INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

NAGOYA, JAPAN

Anomalous Penetration of an Ordinary Wave into a Magnetoplasma Slab

Akio Ishida* and Kazuo Kitao**

IPPJ-154 February 1973

Further communication about this report is to be sent to the Research Information Cneter, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

---

* Present Address: Department of Physics, Hiroshima University, Hiroshima

** Permanent Address: Department of Physics, Niigata University, Niigata

Abstract

The penetration of an ordinary wave into an infinite magnetoplasma slab is calculated by adopting the image-metal technique in treating the electrons reflected at the plasma boundaries. After general expressions are derived, the wave frequency is assumed to be near the electron cyclotron frequency because the influence of the thermal motion of electrons is remarkable only for this case. It is found that the evanescent wave penetrates deeply into the slab compared with the case of a cold plasma, and the size effect appears due to the increase of the wave reflection at the plasma-vacuum boundary. The absorptive power due to collisions, evaluated for the limiting case of infinite thickness of the slab, is much larger than the value for a cold plasma and shows the surface effect. These two effects becomes more noticeable for the higher density and higher temperature of the plasma.

§1. Introduction

Although an ordinary wave whose frequency is below the plasma frequency cannot propagate into a cold plasma, under some conditions it can propagate into a hot magnetoplasma. This is well known as the anomalous propagation of an ordinary wave.[1] However, the penetration problem of an ordinary wave into a bounded plasma has not yet been investigated. In the present paper, therefore, the penetration of ordinary waves into an infinite magnetoplasma slab is studied, particularly for the wave frequencies near the electron cyclotron frequency because the influence of the thermal motion of electrons is significant only in this case.

Weibel[2] studied the conditions of the anomalous skin effect for an electromagnetic wave normally incident on a semi-infinite plasma without a magnetostatic field. His analysis was extended by Drummond[3] to the penetration of a whistler wave. Blevin et al.[4] investigated the case of a plasma slab without a magnetostatic field. In these three works, the specular reflection was assumed for the electrons reflected at the plasma boundary. The application of the specular reflection to the present case makes the calculation too complex. To make the calculation straightforward, we adopt the "image-metal technique" which is commonly used in metal physics.[5]

The phase relation between the electrons reflected at the boundary and the wave is different from that for the gyrating electrons without reflection. Therefore, the spatial variation of the amplitudes of the wave fields in a thin surface layer whose thickness is of the order of the mean Larmor radius differs from that in the interior of the plasma, and the surface effect, for example, for the absorptive power may be expected, especially for high density and high temperature. In the same reason, the usual definition of the penetration depth by the slope of the electric field at the plasma surface is not adequate, and the other reasonable definition should be adopted.

When the wave frequencies are very near but above the electron cyclotron frequency, the evanescent waves penetrate anomalously into the plasma slab with high density and high temperature, and hence the size effect appears due to the increase of the wave reflection at the boundary (see §5). This size effect is different from that owing to the transit time resonance in ref. 4.

In §2, the basic equations are solved and the formal expression of the current density is derived. The expressions of the electric field inside the slab, the coefficients of reflection and transmission, and the absorptive power are derived in §3. In §4, the dispersion relation is solved by making use of an expansion in powers of the ratio of the mean Larmor radius to the slab thickness. By use of the solutions of the dispersion relation, some numerical results are given and discussed in §5.

§2. Basic Equations and Current Density

We assume an infinite plasma slab which extends from $x = 0$ to $x = d$ with a uniform magnetostatic field $\underset{\sim}{B}_0$ along the z-axis, and has a uniform electron density N and a uniform electron temperature T. An ordinary wave normally incident upon the plasma surface from the vacuum $(x < 0)$ is considered, the electric field $E(x)\exp(-i\omega t)$ being parallel to $\underset{\sim}{B}_0$ (see Fig. 1). The wave frequency ω is assumed to be much higher

----

Fig. 1

----

than the ion cyclotron and ion plasma frequencies, so that the motion of ions is neglected. The wave amplitude is assumed small enough to treat the perturbation $f_1(x,\underset{\sim}{v},t)$ of the distribution function by a linear approximation.

$$\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} + (\underset{\sim}{\Omega} \times \underset{\sim}{v}) \cdot \nabla_{\underset{\sim}{v}} f_1 + \nu f_1 = \frac{e}{m} E(x) e^{-i\omega t} \frac{\partial f_0}{\partial v_z}, \qquad (2.1)$$

where m and $-e$ are the electron mass and charge respectively, ν is the effective collision frequency independent of the velocity, $\Omega = eB_0/mc$ is the electron cyclotron frequency and $\underset{\sim}{\Omega} = \underset{\sim}{e}_z \Omega$, and $f_0 = N(m/2\pi T)^{3/2} \exp(-m\underset{\sim}{v}^2/2T)$.

The formal solution of eq. (2.1) is written as $f_1(x,\underset{\sim}{v},t) = f_1(x,\underset{\sim}{v}) \exp(-i\omega t)$, where

$$f_1(x,\underset{\sim}{v}) = \frac{e}{m} \frac{\partial f_0}{\partial v_z} \int_0^\infty E(x') \exp[-(\nu - i\omega)s]\, ds, \qquad (2.2)$$

where x' is the x-coordinate of the unperturbed orbit at time

$t' = t - s$ and given by

$$x' = x - \frac{v_x \sin\Omega s}{\Omega} - \frac{v_y(1 - \cos\Omega s)}{\Omega} , \tag{2.3}$$

v_x, v_y, and v_z being the velocity components at time t.

The Maxwell equations give

$$\frac{d^2E(x)}{d\,x^2} + \left(\frac{\omega}{c} \right)^2 E(x) = - \frac{4\pi i \omega}{c^2} j(x) , \tag{2.4}$$

where $j(x)\exp(- i\omega t)$ is the current density along the z-axis and

$$j(x) = - e \int v_z\, f_1(x,\underset{\sim}{v})\, d^3\underset{\sim}{v} . \tag{2.5}$$

