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On the Physical Picture for the Anomalous Propagation of an Ordinary Wave in a Magnetoplasma

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

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Abstract

The physical picture for the anomalous propagation of an ordinary wave at frequency near but below the electron cyclotron frequency is discussed on the basis of the single-electron equation of motion in the magnetostatic and the wave fields. It is shown that the anomalous propagation originates from the forces along and across the magnetostatic field. The perpendicular force deflects the electrons and gives the density perturbation which contributes to the anomalous propagation, as shown by Fried. It is found, however, that the parallel forces connected with the gyration of the unperturbed motion also contribute to the anomalous propagation and accompany no density perturbation.

§1. Introduction

It is well known that the ordinary wave with frequency near but below the electron cyclotron frequency can propagate in a hot and dense magnetoplasma; this fact was first shown by Drummond.[1)] However, the physical reason of this anomalous propagation unexpected in the cold plasma theory was not presented for a long time until the recent report by Fried.[2)] The physical mechanism considered by him is such that the deflection of thermal electrons by the wave magnetic field produces a velocity perturbation along the wavevector and hence a density perturbation which makes the propagation possible if the frequency is near but below the electron cyclotron frequency. However, regarding the Lorentz force due to the wave magnetic field, he took only account of the component across the static field.

When the effects of gyration of the unperturbed motion are taken into account, the parallel component of the Lorentz force as well as the wave electric field itself contributes to the anomalous propagation. In the present paper, it is found that the contributions from the parallel forces are almost the same as that from the perpendicular force.

In the next section, the basic equations are presented which are necessary for the discussions on the physical mechanism of the anomalous propagation. In §3, we solve the equation of motion for a single electron to derive a expression whose real part gives the wave energy abosrbed by the electrons, and then derive the conductivity from it. In §4, some examination of the dispersion relation derived from the Boltzmann equation is given and the collisional absorption of wave energy under the conditions of the anomalous propagation is found to be much larger than the value obtained in the cold plasma theory.

§2. Fundamental Equation

A homogeneous and unbounded plasma is considered in which the particle density N and the temperature T are spatially uniform and a uniform magnetostatic field $\vec{B}_0$ is in the direction of the z-axis. We consider a weak electromagnetic wave which propagates in the direction perpendicular to $\vec{B}_0$, that is, along the x-axis and is linearly polarized with the electric field along $\vec{B}_0$. The frequency of the electromagnetic wave is assumed to be much higher than the ion cyclotron and ion plasma frequencies, so that the motion of an ion is neglected.

First, the motion is studied for the electrons in the static magnetic field $\vec{B}_0$ as well as in the wave electric and magnetic fields which are denoted by $\vec{E}_1$ and $\vec{B}_1$ respectively. Then the equations of motion of an electron are written for its unperturbed and perturbed velocities $\vec{v}_0$ and $\vec{v}_1$, respectively:

$$\frac{\partial \vec{v}_0}{\partial t} = -\frac{e}{mc}\vec{v}_0 \times \vec{B}_0 \quad , \tag{2.1}$$

$$\frac{\partial \vec{v}_1}{\partial t} + (\vec{v}_0 \cdot \vec{\nabla})\,\vec{v}_1 = -\frac{e}{m}\{\vec{E}_1 + \frac{1}{c}(\vec{v}_1 \times \vec{B}_0 + \vec{v}_0 \times \vec{B}_1)\} - \nu\vec{v}_1 \quad , \tag{2.2}$$

where $-e$ and m are the charge and mass of the electron, ν is the collision frequency which is assumed independent of the velocity, and the perturbed quantities, including $\vec{E}_1$ and $\vec{B}_1$, vary in space only with x .

