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RESEARCH REPORT

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Accessibility of Ion-Ion Hybrid Resonance

H. Toyama and K. Matsuura

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Abstract

The dispersion relation for the ion-ion hybrid resonance has been investigated in the density region of 10^6 cm^{-3} to 10^{17} cm^{-3} with the parallel wave number as the parameter, and the accessibility condition is presented.

§1. Introduction

Buchsbaum[1] has pointed out that when a high-density plasma column, coaxial with a static magnetic field, possesses two ion species with different charge-to-mass ratios, there exists a hybrid resonance. Several experiments on the ion-ion hybrid resonance have already been carried out.[2-7]

When this hybrid resonance is adopted as the heating system of the tokamak, the first problem is to excite the plasma wave at the ion-ion hybrid resonance. Gap excitation is, for example, one of the methods of coupling rf power to the plasma. The experimental plasma column has a certain density profile, so the problem of exciting the plasma at the hybrid resonance becomes one of launching energy into an inhomogeneous plasma from the low density edge. Accessibility to this resonance was discussed by Stix[8] who showed the accessibility condition to be as follows: accessibility is attained if ν^2(ν: perpendicular index of refraction computed from the homogeneous plasma dispersion relation) is positive for all values of the density less than the resonant density. In the case of the ion-ion hybrid resonance, the accessibility condition is

$$\kappa^2 > 2(1 + \gamma_0/\mu_{eff}) \qquad (1)$$

where κ is the parallel index of refraction, μ_{eff} = ion mass/electron mass, and γ_0 is the low-frequency dielectric constant evaluated at the resonat density.

In this paper, the dispersion relation and the condition for accessibility to the ion-ion hybrid resonance are discussed in the density region from 10^6 to 10^{17} cm^{-3} for a 4 MHz wave. The application of this resonance to the heating of the plasma in a tokamak is considered, and the accessibility condition is compared with Stix's result.

§2. Calculation

The dispersion relation for a wave in a cold collisionless two-ion plasma in a static magnetic field $\vec{B}_0$ is given by[8]

$$A\nu^4 - B\nu^2 + C = 0 \tag{2}$$

$$A = S/P \tag{3}$$

$$B = -(1 + S/P)\kappa^2 + S + RL/P \tag{4}$$

$$C = \kappa^4 - 2S\kappa^2 + RL \tag{5}$$

where ν and κ are the perpendicular and parallel indices of refraction, respectively, and S, P, R, and L are terms of the plasma dielectric tensor, which are defined as follows:

$$S \equiv 1/2\ (R + L),$$

$$R \equiv 1 - \sum_k (\Pi_k/\omega)^2 \omega/(\omega + \Omega_k),$$

$$L \equiv 1 - \sum_k (\Pi_k/\omega)^2 \omega/(\omega - \Omega_k),$$

$$P \equiv 1 - \sum_k (\Pi_k/\omega)^2,$$

where ω, Π_k, and Ω_k are the (generator) wave frequency, plasma frequency, and cyclotron frequency, respectively. The perpendicular index of refraction ν is computed from eq.(2) for frequencies in the vicinity of the ion cyclotron frequencies, in the case of a H^+ 50% and D^+ 50% plasma, with the parallel wave number and electron density as the parameters.
Figures 1 and 2 show typical examples of ν^2/κ^2 as a function of ω/Ω_p where Ω_p is the proton cyclotron frequency. We can see resonances at the ion-ion hybrid frequency. As the parallel wave number is decreased, the resonance becomes sharp and all waves except those in the narrow region of the resonance become evanescent. The attenuation in the evanescent region is much larger than in vacuum ($\nu^2/\kappa^2 = -1$), as shown in Fig.2. When the collision term is introduced into the dispersion relation (2), the singularity of ν^2/κ^2 disappears and the dispersion curve is modified as shown in Fig.3. Thus, too narrow a resonance (heavy cutoff except in the narrow region of the resonance) is meaningless from the standpoint of a heating experiment. Figure 4 shows a typical example of $\Delta(\omega/\Omega_p)$ versus $k_\parallel$ for an electron density of 10^{12} cm^{-3}, where $\Delta(\omega/\Omega_p)$ and $k_\parallel$ are the width of the region

where ν^2 is positive and the parallel wave number, respectively. As the value of $k_\parallel$ decreases below 10^{-1} cm^{-1}, $\Delta(\omega/\Omega_p)$ decreases rapidly. The critical value of $k_\parallel$ is defined as the value of $k_\parallel$ at which $\Delta(\omega/\Omega_p)$ assumes the halfvalue of $\Delta(\omega/\Omega_p)$ for sufficient high $k_\parallel$. This critical value of $k_\parallel$ is considered as the practical criterion for accessibility. This accessibility condition is shown in Fig.5 as a function of electron density n_e, with the region bounded above curve (A) being accessible. The resonance frequency is shown in Fig.6 as a function of electron density. At very low values of $k_\parallel$ ($k_\parallel \leq 10^{-3}$ cm^{-1}, when n_e is 10^{12} cm^{-3}), the dispersion relation differs from that for high values of $k_\parallel$ as shown in Fig.7.