In performing the integration of eq.(2.2) along the unperturbed orbit, we apply the "image-metal technique", that is, we assume that the electron velocity changes from (v_x,v_y, v_z) to ($-v_x$,$-v_y$,v_z) at the plasma boundaries.[5] While application of the other conditions for electrons reflected from the boundaries, such as specular or diffuse relfection, makes the present analysis very difficult, the calculation becomes straightforward by use of the image-metal technique provided that E(x) is specified to be symmetric at the boundaries. Then we can regard the electron reflected at the boundary as if it came from the vacuum into the plasma slab. And we can expand E(x) in a Fourier cosine series

$$E(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{d}\right) , \tag{2.6}$$

and hence we obtain

$$\frac{d^2E(x)}{d\,x^2} = -\frac{E'(0) - E'(d)}{d} - \sum_{n=1}^{\infty} [\frac{2}{d}\{E'(0) - (-1)^n E'(d)\}$$

$$+ (\frac{n\pi}{d})^2 a_n] \cos(\frac{n\pi x}{d}) , \qquad (2.7)$$

where

$$E'(0) = \lim_{x\to+0} \frac{dE(x)}{dx} , \qquad E'(d) = \lim_{x\to d-0} \frac{dE(x)}{dx} . \qquad (2.8)$$

Substituting eq.(2.2) into eq.(2.5), and integrating over v_z, we obtain

$$j(x) = C \int_0^{\infty} \exp(-\frac{mv_\perp^2}{2T}) v_\perp dv_\perp \int_0^{2\pi} d\phi \int_0^{2\pi} E(x') \exp(-\frac{\nu - i\omega}{\Omega}\Theta) d\Theta , \qquad (2.9)$$

where $\Theta = s\Omega$, $v_x = v_\perp\cos\phi$, $v_y = v_\perp\sin\phi$, and

$$C = \frac{e^2N}{2\pi T\Omega} [1 - \exp\{-\frac{2\pi(\nu - i\omega)}{\Omega}\}]^{-1} . \qquad (2.10)$$

Substituting eq.(2.6) into eq.(2.9), and carrying out the integration over Θ, ϕ, and $v_\perp$, we obtain

$$j(x) = \sigma(0,\omega)\frac{a_0}{2} + \sum_{n=1}^{\infty} \sigma(n,\omega)\, a_n \cos(\frac{n\pi x}{d}) , \qquad (2.11)$$

where $\sigma(n,\omega)$ is the electric conductivity and expressed as

$$\sigma(n,\omega) = \frac{\omega_p^2}{4\pi} \sum_{\ell=-\infty}^{\infty} \frac{\exp(-b_n)\, I_\ell(b_n)}{\nu - i(\omega+\ell\Omega)} , \quad (n = 0, 1, 2, \cdots) , \qquad (2.12)$$

where $\omega_p^2 = 4\pi Ne^2/m$ and $b_n = (T/m\Omega^2)(n\pi/d)^2$. For $T = 0$, eq.(2.12) is reduced to the result for an infinite cold plasma.

§3. Wave Fields within the Plasma

Substituting eqs.(2.6), (2.7) and (2.11) into eq.(2.4), and setting the coefficient of $\cos(n\pi x/d)$ equal to zero, we obtain the Fourier coefficients

$$a_n = -\frac{2}{d}\frac{E'(0) - (-1)^n E'(d)}{D(n,\omega)}, \quad (n = 0, 1, 2, \cdots), \qquad (3.1)$$

where

$$D(n,\omega) = \left(\frac{n\pi}{d}\right)^2 - \left(\frac{\omega}{c}\right)^2 - \frac{4\pi i\omega\sigma(n,\omega)}{c^2}. \qquad (3.2)$$

If we identify $n\pi/d$ with the wavenumber k, the equation, $D(n,\omega) = 0$, gives the usual dispersion relation for an ordinary wave in an infinite magnetoplasma.[6)]

Because the image-metal technique does not allow any surface current and any surface density, the tangential components of the electromanetic fields must be contineous at the plasma boundaries

$$\begin{aligned} &E(-0) = E(+0), \qquad B(-0) = B(+0), \\ &E(d-0) = E(d+0), \qquad B(d-0) = B(d+0), \end{aligned} \qquad (3.3)$$

where $B(x)\exp(-i\omega t)$ is the wave magnetic field. In the vacuum region where $x < 0$, the wave electric and magnetic fields are written

$$E(x) = E_I e^{i\omega x/c} + E_R e^{-i\omega x/c}, \qquad (3.4)$$

$$B(x) = -E_I e^{i\omega x/c} + E_R e^{-i\omega x/c}, \qquad (3.5)$$

where E_I and E_R are the amplitudes of the incident and reflected waves, respectively. In the vacuum region where $x > d$, we write

$$E(x) = E_T e^{i\omega x/c}, \tag{3.6}$$

$$B(x) = - E_T e^{i\omega x/c}, \tag{3.7}$$

where E_T is the amplitude of the transmitted wave.

The four boundary conditions given by eq.(3.3) yield

$$E_T = \frac{1}{2}\{\frac{cE'(0)}{i\omega} + \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\}, \tag{3.8}$$

$$E_R = -\frac{1}{2}\{\frac{cE'(0)}{i\omega} - \frac{1}{2}a_0 - \sum_{n=1}^{\infty} a_n\}, \tag{3.9}$$

$$E_T = \exp(-i\frac{\omega d}{c})\{\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (-1)^n a_n\}, \tag{3.10}$$

$$E'(d) = \frac{i\omega}{c}\{\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (-1)^n a_n\}. \tag{3.11}$$

Substitution of eq.(3.1) into eq.(3.11) determines the relation between $E'(0)$ and $E'(d)$, that is,

$$E'(d) = \frac{Q}{P + \frac{ic}{\omega}} E'(0), \tag{3.12}$$

where

$$P = \frac{1}{D(0,\omega)d} + \frac{2}{d}\sum_{n=1}^{\infty} \frac{1}{D(n,\omega)},$$

$$Q = \frac{1}{D(0,\omega)d} + \frac{2}{d}\sum_{n=1}^{\infty} \frac{(-1)^n}{D(n,\omega)}. \tag{3.13}$$

Thus, substitution of eqs.(3.1) and (3.12) into eq.(2.6) gives the electric field inside the slab

$$\frac{E(x)}{E(0)} = [(P + \frac{ic}{\omega}) \{ \frac{1}{D(0,\omega)d} + \frac{2}{d} \sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi x}{d})}{D(n,\omega)} \} - Q \{ \frac{1}{D(0,\omega)d}$$

$$+ \frac{2}{d} \sum_{n=1}^{\infty} \frac{(-1)^n \cos(\frac{n\pi x}{d})}{D(n,\omega)} \}] \{P(P + i\frac{c}{\omega}) - Q^2\}^{-1} . \tag{3.14}$$

In particular, we have

$$\frac{E(d)}{E(0)} = \frac{i\frac{c}{\omega} Q}{P(P + i\frac{c}{\omega}) - Q^2} . \tag{3.15}$$

From eqs.(2.6), (3.1) and (3.12) we also obtain

$$\frac{E'(0)}{E(0)} = - \frac{P + i\frac{c}{\omega}}{P(P + i\frac{c}{\omega}) - Q^2} , \qquad \frac{E'(d)}{E(d)} = i\frac{\omega}{c} , \tag{3.16}$$