The solutions of eq.(2.1) are

$$\begin{aligned} v_{0x}(t) &= v_\perp \cos(\Omega t + \phi_0) \quad , \\ v_{0y}(t) &= v_\perp \sin(\Omega t + \phi_0) \quad , \\ v_{0z}(t) &= \text{const} \quad , \end{aligned} \tag{2.3}$$

where $v_\perp$ and ϕ_0 are constant, and $\Omega = eB_0/mc$ is the electron cyclotron frequency. We rewrite eq.(2.2) by its components for the convenience of the following treatments.

$$\frac{dv_{1x}}{dt} + \Omega v_{1y} + \nu v_{1x} = \frac{e}{mc} v_{0z}B_1 \quad , \tag{2.4}$$

$$\frac{dv_{1y}}{dt} - \Omega v_{1x} + \nu v_{1y} = 0 \ , \tag{2.5}$$

$$\frac{dv_{1z}}{dt} + \nu v_{1z} = - \frac{e}{m} (E_1 + \frac{1}{c} v_{0x} B_1) \ , \tag{2.6}$$

where $d/dt = \partial/\partial t + v_{0x}\partial/\partial x$ is the derivative along the unperturbed trajectory given by eq.(2.3).

It is assumed that the electromagnetic wave fields are switched on at $t = 0$ and written for $t > 0$ as follows:

$$\begin{aligned} E_1(x,t) &= \bar{E}\ e^{i(kx-\omega t)} \ , \\ B_1(x,t) &= \bar{B}\ e^{i(kx-\omega t)} \ , \end{aligned} \tag{2.7}$$

where k and ω are assumed real, and $\bar{E}$ and $\bar{B}$ are constant. Because we are concerned with the stationary behaviour, the wave energy absorbed by the plasma per unit time and per unit volume can be represented as the real part of the following expression:

$$W = - \frac{eN}{2} \int d^3\vec{v}_0(0)\ \rho_0(\vec{v}_0(0)) \times \lim_{t\to\infty} \frac{1}{t}\int_0^t dt'(\vec{v}_0 + \vec{v}_1)\cdot\{\vec{E}_1^* + \frac{1}{c}(\vec{v}_0 \times \vec{B}_1^* + \vec{v}_1^* \times \vec{B}_0)\} \ , \tag{2.8}$$

where $d^3\vec{v}_0(0) = v_\perp dv_\perp dv_{0z} d\phi_0$ and $\rho_0(\vec{v}_0(0))$ is the distribution function of electrons in the equilibrium state and normalized as $\int \rho_0(\vec{v}_0(0))\, d^3\vec{v}_0(0) = 1$. Then the ratio of W to $\frac{1}{2}|\bar{E}|^2$ gives the electric conductivity for the relevant wave. Since the plasma is assumed to be the Maxwellian, eq.(2.8) is reduced to

$$W = -\frac{eN}{2}\int d^3\vec{v}_0(0)\ \rho_0(\vec{v}_0(0))$$

$$\times \lim_{t\to\infty}\int_0^t dt'\{v_{1z}(E_1^* + \frac{1}{c}\, v_{0x}B_1^*) - \frac{1}{c}\, v_{0z}v_{1x}B_1^*\}\ . \qquad (2.9)$$

Here it should be noted that the contribution from the term $\vec{v}_0\cdot(\vec{v}_1^*\times\vec{B}_0)/c$ vanishes. In the next section, in evaluating W we shall split it into two parts such that $W = W_1 + W_2$ where W_1 and W_2 come from the first two terms and the last one on the right-hand side of eq.(2.9), respectively.

§3. Derivation of Conductivity

In this section, first solving the equations of motion and then substituting these solutions into eq.(2.9) we evaluate the expressions of W and the conductivity. In what follows, for simplicity we omit the suffix '1' for the perturbed quantities. With the initial condition $v_z = 0$ at $t = 0$, the formal solution of eq.(2.6) is written as

$$v_z(t) = e^{-\nu t} \int_0^t dt' \, e^{\nu t'} \, F(t', x_0, \vec{v}_0(0)) \ , \tag{3.1}$$

where

$$F(t, x_0, \vec{v}_0(0)) = - \frac{e}{m} \{E(x,t) + \frac{1}{c} v_{0x} B(x,t)\} \ , \tag{3.2}$$

and the integration is carried out along the unperturbed orbit given by

$$x = x_0 + \frac{v_\perp}{\Omega} \{\sin(\Omega t + \phi_0) - \sin\phi_0\} \ . \tag{3.3}$$

