INSTITUTE OF PLASMA PHYSICS

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RESEARCH REPORT

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Collisional Absorption of Ion-Ion Hybrid Wave

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Abstract

The damping length of the ion-ion hybrid wave is calculated numerically in the density region of 10^{10} to 10^{17} cm^{-3} using the Langevin collision model, with the ion-ion collision frequency, the parallel wave number, and the D-T concentration ratio as the parameters. It is shown that the heating is effective even for very low collision frequencies encountered in thermonuclear plasmas.

§1. Introduction

Ion-ion hybrid resonance heating presently appears to be a highly attractive method for heating D-T plasma in tokamaks. Both theoretical [1-4] and experimental [5-11] work on this ion-ion hybrid resonance has been done in recent years.

In the previous report [12], it is shown that the accessibility condition is easily satisfied when ion-ion hybrid heating is applied to tokamak plasma. In this paper, the damping length of the ion-ion hybrid wave is calculated numerically in the density region of 10^{10} to 10^{17} cm^{-3}, with the ion-ion collision frequency, the parallel wave number, and the concentration ratio of the two ion species as the parameters. To simplify the discussion, attention is restricted in this note to a D-T plasma. The damping mechanism considered is momentum transfer collisions between the D^+ ions and T^+ ions using the Langevin model.

§2. Calculation

The dielectric tensor of the collisionless two-ion plasma is modified using the Langevin collision model as follows [13]:

$$\hat{K} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \tag{1}$$

with $\quad S \equiv \frac{1}{2}(R + L), \quad D \equiv \frac{1}{2}(R - L)$

$$R = 1 - \frac{\Pi_e^2}{\omega^2}\frac{\omega}{\omega + \Omega_e} - \frac{\Pi_1^2}{\omega^2}\frac{\omega}{\omega + i\nu_1 + \Omega_1} - \frac{\Pi_2^2}{\omega^2}\frac{\omega}{\omega + i\nu_2 + \Omega_2}$$

$$L = 1 - \frac{\Pi_e^2}{\omega^2}\frac{\omega}{\omega - \Omega_e} - \frac{\Pi_1^2}{\omega^2}\frac{\omega}{\omega + i\nu_1 - \Omega_1} - \frac{\Pi_2^2}{\omega^2}\frac{\omega}{\omega + i\nu_2 - \Omega_2}$$

$$P = 1 - \frac{\Pi_e^2}{\omega^2} - \frac{\Pi_1^2}{\omega(\omega + i\nu_1)} - \frac{\Pi_2^2}{\omega(\omega + i\nu_2)}$$

and

$$\Omega_k \equiv \frac{e_k B_0}{m_k}, \quad \Pi_k^2 = \frac{n_k e_k^2}{m_k e_k^2},$$

where subscripts 1, 2, and e stand for D^+ ions, T^+ ions, and electrons, respectively. The first-order fields are assumed to vary as $\exp[i(k_{\parallel}Z - \omega t)]$. Only ion-ion collisions are taken into account for simplicity. The frequencies ν_1 and ν_2 are the D^+ ion - T^+ ion momentum transfer collision frequency and the T^+ ion - D^+ ion collision frequency, respectively. Because the drag on D^+ ions due to T^+ ions is equal and opposite to the drag on T^+ ions due to D^+ ions,

$$n_1 m_1 \nu_1 = n_2 m_2 \nu_2. \quad (2)$$

The charge neutrality condition requires $n_1 + n_2 = n_e$. With the dielectric tensor defined above, the wave equation is,

$$\nabla \times \nabla \times \vec{E} - \frac{\omega^2}{c^2} \hat{K}\cdot\vec{E} = 0. \quad (3)$$