§4. Discussion

(A) In the high density region ($n > 10^{10}$ cm^{-3}), the value of ν^2/κ^2 increases slightly as n_e increases for $\frac{\omega}{\Omega_0}$ just above the resonance and for sufficient large value of $k_\parallel$ as shown in Fig.8.

(B) The resonance frequency shifts from the geometric mean gyrofrequency $(\Omega_1\Omega_2)^{1/2}$ and approaches the ion cyclotron frequency in the low density region as shown in Fig.6; the contribution to S from the vacuum cannot be negligible in the density region where $\Pi_i/\omega \sim 1$ which corresponds to n_e of 5×10^8 cm^{-3}.

(C) In the low density region, the value of D becomes less than 1, and S is approximately 1 at $\omega = (\Omega_1\Omega_2)^{1/2}$. Then,

$$\frac{\nu^2}{\kappa^2} = P(\kappa^{-2} - 1). \tag{6}$$

From the considerations (A)∿(C), the behavior of the rf wave is described as follows. When the rf wave ($\omega = (\Omega_1\Omega_2)^{1/2}$) for sufficient large $k_\parallel$ is launched from the antenna system into the plasma with the inhomogeneous density profile as shown in Fig.9, the wave is at first evanescent in the thin region of very low density ($n<2 \times 10^5$ cm^{-3} for 4 MHz) at the edge of the plasma ($P>0$). Past this region, the wave becomes propagating. The wavelength becomes shorter, as the density increases ($-P$ increases) and enters the resonance region ($n_e>10^{10}$ cm^{-3} for 4 MHz).

As the parallel wave number decreases, C in eq.(5) becomes so small that solutions of eq.(2) are approximately $\nu^2 = B/A$ and 0. Except for the narrow region of the resonance,

$$\nu^2 = B/A$$

$$\sim \frac{P}{S}(-\kappa^2 + S). \tag{7}$$

For small wave numbers ($\kappa^2 << S$),

$$\nu^2 \sim P. \tag{8}$$

Above an electron density of 2×10^5 cm^{-3} (for 4 MHz wave), P is negative and $|P|$ is very large (for example, $P = -5 \times 10^6$ for an electron density of 10^{12} cm^{-3}). Equation (8) means that the wave is evanescent and that the attenuation in the evanescent region is very large. Thus, the practical accessibility condition is

$$\kappa^2 > S. \tag{9}$$

The condition (9) is approximately

$$k_{\parallel} > 7 \times 10^{-8}\, n^{1/2} \qquad \text{(cf. Fig.5).}$$

In the lower density region, there appears the effect of the displacement current, and hence the accessible condition is modified.

In the neighborhood of the resonance,

$$B \sim -\kappa^2 - D^2/P. \tag{10}$$

As the parallel wave number decreases ($\kappa^2 < -D^2/P$), B becomes positive, and the behavior of the dispersion curve changes as shown in Fig.7. Stix's accessibility condition (eq.(1) and (B) in Fig.5) corresponds to this change in sign of B and, as seen from Fig.5, in the high density region is two orders of magnitude smaller than (A).

When the electron density is 10^{14} cm^{-3}, the parallel wavelength should be less than 14 cm for 4 MHz wave; this is easily attained.

In conclusion, the accessibility condition is easily satisfied when ion-ion hybrid resonance heating is applied to tokamak heating.

Acknowledgments

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Figure Captions

Fig.1. The square of the perpendicular index of refraction normalized by the square of the parallel index of refraction versus the ratio of rf frequency to proton cyclotron frequency. Concentration: H^+ ion 50%, D^+ ion 50%. RF frequency: 4 MHz. Electron density: 10^{12} cm^{-3}. Parallel wave number $k_{\parallel}$: 1 cm^{-1}.

Fig.2. The square of the perpendicular index of refraction normalized by the square of the parallel index of refraction versus the ratio of rf frequency to proton cyclotron frequency. Parallel wave number $k_{\parallel}$: 10^{-2} cm^{-1}. The other conditions are the same as in Fig.1.

Fig.3. Collisional effect on the dispersion relation.

Fig.4. A typical example of the width of the region where the perpendicular index of refraction is real as a function of the parallel wave number. n_e : 10^{12} cm^{-3}. Wave frequency: 4 MHz.

Fig.5. The accessibility criterion as a function of electron density and parallel wave number for 4 MHz wave. Accessibility is attained in the upper region, bounded above curve (A). The curve (B) is Stix's criterion.