Equations (3.8)-(3.10) yield the coefficients of reflection and transmission for the amplitudes, which are denoted by γ_R and γ_T respectively,

$$\gamma_R = \frac{E_R}{E_I} = \frac{P^2 - Q^2 + (\frac{c}{\omega})^2}{(P + i\frac{c}{\omega})^2 - Q^2} , \tag{3.17}$$

$$\gamma_T = \frac{E_T}{E_I} = \frac{\frac{2ic}{\omega} Q \exp(-i\frac{\omega d}{c})}{(P + i\frac{c}{\omega})^2 - Q^2} . \tag{3.18}$$

From eqs.(2.6) and (2.11) we obtain the power absorbed by the plasma slab

$$W = \frac{d}{8}\left[\frac{1}{2}\{\sigma(0,\omega) + \sigma^*(0,\omega)\}|a_0|^2 + \sum_{n=1}^{\infty}\{\sigma(n,\omega) + \sigma^*(n,\omega)\}|a_n|^2\right], \tag{3.19}$$

which is expressed as the value per unit cross section of the slab. Then the absorptive power is given by

$$\alpha = \frac{W}{\frac{c}{8\pi}|E_I|^2}$$

$$= \frac{-\frac{2ic}{\omega}\left[\{|P|^2+(\frac{c}{\omega})^2\}(P-P^*) - i\frac{c}{\omega}(P-P^*)^2 + i\frac{c}{\omega}(Q-Q^*)^2 + (PQ^{*2}-P^*Q^2)\right]}{|P + Q + i\frac{c}{\omega}|^2 \; |P - Q + i\frac{c}{\omega}|^2} \tag{3.20}$$

where the asterisk denotes the complex conjugate quantities. Naturally we have $\alpha + |\gamma_R|^2 + |\gamma_T|^2 = 1$. If P and Q are real, then we have $\alpha = 0$, that is, the plasma is nondissipative.

Introducing X and Y defined by

$$X = \frac{1}{d^2}\sum_{m=1}^{\infty}\frac{1}{D(2m,\omega)}, \qquad Y = \frac{1}{d^2}\sum_{m=1}^{\infty}\frac{1}{D(2m-1,\omega)}, \tag{3.21}$$

we can express P and Q as follows:

$$P = \frac{1}{D(0,\omega)d} + 2d(X+Y), \qquad Q = \frac{1}{D(0,\omega)d} + 2d(X-Y). \tag{3.22}$$

We also introduce the dispersion function defined by

$$g(z) = z^2 - (\frac{\omega d}{c})^2 + (\frac{\omega_p d}{c})^2 \sum_{\ell=-\infty}^{\infty} \frac{I_\ell(b_n)\exp(-b_n)}{1 + \ell\frac{\Omega}{\omega} + i\frac{\nu}{\omega}} , \quad (3.23)$$

where $b_n = Tz^2/md^2\Omega^2$. If we put $z = kd$, $g(z)$ becomes the dispersion function for an infinite magnetoplasma.

Since $D(-n,\omega) = D(n,\omega)$ and $g(n\pi) = D(n,\omega)d^2$, we can express X and Y in the integral form

$$X = \frac{1}{4\pi}\int_C \frac{dz}{(e^{iz} - 1)g(z)} , \quad Y = -\frac{1}{4\pi}\int_C \frac{dz}{(e^{iz} + 1)g(z)} , \quad (3.24)$$

where the contour C on the z-plane is illustrated in Fig. 2. Here we have assumed that the dispersion relation $g(z) = 0$ has no solution inside the contour C.

Fig. 2

The dispersion relation $g(z) = 0$ has in general an infinite number of solutions, which are denoted by z_r. By making use of the theorem of residues, we can write

$$X = -\frac{i}{2}\left[\sum_r \frac{1}{\{\exp(iz_r) - 1\}g'(z_r)} + \frac{1}{ig(0)}\right] ,$$

$$Y = \frac{i}{2}\sum_r \frac{1}{\{\exp(iz_r) + 1\}g'(z_r)} \quad (3.25)$$

§4. Dispersion Relation

As we have seen in §3, we can derive information on the wave penetration from the dispersion function g(z) defined by eq.(3.23). In this section, therefore, the properties of g(z) are studied. We are concerned only with the waves whose frequencies are near the electron cyclotron frequency and satisfy $\nu \ll |\omega - \Omega|$. Assuming $b_n \ll 1$, we expand g(z) as a power series of b_n

$$g(z) = z^2 - (\frac{\omega d}{c})^2 + (\frac{\omega_p d}{c})^2 \left[1 - i\frac{\nu}{\omega} + \frac{(\frac{\Omega}{\omega})^2 b_n}{1 - (\frac{\Omega}{\omega})^2} \{1 - i\frac{\nu}{\omega} \frac{1 + (\frac{\Omega}{\omega})^{-2}}{1 - (\frac{\Omega}{\omega})^2} \} + \dots \right] . \tag{4.1}$$

The zeroth-order solutions of g(z) = 0 are denoted by $\pm z_\nu$ and given by

$$z_\nu = z_0 (1 + i\Delta) . \tag{4.2}$$

Here z_0 is the solution for $\nu = 0$

$$z_0 = \frac{\omega d}{c} \left\{ \frac{1 - (\frac{\omega_p}{\omega})^2}{1 + \beta} \right\}^{\frac{1}{2}} , \tag{4.3}$$

and the correction term due to collisions, denoted by Δ, is given by

$$\Delta = \frac{\beta}{1 + \beta} \frac{\frac{\nu}{\omega}}{1 - (\frac{\Omega}{\omega})^2} , \tag{4.4}$$

where we have put

$$\beta = \frac{\frac{T}{mc^2}\left(\frac{\omega_p}{\omega}\right)^2}{1-\left(\frac{\Omega}{\omega}\right)^2}, \tag{4.5}$$

and approximated as $1 + (\Omega/\omega)^2 = 2$ in deriving Δ. From eq.(4.1) then we have

$$g'(\pm z_\nu) = \pm 2z_0(1+\beta)(1-i\Delta) \ . \tag{4.6}$$

The above expansions are allowable under the following conditions

$$(T/md^2\Omega^2)\ |z_0|^2 \ll 1 \qquad \text{and} \qquad |\Delta| \ll 1. \tag{4.7}$$

It is also noted that the above results obtained from the expansion of eq.(4.1) are not valid for $\omega \sim 2\Omega$.