Equation (3) gives the dispersion relation,

$$An^4 - Bn^2 + C = 0, \quad (4)$$

with

$$A = S/P,$$

$$B = -(1 + S/P)n_{\parallel}^2 + S + RL/P,$$

$$C = n_{\parallel}^4 - 2sn_{\parallel}^2 + RL,$$

where $n_\perp$ and $n_\parallel$ are the perpendicular and parallel indices of refraction, respectively. The complex value of the perpendicular index of refraction $n_\perp$ is computed from eq.(4) for frequencies in the vicinity of the geometric mean gyrofrequency $\Omega_0 = (\Omega_1\Omega_2)^{1/2}$ with the parallel wave number $k_\parallel$, electron density n_e, relative collision frequency $\gamma = \nu_1/\omega$, and relative concentration of the two ion species as the parameters. Figures 1 and 2 show typical examples of refractive index behavior for plasma parameters in the range being considered as a function of ω/Ω_0. The electron density n_e is 10^{13} cm^{-3} and $\gamma = 10^{-4}$ in the case of Fig.1. In Fig.2, $n_e = 10^{17}$ cm^{-3} and $\gamma = 10^{-2}$. The curves of $Re(n_\perp)/n_\parallel$ and $Im(n_\perp)/n_\parallel$ versus ω/Ω_0 are independent of parallel wave number $k_\parallel$ and electron density n_e in the region where the accessible condition is satisfied [12]. In the case of the collisionless plasma, the ion-ion hybrid wave is evanescent in low frequency side ($\omega < \Omega_0$) and propagating (with no damping) in high frequency side ($\omega > \Omega_0$) for sufficient high $k_\parallel$ [12]. Thus, the low frequency side of $Im(n_\perp/n_\parallel)$ in Figs.1 and 2 represent the evanescent character for low collision frequency. On the high frequency side, the wave has a damping term when the collision is taken into account. In order to know what portion is absorbed, $Q = (\omega/8\pi)\,\varepsilon''_{\alpha\beta} E^*_\alpha E_\beta$ must be calculated, where $\varepsilon''_{\alpha\beta}$ is the anti= Hermitian part of the tensor $\hat{K}$. However, assuming the dissipation due to collisions contributes mainly to the damping term on the high frequency side, the damping length, defined by

$$\lambda_d = [I_m(n_\perp \omega/c)]^{-1},$$

is calculated as a function of $k_\parallel$, n_e, and α. Figure 3 shows normalized damping length $\lambda_d/\lambda_\parallel$ versus relative collision frequency in the case of $n_e = 10^{13}$ cm^{-3} and $k_\parallel = 0.1$ cm^{-1}, where $\lambda_\parallel = 2\pi/k_\parallel$. The curves (1), (2) and (3) in Fig.3 show the case of $\omega/\Omega_0 = 1.005$, 1.004, and 1.013

at which $\mathrm{Re}(n_\perp/n_\parallel)$ assumes the peak value for $\gamma = 10^{-3}$, 10^{-2}, and 4×10^{-2}, respectively. As the collision frequency increases at the fixed value of ω/Ω_0, the damping length decreases at first and becomes a minimum near the point where $\mathrm{Re}(n_\perp/n_\parallel)$ assumes the peak value.

§3. Discussion

The real part of $n_\perp^2$ is zero when $\omega/\Omega_0 = 1$ and becomes negative for $\omega/\Omega_0 < 1$. Then, the peak in the imaginary part of $n_\perp$ shifts to lower values of ω/Ω_0 compared to the peak in the real part of $n_\perp$. In the low density region the peak of $\mathrm{Re}(n_\perp)$ shifts to lower values of ω/Ω_0. Keeping the electron density n_e and the equivalent temperature (according to Spitzer's collision formula [14]) constant, the perpendicular index of refraction $n_\perp$ is calculated as a function of the D-T concentration ratio. Figure 4 shows the maximum of $\mathrm{Im}(n_\perp/n_\parallel)$ and the value of ω/Ω_0 at which $\mathrm{Im}(n_\perp/n_\parallel)$ assumes the maximum value. The value of ω/Ω_0 decreases as the concentration of the deuterium increases. The value of $\mathrm{Im}(n_\perp/n_\parallel)$ becomes a maximum at the condition that the collision frequency ν_1 is equal to ν_2. Thus, power absorption will be most effective in the concentration ratio

$$\frac{n_1}{n_2} = \frac{m_2}{m_1}.$$

As the refractive index behavior is independent of $k_{\parallel}$ in the accessible region, an exciting system of short wavelength structure is advantageous for plasma heating.