Substituting eq.(3.3) into eq.(3.2) and expanding exp(ikx) by the Bessel functions, we obtain

$$F(t, x_0, \vec{v}_0) = - \frac{e}{m} \overline{E} \, e^{ikx_0} \sum_{\ell_1, \ell_2 = -\infty}^{\infty} J_{\ell_1}(\lambda) \, J_{\ell_2}(\lambda)$$

$$\times \{I_1(t;\ell_1,\ell_2) + I_2(t;\ell_1,\ell_2) + I_3(t;\ell_1,\ell_2)\} \tag{3.4}$$

where

$$\lambda = k \, v_\perp / \Omega \ ,$$

$$I_1(t;\ell_1,\ell_2) = \exp(i\ell_1\phi_0) \, \exp[-i\{\omega - (\ell_1 - \ell_2)\Omega\}t] \ , \tag{3.5}$$

$$I_2(t;\ell_1,\ell_2) = -\frac{kv_\perp}{2\omega}\exp\{i(\ell_1+1)\phi_0\}$$

$$\times \exp[-i\{\omega-(\ell_1-\ell_2+1)\Omega\}t] \ , \qquad (3.6)$$

$$I_3(t;\ell_1,\ell_2) = -\frac{kv_\perp}{2\omega}\exp\{i(\ell_1-1)\phi_0\}$$

$$\times \exp[-i\{\omega-(\ell_1-\ell_2-1)\Omega\}t] \ . \qquad (3.7)$$

By substituting eqs.(3.1) and (3.4) into eq.(2.9), we evaluate W_1 which is the contribution from F given by eq.(3.2). After some calculations we obtain

$$W_1 = \frac{Ne^2}{2m}\int d^3\vec{v}_0(0)\ \rho_0(\vec{v}_0(0))\ |\bar{E}|^2$$

$$\times \sum_{\ell_1,\ell_2,\ell_3,\ell_4} J_{\ell_1}(\lambda)\ J_{\ell_2}(\lambda)\ J_{\ell_3}(\lambda)\ J_{\ell_4}(\lambda) \sum_{i,j=1}^{3} A_{ij} \ ,$$

(3.8)

where

$$A_{ij} = \lim_{t\to\infty}\frac{1}{t}\int_0^t dt'\ e^{-\nu t'}\ I_i^*(t';\ell_1,\ell_2)$$

$$\times \int_0^{t'} dt''\ e^{\nu t''}\ I_j(t'';\ell_3,\ell_4) \ ,$$

and

$$A_{11} = \frac{\exp\{i(\ell_3 - \ell_1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4} \, ,$$

$$A_{12} = -\frac{kv_\perp}{2\omega} \frac{\exp\{i(\ell_3 - \ell_1 + 1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4+1} \, ,$$

$$A_{13} = -\frac{kv_\perp}{2\omega} \frac{\exp\{i(\ell_3 - \ell_1 - 1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4-1} \, ,$$

$$A_{21} = -\frac{kv_\perp}{2\omega} \frac{\exp\{i(\ell_3 - \ell_1 - 1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 + 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4-1} \, ,$$

$$A_{22} = \left(\frac{kv_\perp}{2\omega}\right)^2 \frac{\exp\{i(\ell_3 - \ell_1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 + 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4} \, ,$$

$$A_{23} = \left(\frac{kv_\perp}{2\omega}\right)^2 \frac{\exp\{i(\ell_3 - \ell_1 - 2)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 + 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4-2} \, ,$$

$$A_{31} = -\frac{kv_\perp}{2\omega} \frac{\exp\{i(\ell_3 - \ell_1 + 1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 - 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4+1} \, ,$$

$$A_{32} = \left(\frac{kv_\perp}{2\omega}\right)^2 \frac{\exp\{i(\ell_3 - \ell_1 + 2)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 - 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4+2} \, ,$$

$$A_{33} = \left(\frac{kv_\perp}{2\omega}\right)^2 \frac{\exp\{i(\ell_3 - \ell_1)\phi_0\}}{\nu - i\{\omega - (\ell_1 - \ell_2 - 1)\Omega\}} \, \delta_{\ell_1-\ell_2,\ell_3-\ell_4} \, ,$$