Equation (4.3) is equivalent to the dispersion relation for an infinite magnetoplasma and reduced to eq.(3.4.2) of ref.1 if we put $n\pi/d = k$, that is, $z_0 = kd$. A real value of z_0 means undamped propagation of an ordinary wave, whereas a purely imaginary value of z_0 means evanescence of the wave. It is found from eq.(4.3) that in $(\Omega/\omega)^2 - (\omega_p/\omega)^2$ diagram (Fig. 3) there exists also an evanescent region V for $\omega > \omega_p$ while there exists a propagation region II for $\omega < \omega_p$ (this is called the anomalous propagation[1]).

Fig. 3

These results are unexpected in the cold plasma approximation. The physical picture for this anomalous propagation was recently examined by Fried[7] and more correctly by the present authors.[8]

Substitution of eqs.(3.25) and (4.6) into eq.(3.22) leads to

$$P = - \frac{d(1 + i\Delta)}{z_0(1 + \beta)} \cot z_\nu \, , \tag{4.8}$$

$$Q = - \frac{d(1 + i\Delta)}{z_0(1 + \beta)} \operatorname{cosec} z_\nu \, , \tag{4.9}$$

where we have only taken $\pm z_\nu$ as the solutions of $g(z) = 0$ because the other solutions make only minor contributions to X and Y (eq.(3.24)). When $\nu = 0$, both P and Q are real and $\alpha = 0$, and hence $|\gamma_R|^2 + |\gamma_T|^2 = 1$.

§5. Numerical Results and Discussion

The penetration of the ordinary waves is investigated in more detail by use of the results obtained in the previous sections. In what follows, although some general expressions are derived, we have more interest on the evanescent waves and the numerical calculations are confined to the evanescent cases A, B, and C which satisfy the condition (4.7) (see Table I).

Table I.

Since the analysis becomes simple for $(\omega_p/\omega)^2 \gg 1$ and for the limit of $d \to \infty$, the following expressions are derived on these conditions. Substituting eqs.(4.8) and (4.9) into eq.(3.16), we obtain

$$\frac{E'(0)}{E(0)} = \frac{iz_0|1+\beta|}{d(1+i\Delta)}, \qquad \text{in regions I, II, III.} \qquad (5.1)$$

When $\nu = 0$, eq.(5.1) is negative for the evanescent waves (regions I and III) and purely imaginary for the propagated waves (region II).

Let us define the penetration depth by $\delta = d/\ln|E(0)/E(d)|$. From eq.(3.15), then we have

$$\delta_\infty = \begin{cases} \dfrac{d}{z_0|\Delta|} > 0, & \text{in region II,} \qquad (5.2a) \\ \dfrac{d}{|z_0|} = \dfrac{c}{\omega_p}(1+\beta)^{1/2} > 0, & \text{in regions I, III,} \qquad (5.2b) \end{cases}$$

where the suffix ∞ means the limiting value for $d \to \infty$. Equations (5.2a) and (5.2b) agree with the results for a semi-infinite case. In region II, we have $\delta_\infty \to \infty$ as $\nu \to 0$. In regions I and III, the factor $(1 + \beta)^{1/2}$ gives the deviation from the penetration depth for collisionless cold plasmas, $\delta_c = c/\omega_p$. Thus, if $(\Omega/\omega)^2 \to 1\text{-}0$ and N and T increase, the evanescent waves penetrate anomalously into the plasma. It is noted that we have $\delta_\infty/\delta_c = 4.6$ in case A.

By use of eq.(5.2b), for $\nu = 0$ eq.(5.1) is rewritten as

$$\left|\frac{E'(0)}{E(0)}\right| = \frac{1 + \beta}{\delta_\infty}, \quad \text{in regions I, III,} \tag{5.3}$$

which increases with $(1 + \beta)^{1/2}$. From eqs.(5.2b) and (5.3) we can say that for large values of $(1 + \beta)$ the electric field decreases steeply near the surface and then decreases slowly to penetrate deeply into the plasma. This is the reason why we have defined the penetration depth by $\delta = d/\ln|E(0)/E(d)|$, differently from the usual definition. Such spatial variation of E(x) comes from the fact that the phase relation between the electrons reflected at the boundary and the wave differs from that for the gyrating electrons without reflection, and the effect of this difference is more remarkable for higher density and higher temperature (cf. the discussion in ref.8). Therefore, such a boundary region extends only to a distance of order r_L from the surface, where r_L is the mean Larmor radius of electrons.

Substitution of eqs.(4.8) and (4.9) into eq.(3.20) yields

$$\alpha = -\frac{4\omega}{c}\,\mathrm{Im}\left(\frac{E(0)}{E'(0)}\right) = \begin{cases} \dfrac{4\omega}{c}\dfrac{d}{z_0}\dfrac{1}{|1+\beta|} > 0, \text{ in region II,} & (5.4a) \\[2ex] \dfrac{4\omega}{c}\dfrac{d}{|z_0|}\dfrac{\Delta}{1+\beta} > 0, \text{ in regions I, III.} & (5.4b) \end{cases}$$

For $\nu = 0$, eq.(5.4a) is not applicable, and $\alpha = 0$ as mentioned in §4. In the evanescent regions I and III, the value of α is propotional to ν because δ_∞ is independent of ν, and the relation $\alpha + |\gamma_R|^2 = 1$ is satisfied. The numerical values of α/α_c are shown in Table I, where $\alpha_c = 2\nu/\omega$ is the absorptive power for cold plasmas. It is found that if the frequencies are nearly equal to the electron cyclotron frequency the magnetoplasma is strongly dissipative for the ordinary waves in the evanescent regions, particularly in case C.

Equation (5.4b) can be expressed in a different form by use of the result for a semi-infinite case (see Appendix)

$$\alpha = \frac{\alpha_\infty}{1+\beta}, \qquad (5.5)$$

where $\alpha_\infty = -(4\omega/c)\lim_{x\to\infty} \mathrm{Im}\, E(x)/E'(x)$, $E(x)$ being the value for a semi-infinite case. Since the surface effect is eliminated from α_∞, we can interprete that the factor $(1+\beta)$ indicates the surface effect owing to the image-metal approach, as in eq.(5.3). Whether the image-metal approach is a good approximation or not may be examined by the measurement of $E'(0)/E(0)$ or $|\gamma_R|^2 = 1 - \alpha$.

To show the size effect, for the finite values of d the numerical results of δ/δ_∞, $|E'(0)/E(0)|$, and $|\gamma_T|^2$ are depicted in Figs. 4, 5, and 6, respectively. It is found that the size effect appears in cases A and B, but does not appear in case C. Thus we can say that the size effect reveals itself as a result of large δ_∞/δ_c, consequently, of the increase of the wave reflection at $x = d$.