Fig.6. The resonance frequency normalized by the geometric mean gyrofrequency $\Omega_0 = (\Omega_1\Omega_2)^{1/2}$ as a function of electron density.

Fig.7. The square of the perpendicular index of refraction normalized by the square of the parallel index of refraction versus the ratio of rf frequency to proton cyclotron frequency. Parallel wave number $k_{\parallel}$: 10^{-4} cm^{-1}. The other conditions are the same as in Fig.1.

Fig.8. The variation of ν^2/κ^2 versus ω/Ω_0 for increasing density and decreasing $k_{\parallel}$, where ν: perpendicular index of refraction, κ: parallel index of refraction, ω: applied frequency, $\Omega_0 = (\Omega_1\Omega_2)^{1/2}$: gm. gyro-frequency.

Fig.9. Schematic diagram of the behavior of an rf wave (4 MHz) incident on a plasma with an inhomogeneous density profile.

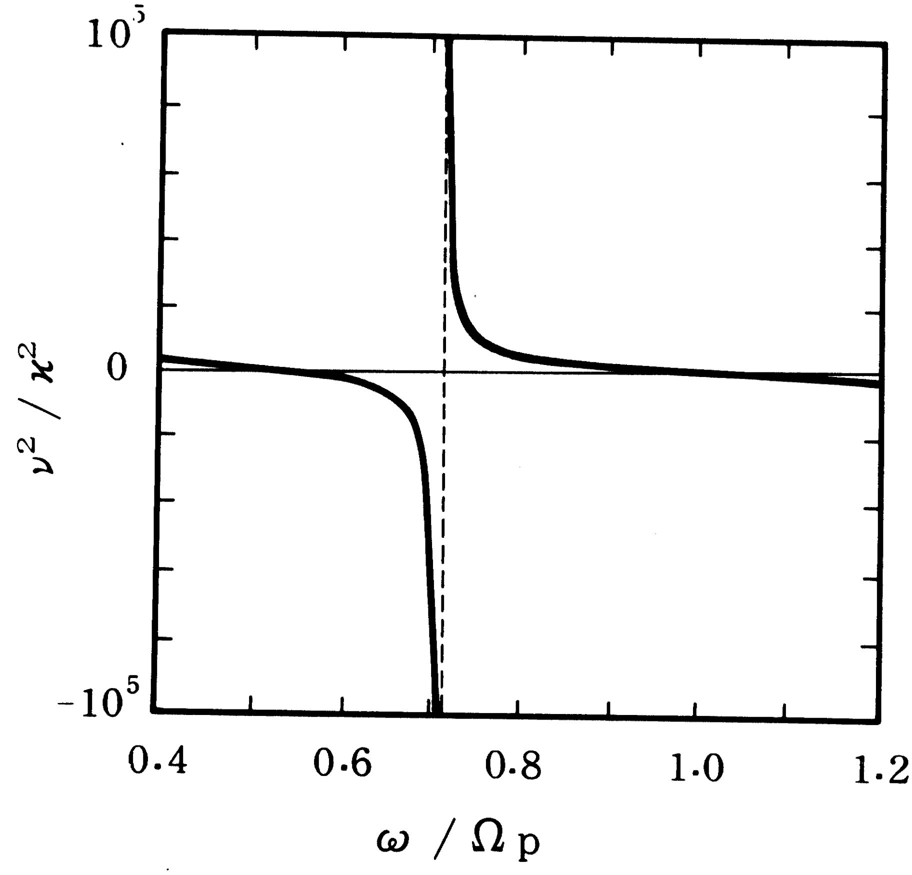


Fig. 1

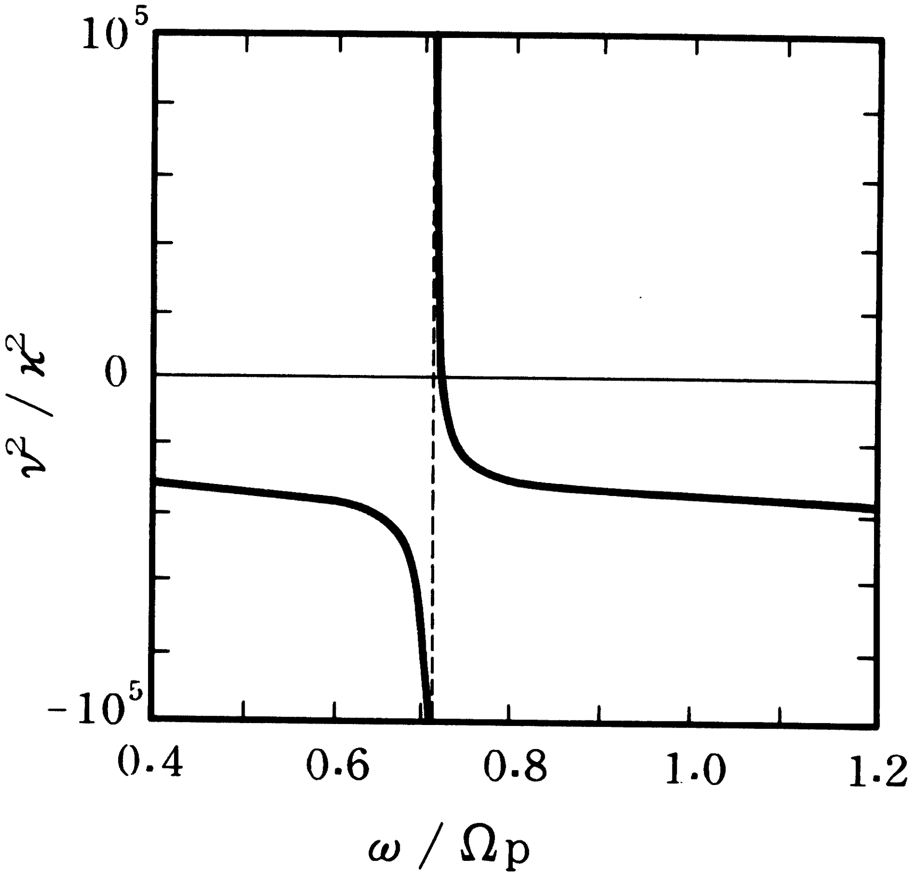


Fig. 2

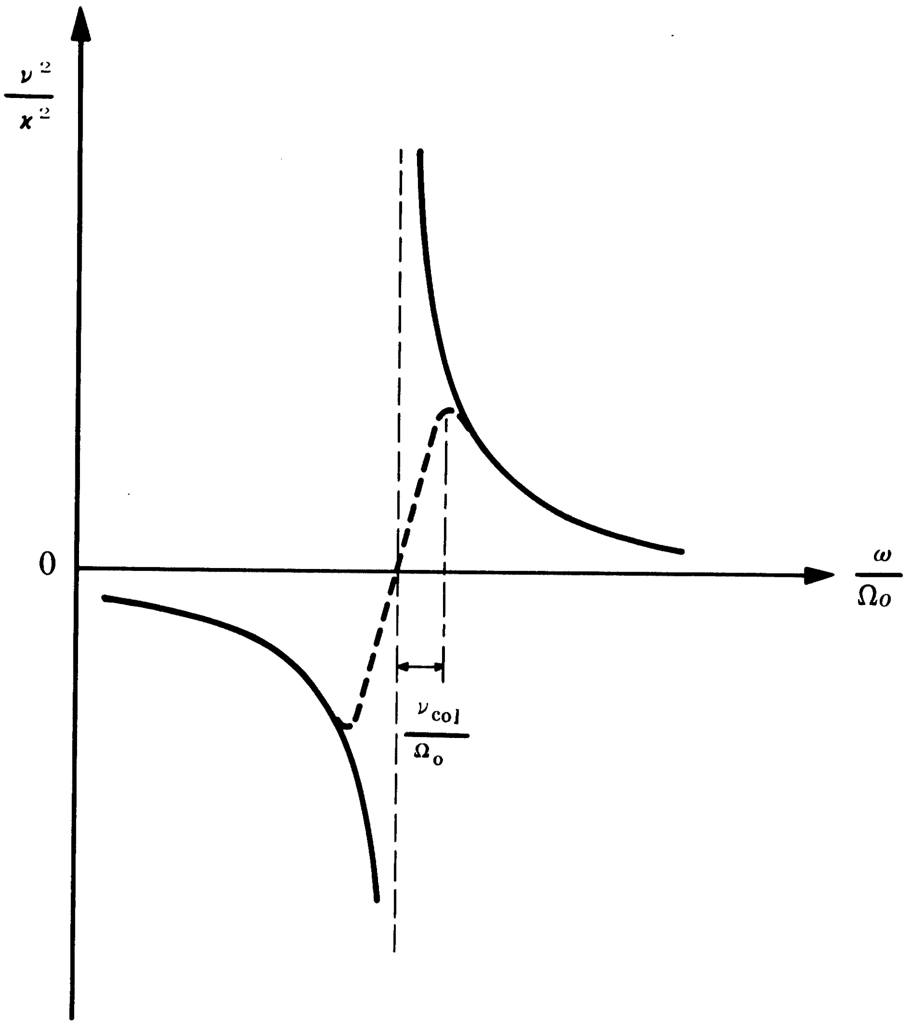


Fig. 3

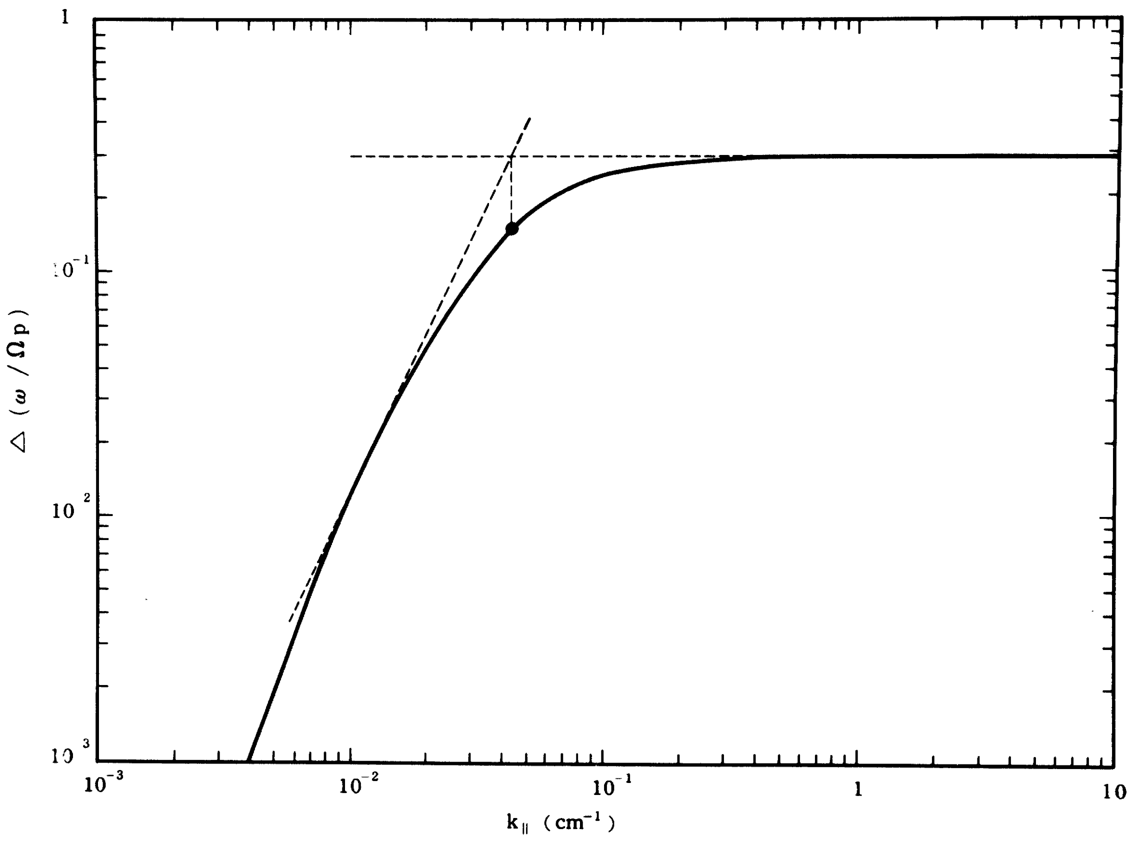
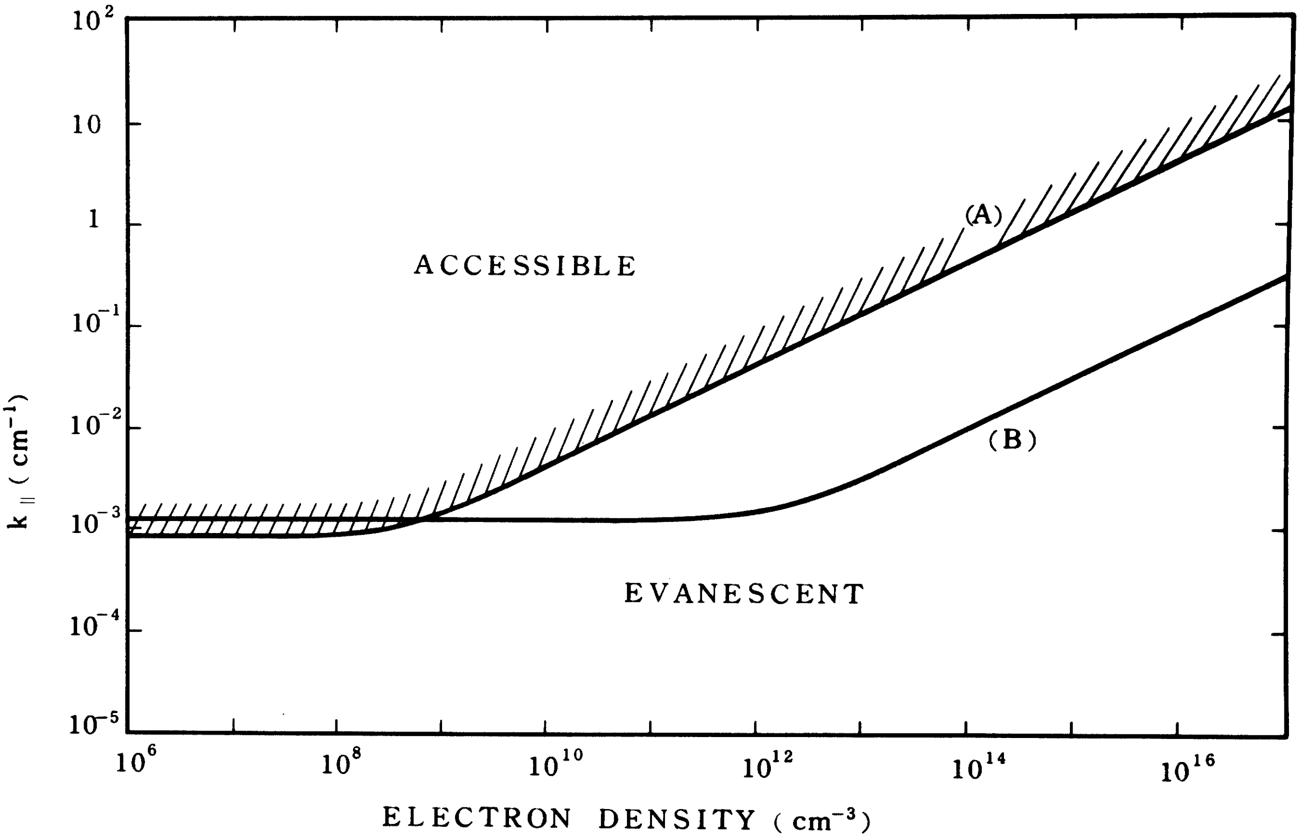

