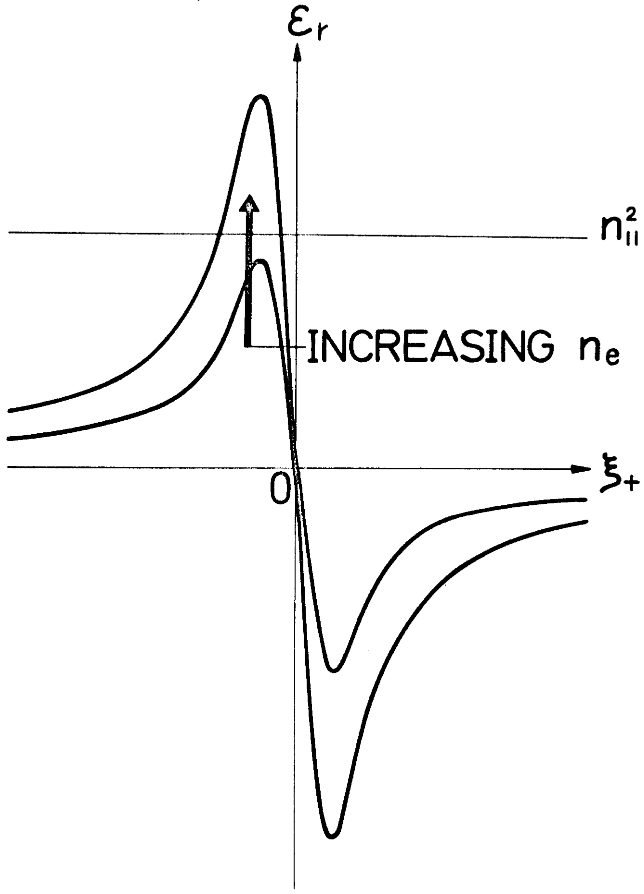


Fig. 9