Figs. 4, 5, 6.

In ref. 4 where the relation, $\delta < d <$ the mean free path, is satisfied, however, the size effect comes from the transit time resonance, which occurs when the transit time of electrons through the slab equals a half-period of the wave.

In Fig. 6, the transmission coefficient of the intensities $|\gamma_T|^2$ shows the behaviour consistent with that in Fig. 4. The values of $E(x)$ at $x = \frac{1}{2}d$ and $x = d$ are given for $\omega_p d/c = 3$ in Table II.

Table II

Acknowledgements

One of the authors (A. I.) is grateful to Prof. K. Nishikawa, Dr. K. Kawamura, and Mr. K. Mima of Hiroshima University for their continual encouragement and helpful discussions.

Appendix A Semi-Infinite Magnetoplasma

Some calculations for a semi-infinite magnetoplasma are presented to elucidate the surface effect. Instead of the Fourier series expansion, the basic equations (2.1)-(2.5) for $E(x)$ within the plasma are solved by the Fourier transformation under the assumption $E(-x) = E(x)$. That is,

$$E(k,\omega) = \int_{-\infty}^{\infty} E(x)\, e^{-ikx}\, dx \,. \tag{A.1}$$

Then we have

$$\int_{-\infty}^{\infty} d^2E(x)/dx^2 \exp(-ikx)\, dx = -2E'(0) - k^2E(k,\omega) \,. \tag{A.2}$$

Following the similar procedure as in the text, we obtain the Fourier transform of the current density

$$j(k,\omega) = \sigma(k,\omega)\, E(k,\omega), \tag{A.3}$$

where

$$\sigma(k,\omega) = \frac{\omega_p^2}{4\pi} \sum_{\ell=-\infty}^{\infty} \frac{\exp(-b)\, I_\ell(b)}{\nu - i(\omega + \ell\Omega)} \tag{A.4}$$

and $b = Tk^2/m\Omega^2$.

Substitution of eqs.(A.1)-(A.3) into eq.(2.4) leads to

$$E(k,\omega) = -\frac{2E'(0)}{D(k,\omega)}, \tag{A.5}$$

where

$$D(k,\omega) = k^2 - \frac{\omega^2}{c^2} - \frac{4\pi i\omega}{c^2}\sigma(k,\omega). \tag{A.6}$$

After some calculations, the absorbed wave energy per unit cross section

$$W = \frac{1}{2}\,\mathrm{Re}\int_0^\infty j(x)\ E(x)^*dx = \frac{1}{4}\,\mathrm{Re}\int_{-\infty}^\infty j(x)\ E(x)^*dx \tag{A.7}$$

is written as

$$W = -\frac{c^2\ |E'(0)|^2}{8\pi\omega}\ \mathrm{Im}\left(\frac{E(0)}{E'(0)}\right). \tag{A.8}$$

Thus the absorptive power is given by

$$\alpha = \frac{W}{\frac{c}{8\pi}|E_I|^2} = \frac{-\frac{4c}{\omega}\ \mathrm{Im}\left(\frac{E(0)}{E'(0)}\right)}{\left|\frac{1}{\pi}\int_{-\infty}^\infty \frac{dk}{D(k,\omega)} + i\frac{c}{\omega}\right|^2}, \tag{A.9}$$

Here the wave fields in the vacuum are written as eqs.(3.4) and (3.5). When $(\omega_p/\omega)^2 \ll 1$, eq.(A.9) is reduced to $\alpha = -(4\omega/c)\ \mathrm{Im}\ (E(0)/E'(0))$, which is just eq.(5.4).

The inverse Fourier transform is easily obtained from eqs.(A.5) and (A.6), and we have

$$\frac{E(0)}{E'(0)} = -\frac{i}{k_\nu(1+\beta)}, \qquad \lim_{x\to\infty}\frac{E(x)}{E'(x)} = -\frac{i}{k_\nu}, \tag{A.10}$$

where k_ν is the root of $D(k,\omega) = 0$ and equivalent to eq.(4.2)

Thus we have

$$\frac{E(0)}{E'(0)} = \frac{1}{1+\beta} \lim_{x\to\infty} \frac{E(x)}{E'(x)} . \qquad \text{(A.11)}$$

On the other hand, it is easily found that the absorptive power far enough from the plasma boundary is given by $\alpha_\infty = -(4\omega/c) \lim_{x\to\infty} \mathrm{Im}\,(E(x)/E'(x))$. Hence we obtain $\alpha/\alpha_\infty = 1/(1+\beta)$.

References

1) J. E. Drummond: Phys. Rev. 110 (1958) 293.

2) E. S. Weibel: Phys. of Fluids 10 (1967) 741.

3) J. E. Drummond: Phys. of Fluids 11 (1968) 1196.

4) H. A. Blevin, J. A. Reynolds, and P. C. Thonemann: Phys. of Fluids 13 (1969) 1259.

5) R. G. Chambers: The Physics of Metals 1. Electrons ed. J. M. Ziman (Camb. Univ. Press, 1969) Chap. 4.

6) Yu. N. Dnestrovskii and D. P. Kostmarov: Soviet-Physics JETP 13 (1970) 986.

7) E. D. Fried: Research Report of the Institute of Plasma Physics, Nagoya University, Nagoya, IPPJ-130 June 1972.

8) A. Ishida and K. Kitao: submitted to J. Phys. Soc. Japan.

Figure Captions

Fig. 1. Schematic diagram showing the geometry.

Fig. 2. Integration contour C in the z-plane of eq.(3.24).

Fig. 3. Schematic plot of $(\Omega/\omega)^2$ vs $(\omega_p/\omega)^2$ showing the evanescent regions I, III, V, and the propagation regions II, IV, VI. The plasma parameters of three cases A, B, and C are shown in Table 1.

Fig. 4. Penetration depth defined by $\delta = d/\ln|E(0)/E(d)|$ (normalized to δ_∞) vs the normalized slab thickness d/δ_c, where δ_∞ is the penetration depth for a semi-infinite case and $\delta_c = c/\omega_p$ is that for collisionless cold plasmas.

Fig. 5. Slope of the wave electric field at $x = 0$ vs the normalized slab thickness d/δ_c.

Fig. 6. Transmission coefficient of the intensities $|\gamma_T|^2$ vs the normalized slab thickness d/δ_c.

Table I. Comparison of the values of δ_∞/δ_c and α/α_c in three typical cases A, B, and C in the evanescent regions I and III.