The relative collision frequency γ is 10^{-3} according to Spitzer [14] in the case of the following plasma parameters: $n_e = 10^{13}$ cm^{-3}, $T_i = 1$ keV, $\omega/2\pi = 4$ MHz, and the concentration ratio of D^+ ions to T^+ ions = 1. Figure 3 gives the normalized damping length $\lambda_d/\lambda_{\parallel} = 3.3 \times 10^{-3}$ and 2.3×10^{-1} at $\omega/\Omega_0 = 1.0005$ and 1.013, respectively. Thus, rf power will be absorbed efficiently even at the very low collision frequencies encountered in thermonuclear plasmas.

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Figure Captions

Fig.1. The real and imaginary parts of the perpendicular index of refraction $n_\perp$ normalized by the parallel index of refraction $n_\parallel$ versus the ratio of the applied frequency ω to the geomertic mean gyrofrequency $\Omega_0 = (\Omega_1\Omega_2)^{1/2}$ for D-T plasma. Here, electron density $n_e = 10^{13}$ cm^{-3}, $k_\parallel = 0.1$ cm^{-1}, $\gamma = 10^{-4}$, $\omega/2\pi = 4$ MHz, and the concentration ratio of the D^+ ions to T^+ ions = 1.

Fig.2. $Re(n_\perp)/n_\parallel$ and $Im(n_\perp)/n_\parallel$ vs ω/Ω_0. Here, $n_e = 10^{17}$ cm^{-3}, $k_\parallel = 100$ cm^{-1}, and $\gamma = 10^{-2}$. The other conditions are the same as in Fig.1.

Fig.3. The damping length λ_d normalized by $\lambda_\parallel$ as a function of $\gamma = \nu_1/\omega$, where ν_1 is the D^+ ion $-T^+$ ion momentum transfer collision frequency and $\lambda_\parallel = 2\pi/k_\parallel$. Here, $n_e = 10^{13}$ cm^{-3}, $k_\parallel = 0.1$ cm^{-1}, $\omega/2\pi = 4$ MHz, and $n_1/n_2 = 1$.

Fig.4. $Im(n_\perp/n_\parallel)$ and ω/Ω_0 at which $Im(n_\perp/n_\parallel)$ assumes the peak value vs D^+ ion concentration. Here, $n_e = 10^{13}$ cm^{-3}, $k_\parallel = 10$ cm^{-1}, and $\omega/2\pi = 4$ MHz.