In order to compare with the result obtained by Drummond, in the following calculations we assume $\lambda << 1$ and retain the terms up to the second order of λ. After integration over ϕ_0 we have

$$\int_0^{2\pi} d\phi_0 \sum_{\ell_1,\ell_2,\ell_3,\ell_4} J_{\ell_1} J_{\ell_2} J_{\ell_3} J_{\ell_4} A_{11} = 2\pi \sum_{\ell_1,\ell_2} \frac{J^2_{\ell_1}(\lambda)\, J^2_{\ell_2}(\lambda)}{\nu - i\{\omega - (\ell_1 - \ell_2)\Omega\}}$$

$$= 2\pi \left[\frac{1}{\nu - i\omega} + \frac{1}{2}\lambda^2 \left\{\frac{1}{\nu - i(\omega - \Omega)} + \frac{1}{\nu - i(\omega + \Omega)}\right\}\right] , \tag{3.10}$$

$$\int_0^{2\pi} d\phi_0 \sum_{\ell_1,\ell_2,\ell_3,\ell_4} J_{\ell_1} J_{\ell_2} J_{\ell_3} J_{\ell_4} (A_{12} + A_{13} + A_{21} + A_{31})$$

$$= -\frac{4\pi\Omega}{\omega} \sum_{\ell_1,\ell_2} \frac{\ell_1\, J^2_{\ell_1}(\lambda)\, J^2_{\ell_2}(\lambda)}{\nu - i\{\omega - (\ell_1 - \ell_2)\Omega\}}$$

$$= -\frac{\pi\Omega\lambda^2}{\omega} \left\{\frac{1}{\nu - i(\omega - \Omega)} - \frac{1}{\nu - i(\omega + \Omega)}\right\} , \tag{3.11}$$

$$\int_0^{2\pi} d\phi_0 \sum_{\ell_1,\ell_2,\ell_3,\ell_4} J_{\ell_1} J_{\ell_2} J_{\ell_3} J_{\ell_4} (A_{22} + A_{23} + A_{32} + A_{33})$$

$$= 2\pi \left(\frac{kv_\perp}{2\omega}\right)^2 \left\{\frac{1}{\nu - i(\omega - \Omega)} + \frac{1}{\nu - i(\omega + \Omega)}\right\} , \tag{3.12}$$

where the contributions from A_{23} and A_{32} vanish. After further integration over $v_\perp$ and v_{0z} , finally we obtain the expression for W_1 to second order of λ as follows:

$$W_1 = \frac{i\omega_p^2 \, |\bar{E}|^2}{8\pi\omega} \left[\frac{1}{1 + i\frac{\nu}{\omega}} + \frac{T}{m} \left(\frac{k}{\Omega}\right)^2 \times \left\{ \frac{1 - \frac{\Omega}{\omega} + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2}{1 - \frac{\Omega}{\omega} + i\frac{\nu}{\omega}} + \frac{1 + \frac{\Omega}{\omega} + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2}{1 + \frac{\Omega}{\omega} + i\frac{\nu}{\omega}} \right\} \right] , \tag{3.13}$$

where $\omega_p^2 = 4\pi Ne^2/m$, and which is reduced to the result for the cold plasma if $T = 0$. That is, the first term in the square bracket on the right-hand side is the contribution only from the wave electric field and independent of the thermal motion. On the other hand, as we shall see below, the second and third-terms come from the effect of gyration of the unperturbed motion, which introduces no density perturbation.

Next, to calculate the second contribution W_2 the perturbation equation of motion is solved along the unperturbed orbit. Under the assumption that $v_x = 0$ at $t = 0$, eqs.(2.4) and (2.5) yield

$$v_x(t,x_0,\vec{v}_0) = \frac{ev_{0z}}{2mc} \left[\frac{B(x_0,0)}{i\Omega} \{e^{-(\nu+i\Omega)t} - e^{-(\nu-i\Omega)t}\} \right.$$

$$+ e^{-(\nu-i\Omega)t} \int_0^t dt' \; B(x',t') \; e^{(\nu-i\Omega)t'}$$

$$\left. + e^{-(\nu+i\Omega)t} \int_0^t dt' \; B(x',t') \; e^{(\nu+i\Omega)t'} \right] . \qquad (3.14)$$