CASE	T/mc^2	$(\omega_p/\omega)^2$	$(\Omega/\omega)^2$	ω_p^2	$1+\beta$	δ_∞/δ_c	α/α_c
A	10^{-4}	10^3	0.995	10^{21}	21.0	4.58	83
B	10^{-5}	10^3	0.995	10^{21}	3.0	1.73	150
C	10^{-5}	10^3	1.014	10^{21}	0.3	0.55	600

Table II. Comparison of the values of $E(d/2)/E(0)$ and $E(d)/E(0)$ in three typical cases A, B, C, and for the cold theory.

$c/\omega_p = 0.95$ cm, $\omega_p d/c = 3$.

CASE	$\lvert E(d/2)/E(0) \rvert$	$\lvert E(d)/E(0) \rvert$
A	0.87	0.82
B	0.49	0.35
C	0.06	0.01
Cold Theory	0.23	0.10

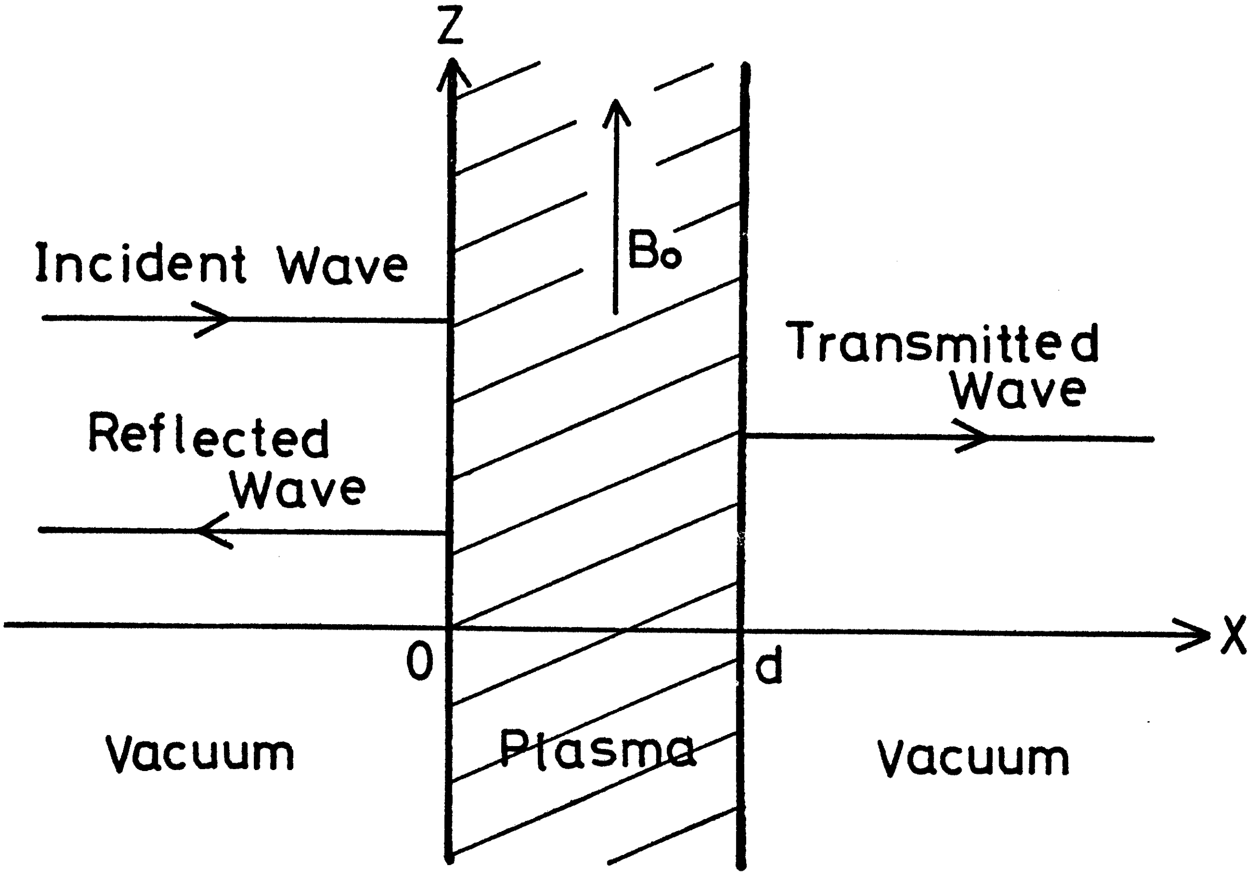

Z
Incident Wave
Reflected
Wave
B₀
Transmitted
Wave
0
d
X
Vacuum
Plasma
Vacuum

Fig. 1

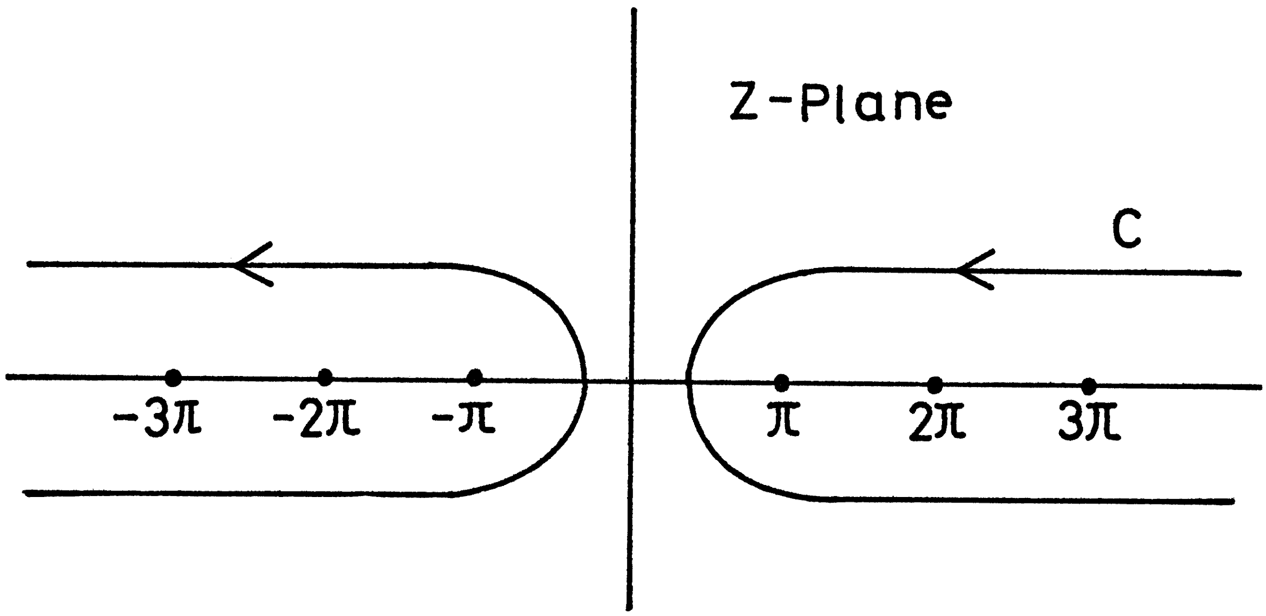
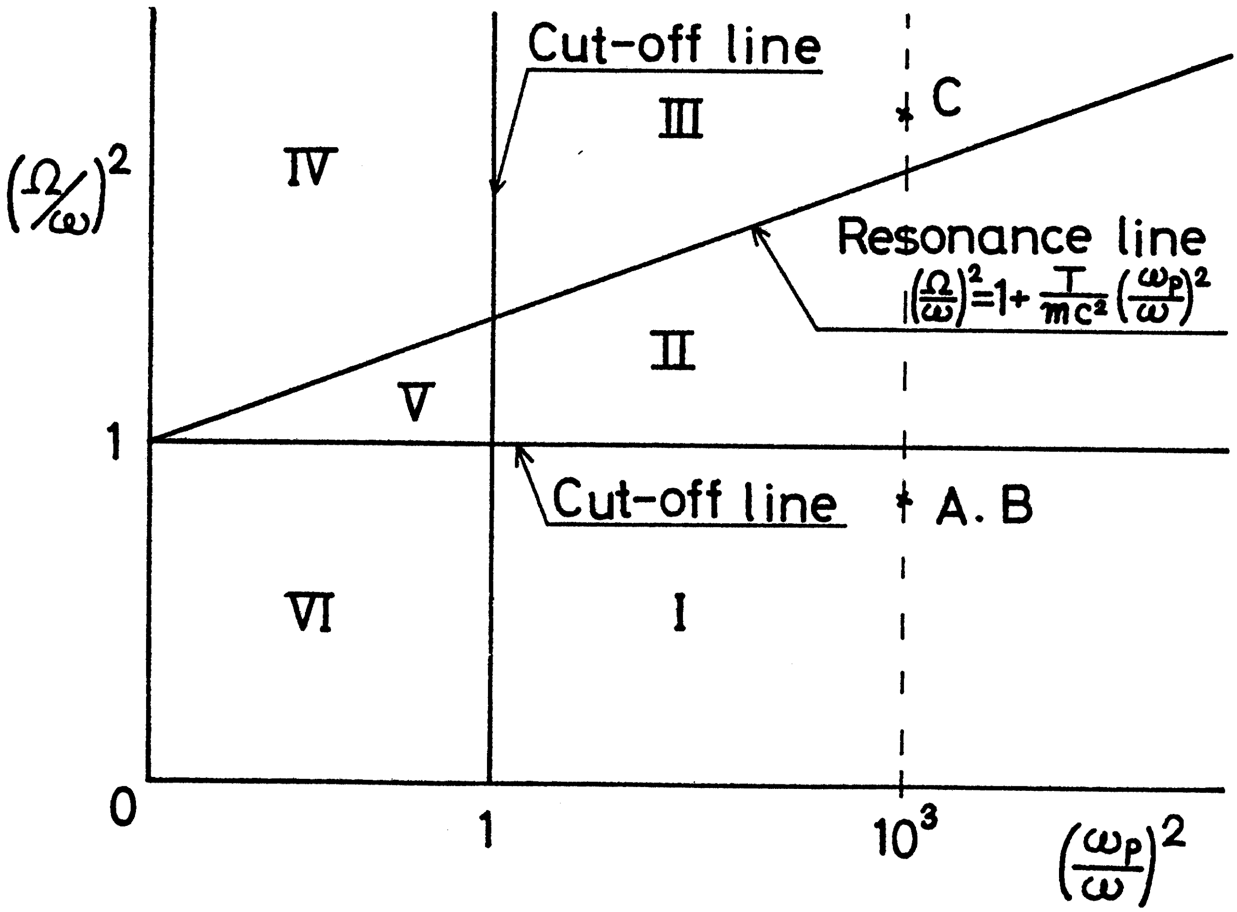


Fig. 2



$(\Omega/\omega)^2$
Cut-off line
III
C
IV
Resonance line
$\left(\frac{\Omega}{\omega}\right)^2 = 1 + \frac{T}{mc^2}\left(\frac{\omega_p}{\omega}\right)^2$
II
V
1
Cut-off line
A. B
VI
I
0
1
10^3
$\left(\frac{\omega_p}{\omega}\right)^2$

Fig. 3

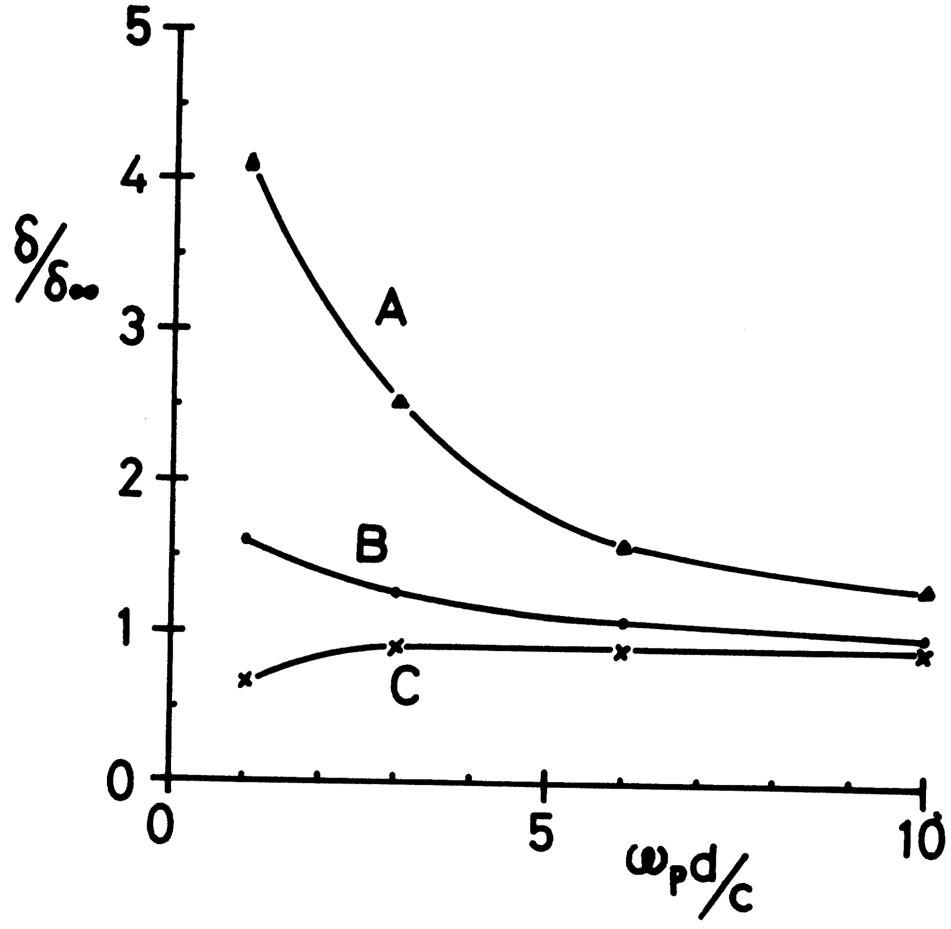


Fig. 4.

