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RESEARCH REPORT

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Paramagnetic Configuration with
Absolute Minimum-B Property

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Abstract

The equilibrium configuration is calculated including the beam-plasma interaction. This type of equilibrium allows the paramagnetic configuration with absolute minimum-B property. Configuration with both high shear and absolute minimum-B is possible without external guide field.

Recently Christofilos and Yoshikawa[1] have clarified the existence of the absolute minimum-B configuration produced by relativistic electron beam below the Alfvén limit. Their minimum-B configuration is created with the help of steep gradient of toroidal magnetic field. Below the Alfvén limit the equilibrium current path deviates from the magnetic surface in the toroidal geometry. The deviation is not convenient for the use of the relativistic beam to confine plasma, since the deviation suggests the strong interaction between the beam and the toroidal vessel wall of the machine. Ikuta and Mohri[2] have pointed out a possibility of a new type of minimum-B configuration produced by relativistic electron beam in the plasma. In this configuration the gradient of toroidal magnetic field is not necessary in creating the minimum-B field, but the interaction between the plasma force free current and the electron beam one is essential to creat the configuration. In general the toroidal correction is not small except for the highly paramagnetic configuration of reverse pinch type.[3] The closed magnetic isobar tends to vanish by the toroidal effect in the case of the weak paramagnetic configuration. If a highly paramagnetic configuration with a minimum-B property is present, this system should offer the ideal magnetic containment in the toroidal geometry at least from the theoretical point of view.

The purpose of the present paper is to report the possibility of the paramagnetic configuration with absolute minimum-B property. This type of configuration is possible only when the beam-plasma interaction is taken into account. The basic Eqs. in M.K.S. unit are

$$\vec{J}_p + \vec{J}_e = \frac{1}{\mu_0} \nabla \times \vec{B} \ , \tag{1}$$

$$m_0 (\vec{v}_e \cdot \nabla) \vec{v}_e = e \ \vec{v}_e \times \vec{B} \ , \tag{2}$$

$$\vec{J}_p \times \vec{B} = \nabla P \ , \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \ , \tag{4}$$

$$\vec{J}_e = ne \ \vec{v}_e \ , \tag{5}$$

where $\vec{J}_p$ and $\vec{J}_e$ are the plasma current and the electron beam one, and n, e, $\vec{v}_e$ and m_o are the uniform electron beam density, the charge of the electron, the velocity of the beam and the relativistic mass of the electron, respectively. The quantity μ_0 is the magnetic permeability of vacuum and P represents the plasma pressure. The electron beam is assumed to be cold.

In the case of cylindrical configuration with vanishing plasma pressure Eq.(2) is reduced to

$$\frac{\delta}{r}\left(\frac{dB_z}{dr} + \alpha \frac{K}{r}\right)^2 = B_z \frac{dB_z}{dr} + \frac{K}{r^2}\frac{dK}{dr} \ , \tag{6}$$

where $\delta = \dfrac{m_0}{ne^2\mu_0}$, $\alpha = \mu_0 \dfrac{\vec{J}_p \cdot \vec{B}}{B^2}$ and $K = rB_\theta$. The electron beam density is assumed to be uniform. Now, we give an analytic model of paramagnetic configuration with a minimum-B property.

We choose

$$\alpha = \alpha_0 = \text{const.}\ (\neq 0), \quad 0 < r < b$$
$$= \quad 0 \quad , \quad b < r < a \quad . \tag{7}$$

$$B_z = B_0 = \text{const.} \quad , \quad 0 < r < b \ ,$$

$$= B_0[1 - \frac{(r-b)^2}{(a-b)^2}] + B_1 \ , \quad b < r < a \ . \tag{8}$$

where a is the beam radius and $0 < b < a$, and B_1 is a constant. Then we have

$$K = K_0 (\frac{r}{b})^{\delta\alpha_0^2}, \qquad 0 < r < b$$

$$K^2 = \frac{2}{3} \frac{\delta\ B_0^2}{(a-b)^2} (r - b)^3 (3r + b)$$
$$+ \frac{B_0(B_0+B_1)}{3(a-b)^2} (r-b)^2 (3r^2 + 2br + b^2)$$
$$- \frac{B_0^2}{15(a-b)^4} (r - b)^4 (10r^2 + 4br + b^2)$$
$$+ K_0^2 \quad , \qquad b < r < a \quad , \tag{10}$$

where K_0 represents a total current in the central core region. For $\delta\alpha_0^2 > 2$ the central core region has apparently the minimum-B property, i.e.