Substituting eq.(3.14) into the last term on the right of eq.(2.9) and following the same procedure as in obtaining eq.(3.13) we obtain

$$W_2 = \frac{i\omega_p^2 \; |\overline{E}|^2}{8\pi\omega} \frac{T}{2m} \left(\frac{k}{\Omega}\right)^2 \left(\frac{\Omega}{\omega}\right)^2$$

$$\times \left\{ \frac{1}{1 - \frac{\Omega}{\omega} + i\frac{\nu}{\omega}} + \frac{1}{1 + \frac{\Omega}{\omega} + i\frac{\nu}{\omega}} \right\} . \qquad (3.15)$$

When $\nu = 0$, the ratio of W_2 to $\frac{1}{2}|\overline{E}|^2$ is reduced to the contribution to the conductivity from the density perturbation derived by Fried.[2)] For $\omega \sim \Omega$, eq.(3.15) is almost the same as the last two terms on the right of eq.(3.13).

From the sum of W_1 and W_2 divided by $\frac{1}{2}|\overline{E}|^2$ we find the expression of the conductivity which is written for $\nu = 0$ as follows:

$$\sigma(\omega) = \frac{i\omega_p^2}{4\pi\omega} \{1 + \frac{\frac{2T}{m}(\frac{k}{\Omega})^2}{1 - (\frac{\Omega}{\omega})^2}\} , \tag{3.16}$$

where the second term in the bracket is different only by the factor $2(\Omega/\omega)^2$ from the usual one derived by the Vlasov equation.

Let us examine W_2 in more detail. If the perturbation of the distribution function, denoted by $\rho(t,x_0,\vec{v}_0)$, is regarded as the density perturbation of an electron beam with drift velocity $\vec{v}_0$ and density ρ_0, then $\rho(t,x_0,\vec{v}_0)$ obeys the continuity equation

$$\frac{\partial\rho}{\partial t} + v_{0x}\frac{\partial\rho}{\partial x} = - \rho_0 \frac{\partial v_x}{\partial x} , \tag{3.17}$$

which was used also in ref.2. If we write the stationary values of ρ and v_x as

$$\rho(x,t) = \bar{\rho}\, e^{i(kx - \omega t)} ,$$
$$v_x(x,t) = \bar{v}_x\, e^{i(kx - \omega t)} , \tag{3.18}$$

we easily find

$$\bar{\rho} = - \lim_{t\to\infty} \frac{1}{t}\int_0^t dt'\, e^{-i(kx' - \omega t')}\int_0^{t'} dt''\, \rho_0 \frac{\partial v_x}{\partial x''} = \frac{k}{\omega}\,\bar{v}_x\rho_0 . \tag{3.19}$$

Substituting eq.(3.19) into

$$W_2 = \frac{eN}{2}\int d^3\vec{v}_0(0)\ \rho_0 \lim_{t\to\infty}\frac{1}{t}\int_0^t \frac{1}{c}\, v_x v_{0z} B^* dt' \ ,$$

and taking account of $B = -(ck/\omega)E$, we can rewrite W_2 in the form

$$W_2 = -\frac{eN}{2}\,\overline{E}^* \int d^3\vec{v}_0(0)\,\overline{\rho}\, v_{0z} \ . \tag{3.20}$$

Equation (3.20) reveals that W_2 comes from the current along the z-axis arising from the density perturbation. It is noted that to second order of λ the value of W_2 is independent of whether $v_{0x}(t)$ is treated as constant or not. In his treatment, Fried neglected the effect of gyration in deriving the formula corresponding to W_2.

On the other hand, if we neglect the effect of the gyrating motion, that is, we regard $v_{0x}(t)$ as constant, then eq.(3.13) reduces to

$$W_1 = \frac{i\omega_p^2}{8\pi\omega}\,|\overline{E}|^2\,\frac{1}{1 + i\,\frac{\nu}{\omega}} \ , \tag{3.21}$$

which is independent of both $\vec{B}_0$ and $\vec{B}$ and just the result obtained in the cold theory. For $\nu = 0$, the sum of eqs.(3.15) and (3.21) divided by $\frac{1}{2}|\overline{E}|^2$ leads to the conductivity obtained by Fried (eq.(10) in ref. 2) provided that we put $T/m = a^2/6$.