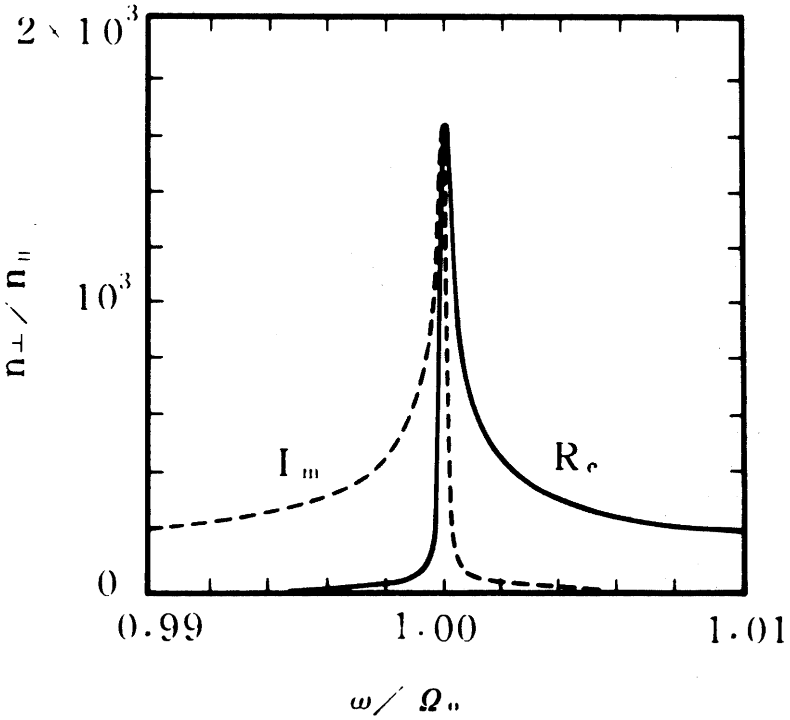

Fig. 1

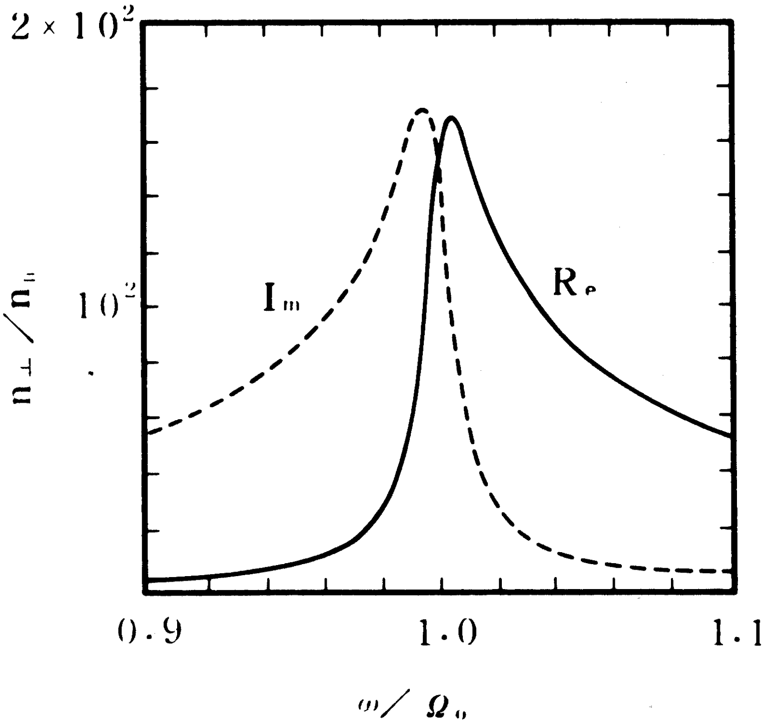

Fig. 2

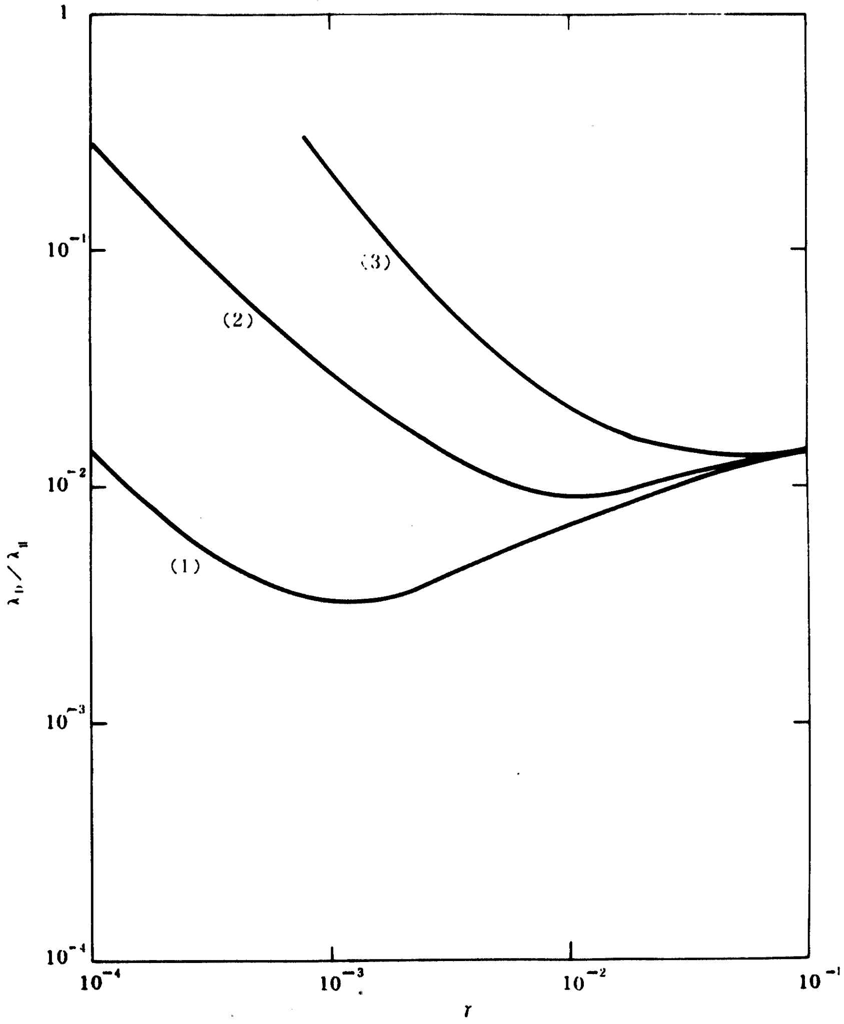