$$\frac{d}{dr}\left(\frac{K}{r^2}\right) > 0. \tag{11}$$

The outside paramagnetic shell structure becomes a high shear configuration for a suitable choice of the ratio B_0/B_1. For a negative B_1/B_0 the structure of the magnetic configuration in the shell region is very similar to that of the reverse pinch.

In the analytic model the assumption is made that α is a step function with respect to r. In the real plasma it seems too artificial for α to be a step function. Therefore it is very important to investigate the configurations with a smooth function of α. To see this it is convenient to rewrite (6) in dimensionless form, with $\delta\alpha_0^2$ appearing explicitly as a decisive parameter. We define dimensionless coordinate x:

$$r = ax \tag{12}$$

The plasma and the beam are inside the circle

$$0 \leq x \leq 1.$$

We introduce dimensionless parameters ξ and η, and functions f, y and h:

$$\xi = \delta\alpha_0^2 - 2 > 0,$$

$$\eta = \alpha_0 a \qquad ,$$

$$B_z = B_0 f ,$$

$$K = aB_0 x^{2+\xi} y,$$

$$\alpha = \alpha_0 h ,$$

where B_0 is the strength of the magnetic field on the z axis. Equation (6) becomes

$$xy \frac{dy}{dx} + (2 + \xi) \{y^2 - (hy + \frac{1}{\eta x^{1+\xi}} \frac{df}{dx})^2\}$$

$$+ \frac{f}{x^{1+2\xi}} \frac{df}{dx} = 0 . \qquad (13)$$

We choose

$$\eta = 1$$

$$h = \frac{1}{1 + (\frac{x}{b})^{\ell}} \qquad (14)$$

and

$$f = 1 - x^m + f_1 ,$$

where ℓ, m, f_1 and b are the arbitrary positive constants. From (13) and (14) we obtain the following inequality to have a solution of y to be a non-vanishing constant near x = 0.

$$m - 3 - 2\xi > 0 \qquad (15)$$

It is apparent from the inequality that the distribution of the axial magnetic field should have a flat profile near $x = 0$ and that the flatness guarantees the absolute minimum-B property of the configuration for $\xi > 0$.

The interesting possibility lies in $f_1 = 0$. The case $f_1 = 0$ suggests that the external guide field is not necessary to construct high shear absolute minimum-B configuration. This type of the configuration is impossible without taking the beam-plasma interaction into account.

The key point to produce the high shear absolute minimum-B configuration is to control the distribution of the force free plasma current. The ratio $J_{p\theta}/B_\theta$ should decrease as we approach the edge of the plasma. A typical radial distribution of the field component B_θ/B_0 and B_z/B_0, and the pitch length $(rB_z)/(aB_\theta)$ of the field structure is shown in Fig.1, where the parameters are $m = 4$, $\ell = 3$, $b = 1.0$, $\xi = 0.3$ and $f_1 = 0$. The value y at $x = 0$ is chosen to be 0.1.