From the above discussions, we can conclude that the effects of the unperturbed gyrating motion as well as the density perturbation induced by the wave magnetic field are equally responsible for the anomalous propagation.

§4. Dispersion Relation and Collisional Absorption

In the above section, on the basis of the single-electron equation of motion we have explained the physical mechanism of the anomalous propagation and given almost the same expression of the conductivity for $\omega \sim \Omega$ as that obtained by the Boltzmann equation.

In this section, by making use of the conductivity derived from the Boltzmann equation, we examine the dispersion relation $(kc)^2 = \omega^2 + 4\pi i\sigma\omega$, which is written as

$$(\frac{kc}{\omega})^2 = \{1 - (\frac{\omega_p}{\omega})^2\}\{1 + \frac{\frac{T}{mc^2}(\frac{\omega_p}{\omega})^2}{1 - (\frac{\Omega}{\omega})^2}\}^{-1} , \qquad (4.1)$$

where $\nu = 0$ is assumed.

Figure 1 which is depicted according to eq.(4.1) shows the regions of propagation and evanescence for the plasma parameters. That is, when

$$(\frac{\Omega}{\omega})^2 < 1 \quad \text{or} \quad (\frac{\Omega}{\omega})^2 > 1 + \frac{T}{mc^2} (\frac{\omega_p}{\omega})^2 \quad \text{for} \quad \omega^2 > \omega_p^2 , \tag{4.2a}$$

and when

$$1 < (\frac{\Omega}{\omega})^2 < 1 + \frac{T}{mc^2} (\frac{\omega_p}{\omega})^2 \quad \text{for} \quad \omega^2 < \omega_p^2 , \tag{4.2b}$$

the wave electric field cannot be fully shielded, so that the wave can propagate.
This is also seen from Fig. 2 which shows the polarizability given by

$$\alpha = - \frac{1}{4\pi} \frac{(\frac{\omega_p}{\omega})^2 \{ 1 - (\frac{\Omega}{\omega})^2 + \frac{T}{mc^2} \}}{1 - (\frac{\Omega}{\omega})^2 + \frac{T}{mc^2} (\frac{\omega_p}{\omega})^2} , \tag{4.2}$$

and is drawn for constant $(\omega_p/\omega)^2$ larger than unity.

Last we examine the absorption of wave energy by collisions. The rate of absorption of wave energy by the plasma is given by Re W and approximately written for $\omega \sim \Omega$

$$\mathrm{Re}\, W = \frac{\nu\omega_p^2|\overline{E}|^2}{8\pi\omega^2}\left\{ 1 + \frac{\frac{T}{2m}\left(\frac{k}{\Omega}\right)^2}{\left(1 - \frac{\Omega}{\omega}\right)^2} \right\} , \qquad (4.3)$$

where we have assumed $\nu \ll |\omega-\Omega|$ and used the dispersion relation. If we use the result obtained in the above section, the second term in the bracket on the right of eq.(4.3) becomes twice. For the waves in the anomalous propagation band, the second term becomes much larger than the first term. However, it should be noted that the energy absorption never exists for $\nu = 0$.