Fig. 3

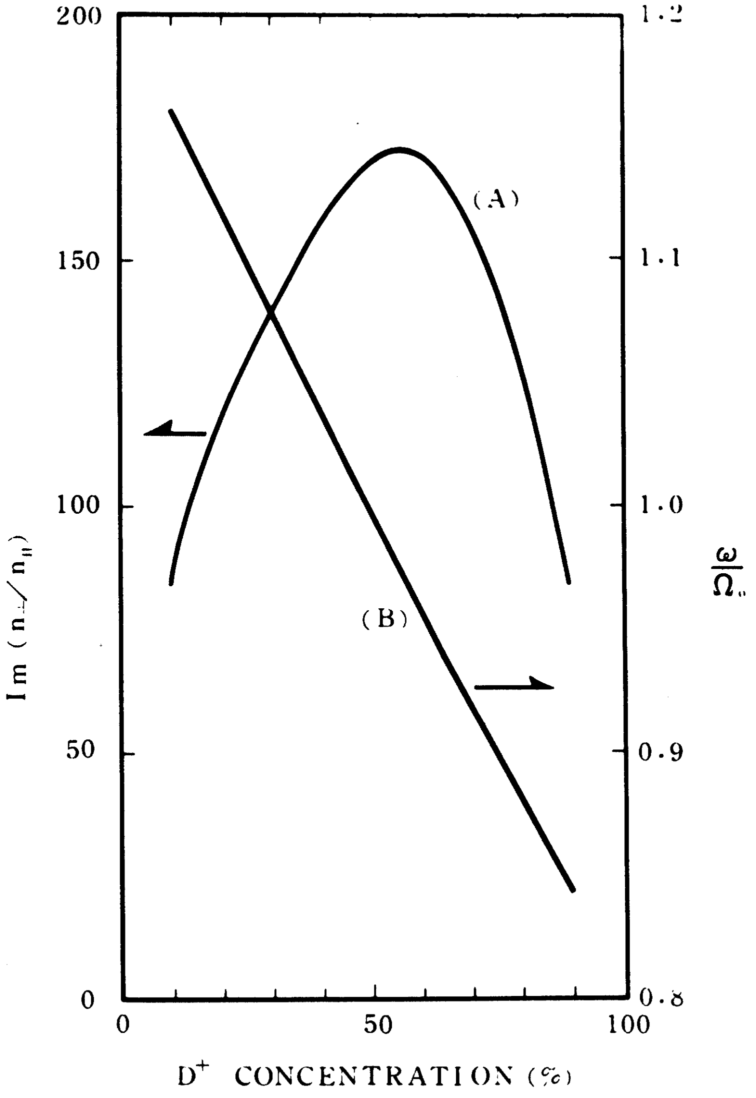
200
150
100
50
0
1.2
1.1
1.0
0.9
0.8
0
50
100
(A)
(B)
Im (n⊥ / n∥)
ω / Ω
D+ CONCENTRATION (%)

Fig. 4