References

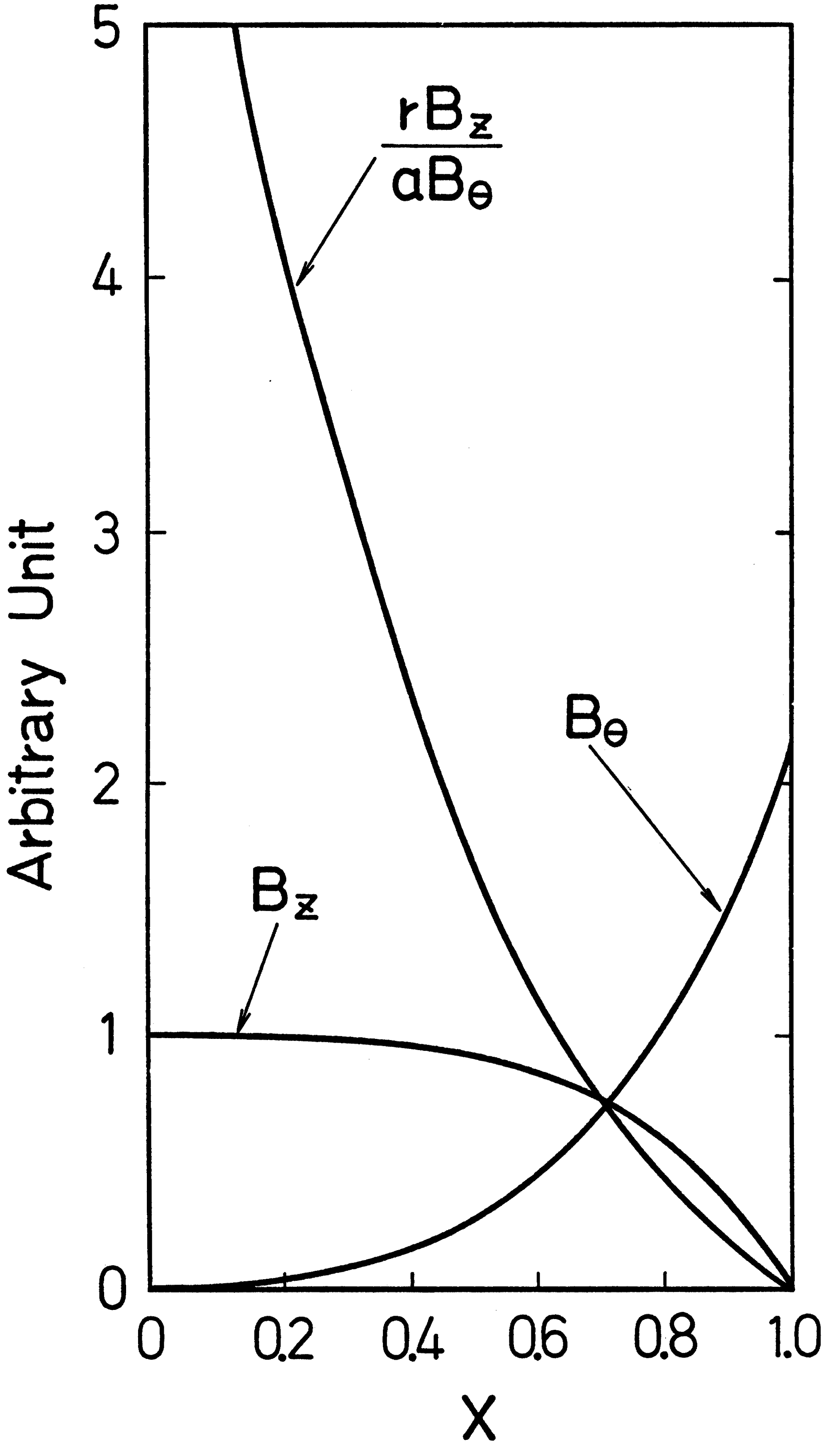
1) S. Yoshikawa and N. Christofilos, in Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, 1971), Vol. 2, p.357.

2) K. Ikuta and A. Mohri, IPPJ-158 of Institute of Plasma Physics, Nagoya University.

3) D.C. Robinson, Plasma Physics 13, 439 (1971).

Figure Caption

The strength of the field components B_θ and B_z, and the pitch length of the line of force versus the radius of the plasma, where the parameters are $m = 4$, $\ell = 3$, $b = 1$ and $\xi = 0.3$.



5
4
3
2
1
0
Arbitrary Unit
$\frac{rB_z}{aB_\theta}$
B_θ
B_z
0
0.2
0.4
0.6
0.8
1.0
X