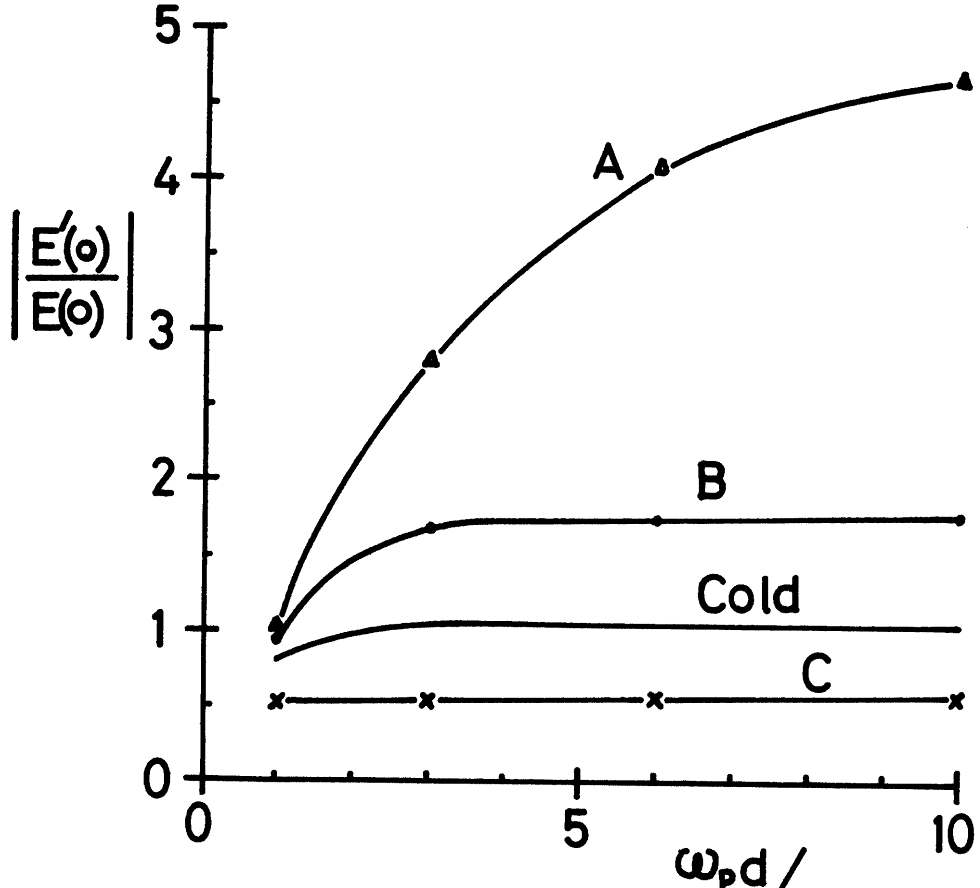


Fig. 5.

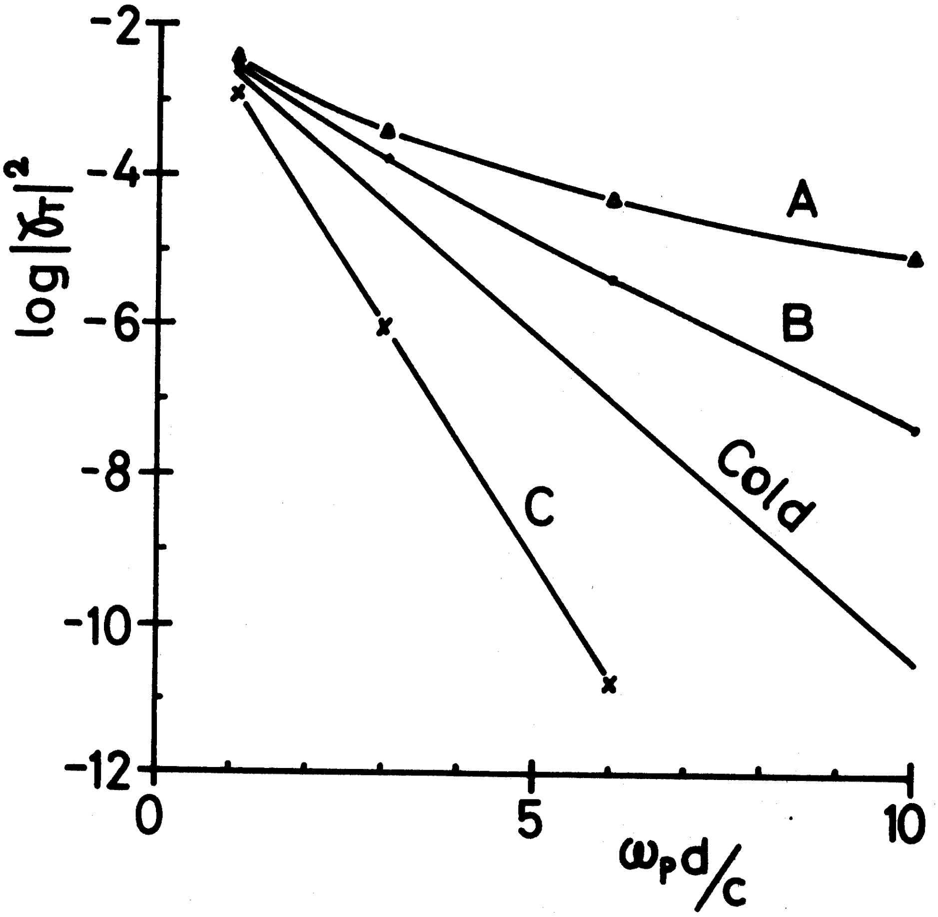


Fig. 6.