1
10^{-1}
10^{-2}
10^{-3}
$\Delta\ (\omega\ /\ \Omega_p)$
10^{-3}
10^{-2}
10^{-1}
1
10
$k_{\parallel}$ (cm^{-1})

Fig. 4



10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$k_{\parallel}$ (cm^{-1})
ACCESSIBLE
EVANESCENT
(A)
(B)
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
ELECTRON DENSITY (cm^{-3})

Fig. 5

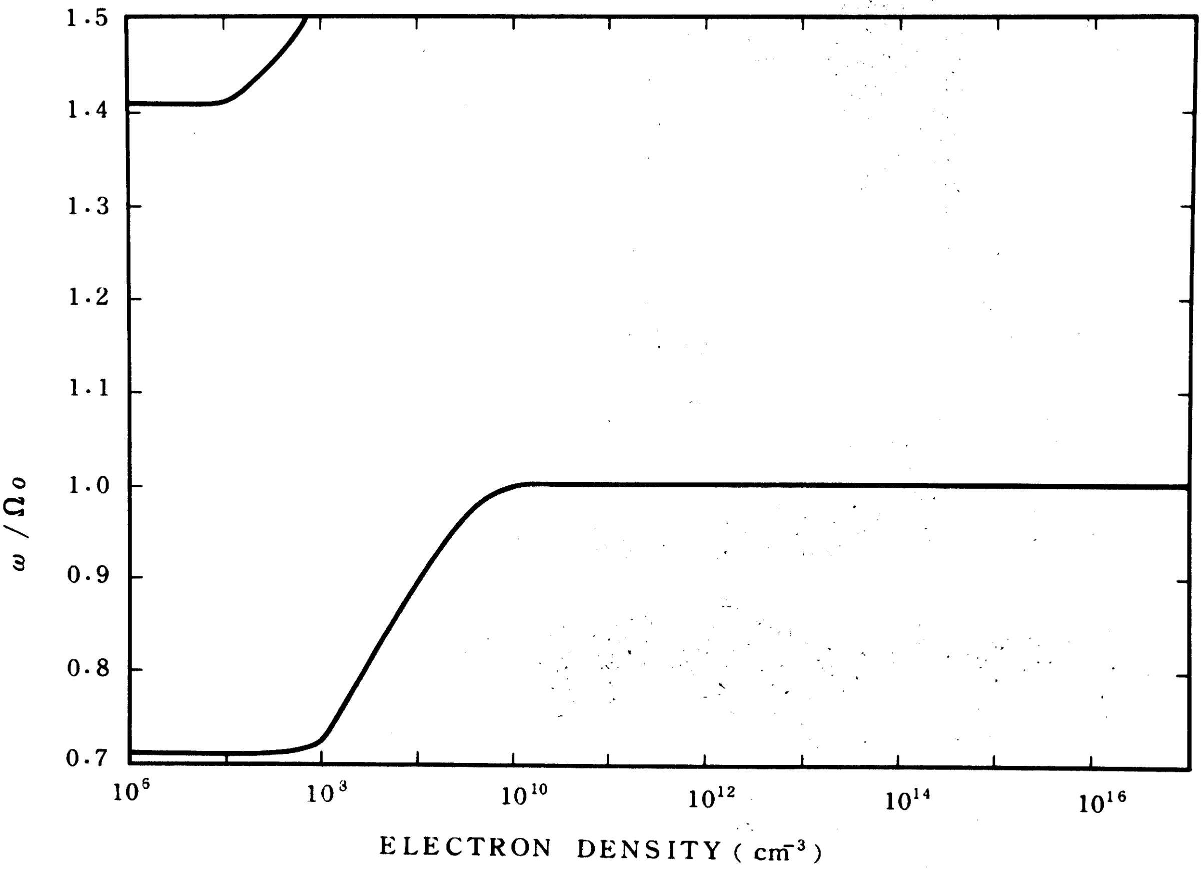

1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
ω / Ω₀
10^6
10^3
10^10
10^12
10^14
10^16
ELECTRON DENSITY (cm^-3)

Fig. 6

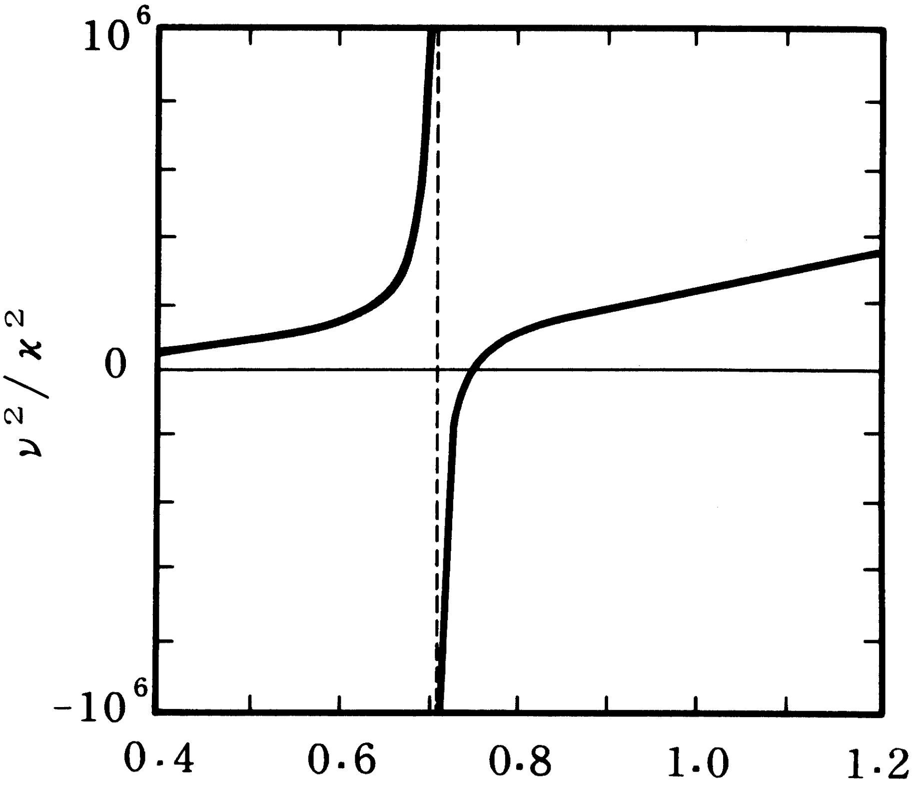
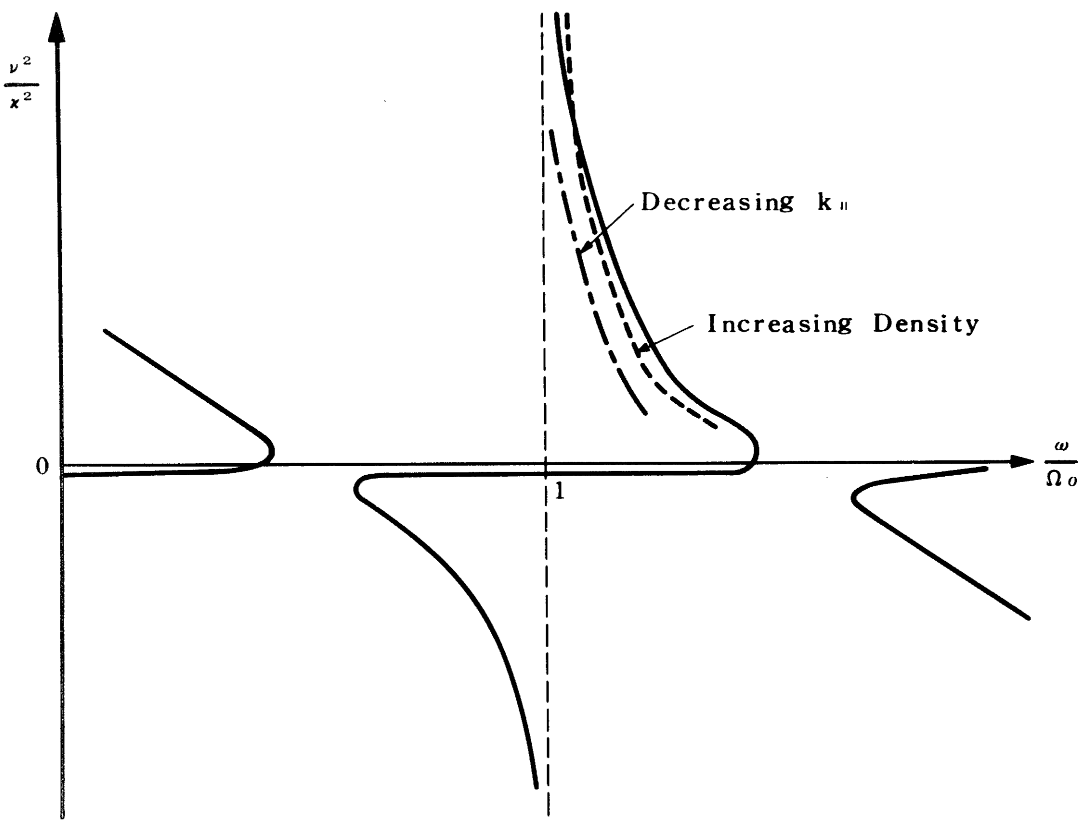