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References

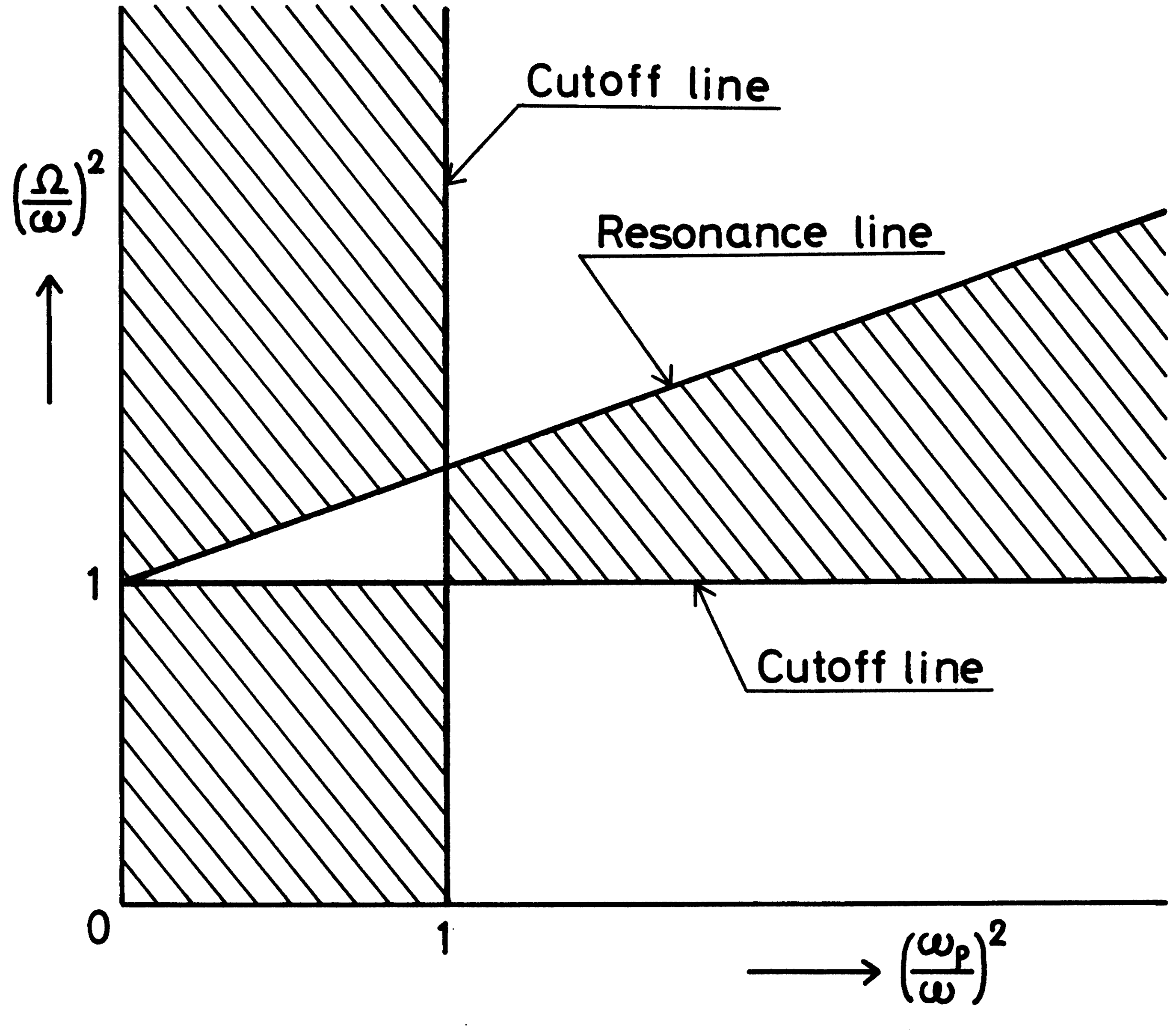
1) J. E. Drummond: Phys. Rev. 110 (1958) 293.

2) E. D. Fried: Research Report of the Institute of Plasma Physics, Nagoya University, Nagoya, IPPJ-130 June 1972.

Figure Captions

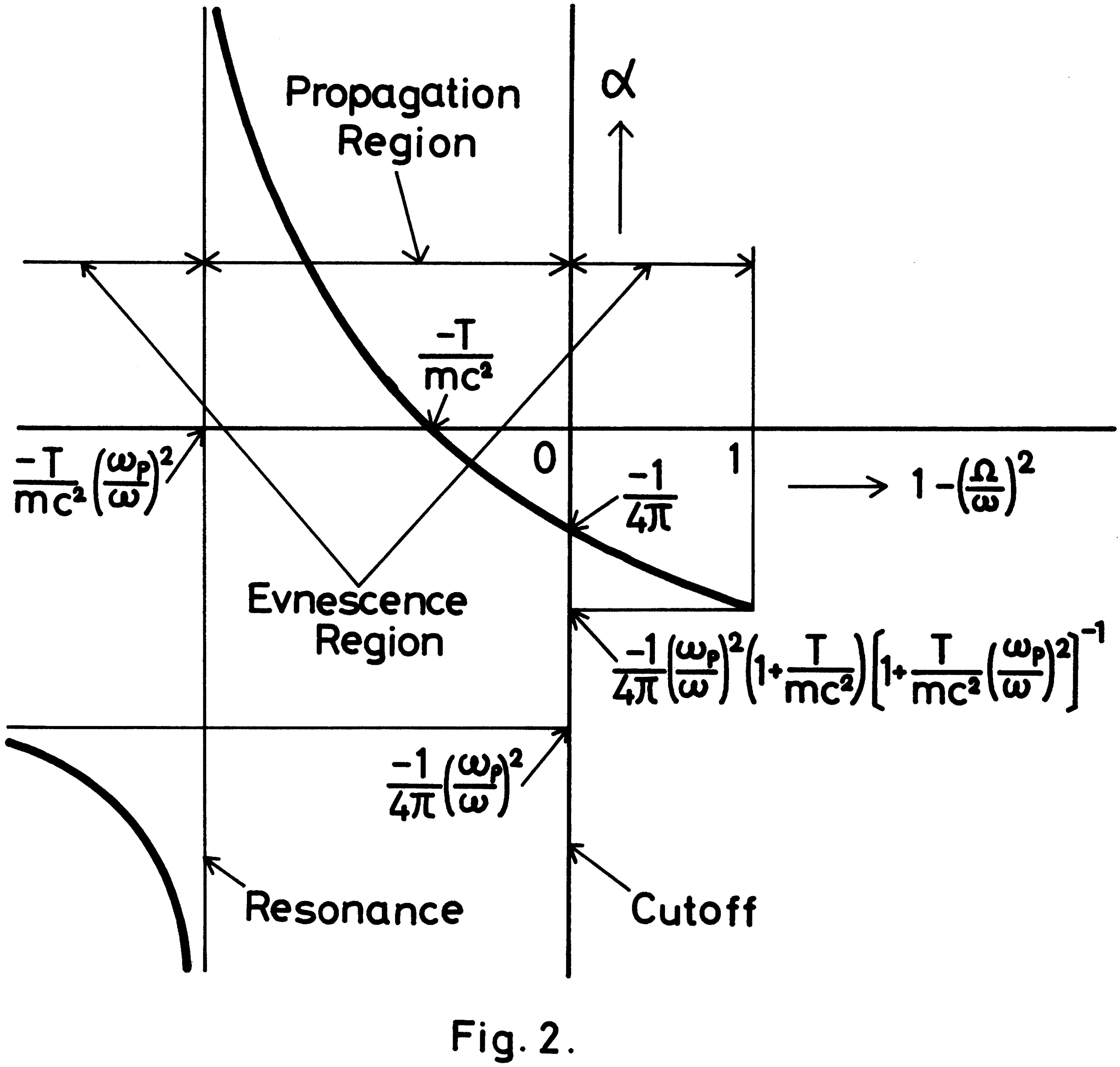
Fig. 1 Schematic plot of $(\Omega/\omega)^2$ vs $(\omega_p/\omega)^2$ shows the propagation regions which are shaded. The resonance line is given by $(\Omega/\omega)^2 = 1 + (T/mc^2)(\omega_p/\omega)^2$.

Fig. 2 Schematic plot of plasma polarizability α given by eq. (4.2) vs $1 - (\Omega/\omega)^2$, where $\omega^2 < \omega_p^2$.



Cutoff line
Resonance line
Cutoff line
$\left(\frac{\Omega}{\omega}\right)^2$
1
0
1
$\left(\frac{\omega_p}{\omega}\right)^2$

Fig. 1.



Propagation Region
α
$\frac{-T}{mc^2}$
$\frac{-T}{mc^2}\left(\frac{\omega_p}{\omega}\right)^2$
0
1
$1-\left(\frac{\Omega}{\omega}\right)^2$
$\frac{-1}{4\pi}$
Evnescence Region
$\frac{-1}{4\pi}\left(\frac{\omega_p}{\omega}\right)^2\left(1+\frac{T}{mc^2}\right)\left[1+\frac{T}{mc^2}\left(\frac{\omega_p}{\omega}\right)^2\right]^{-1}$
$\frac{-1}{4\pi}\left(\frac{\omega_p}{\omega}\right)^2$
Resonance
Cutoff

Fig. 2.