Fig. 7



$\frac{\nu^2}{\kappa^2}$
Decreasing $k_{\parallel}$
Increasing Density
0
1
$\frac{\omega}{\Omega_0}$

Fig. 8

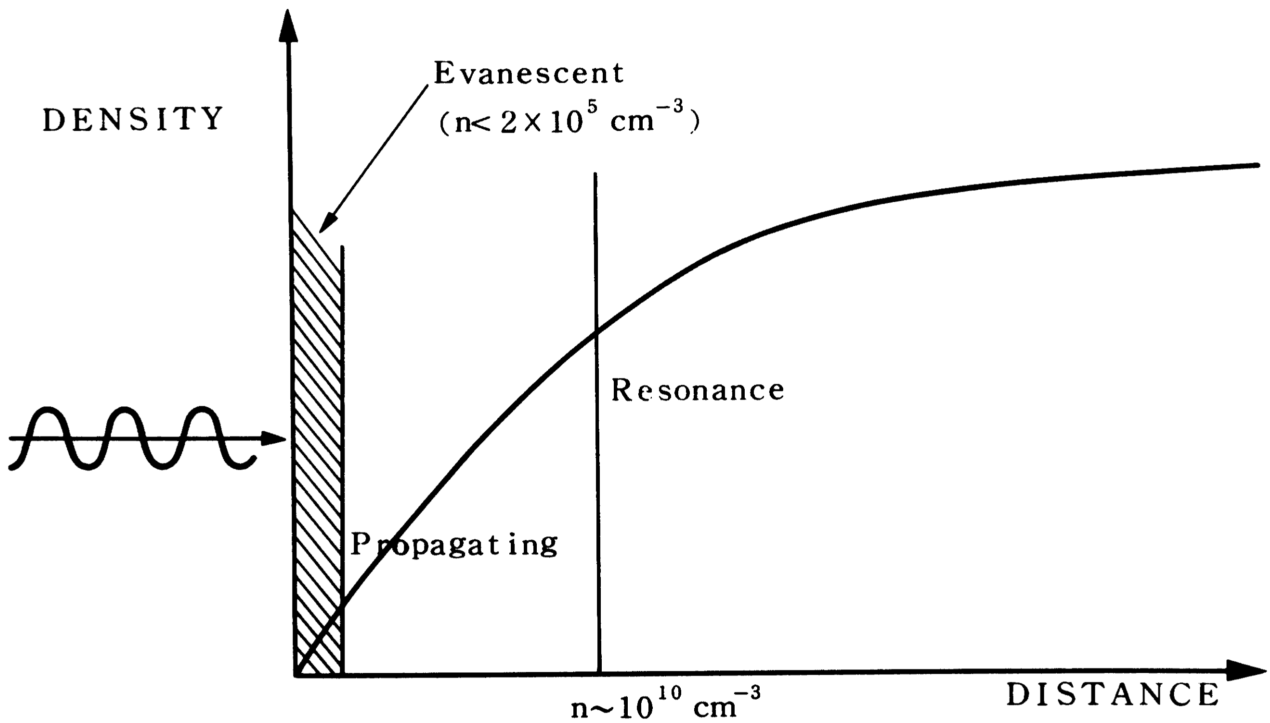

Evanescent
(n< 2×10^5 cm^-3)
DENSITY
Resonance
Propagating
n~10^10 cm^-3
DISTANCE

Fig. 9