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A Method to Make the RF Plugging Efficient in the Point Cusp

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Abstract

It is possible to make the width (radius) of the plasma in the line (point) cusp to the order of the ion thermal Larmor radius, when the plasma is injected from one of the point cusps. To do this, it is indispensable to prevent electrons from flowing back through the line cusp. The thin (slender) plasma is convenient for the rf plugging method. It may be possible to improve greately the containment of the axially symmetric system, by utilizing the cusp geometry together with the rf plugging method.

§1. Introduction

We think of a plasma contained in a certain axially symmetric magnetic field with two open ends, for instance, a mirror system or a theta-pinch system. The main leakage of this type of plasma will be through the two open ends. We can expect that the containment of the plasma in this system will be greatly improved, by introducing an additional magnetic cusp field to each open end and by applying the rf plugging method to both the line and the point cusps (see Fig.1)[1].

It is necessary to use cusp geometry for the efficient operation of rf plugging. As one sees later, it is possible to make the width (radius) of the plasma to the order of the ion thermal Larmor radius in the line (point) cusp field. This kind of thin (slender) plasma has an electrostatic wave mode[2),3)] with the eigen frequency of order ω_{ci}, ion cyclotron frequency, that is quite efficient to produce a strong rf field in the plasma if the power is applied at the eigen frequency. In the case of mirror field alone, for instance, the radius of plasma is generally so much larger than the ion Larmor radius that an external rf field can penetrate little into the mirror plasma. This is the reason why the direct application of an rf field to the mirror end is so inefficient.

There have been a series of experiments (hereafter called the TPD experiment)[4] to study the effect of the rf plugging in the cusp geometry. They have found that the rf plugging is quite efficient in the line cusp, but, unfortunately, inefficient in the point cusp. This fact can be explained mainly due to the large plasma radius in the point cusp compared with ion Larmor radius.

The purpose of this article is to propose a method to reduce the plasma radius in the point cusp to the order of the ion Larmor radius and to make the rf plugging efficient there. It is pointed out that the observed large plasma radius in the point cusp can be explained by the following situation: those electrons which have once escaped from the line cusp will run across the magnetic field on the conductive wall of the vacuum chamber and then flow backwards into the ion rich plasma region along the line of force. Therefore, it is indispensable to prevent electrons from counterflowing through the line cusp, in order to make the plasma slender in the point cusp. One possible way is to make the wall of vacuum chamber out of some insulating material.

§2. Reasons for the inefficiency of the rf plugging in the point cusp

We give now three rational reasons.

1) We know the following elementary argument. The low frequency rf field is applied perpendicular to the magnetic field. Electrons drift in the direction perpendicular both to the magnetic field and to the applied rf field. In the case of cylindrical geometry, this drift causes an electric field in the direction of electrons' drift due to the charge

separation, although in the planar geometry charge separation does not occur. This induced field again causes the secondary drift of electrons in the direction of the applied rf field, resulting in the shielding of the initial applied field. In the linear theory[3], however, there exists a convenient mode for the rf plugging in the both cases, the drift approximation and the Boltzmann distribution for the electron. We can say nothing about the nonlinear theory, now.

2) The plasma density is higher in the point cusp than in the line cusp. The distribution function is assumed to take the form

$$f = \exp(-\frac{M}{2T}v^2)\,g(p_\theta), \qquad (1)$$

where $p_\theta = r(Mv_\theta + \frac{e}{c}A_\theta(r, z))$, angular momentum conjugate to θ, z-axis is the symmetry axis of the cusp field, and r is the radial coordinate. On the z-axis (r = 0), density derived from the above distribution function is independent of z, so that the plasma density in the point cusp is equal to the density in the cusp center (z = r = 0). In the TPD experiment[5] under a certain condition, the density is really about 15 times higher in the point cusp than in the line cusp. This fact, however, will be overcome, if the strength of the magnetic field is increased, because the plasma density effects the rf plugging through the dimensionless parameter $\tilde{n}(\equiv \omega_p^2/\omega_c^2)$.

3) The plasma radius in the point cusp is larger than the ion thermal Larmor radius ρ_i. The photograph of the plasma shape

taken in the TPD experiment[6] shows that the boundary of bright region is nearly identifiable to a magnetic line of force. This shows that the iso-density contour of the plasma, at least of the electron, is along the line of force. Assuming this fact, we can prove the ratio of the plasma radius, Δr, to the ion Larmor radius, $\rho_{p_2 i}$, in the point cusp is given by

$$\frac{\Delta r}{\rho_{p_2 i}} = \sqrt{\frac{B_{p_2}}{B_\ell} \frac{2L}{\rho_{\ell i}}} \quad , \tag{2}$$

for the general axially symmetric cusp field shown in the Fig.2. Here B_ℓ and B_{p_2} are the strength of the magnetic field in the line and the right point cusp, respectively. L and $\Delta\ell$ are defined in the Fig.2. Denote by N the total number of the magnetic line of force within the radius Δr, we have

$$B_\ell = N/2\pi L \Delta\ell \ ,$$

$$B_{p_2} = N/\pi \Delta r^2 \ .$$

As we have the relation $B_{p_2}/B_\ell = \rho_{\ell i}/\rho_{p_2 i}$, $\rho_{\ell i}$ being the thermal ion Larmor radius in the line cusp, we put $\Delta\ell = \rho_{\ell i}$, and obtain the relation (2). Generally, the ratio $L/\rho_{\ell i}$ is much larger than unity, so that $\Delta r/\rho_{p_2 i}$ is larger than unity, as far as B_{p_2} is comparable with B_ℓ. Given the ion temperature, we have a fixed value of $B_\ell \rho_{\ell i}$, so that the small value of B_{p_2} is preferrable in reducing the ratio of $\Delta r/\rho_{p_2 i}$. The magnetic field B_{p_2} is, however, to be strong enough to give an appropriate $\tilde{n}$. Therefore, we cannot reduce $\Delta r/\rho_{p_2 i}$ indefinitely. The large ratio of $\Delta r/\rho_{p_2 i}$ is an attribute of the cusp geometry, provided that the boundary of the plasma is along a magnetic line of force.

§3. Single particle theory and inconsistency with the experiment

We study the trajectories of ions or electrons starting from the left point cusp (see Fig.2). This situation is identical with that of the TPD experiment (symmetric cusp), and is also applicable to study the loss of the main axially symmetric plasma from the line or the point cusp. We regard the magnetic field in the point cusp as nearly homogeneous. The distance R from the z-axis of a gyration center (X,Y) of a particle is given by

$$R^2 = X^2 + Y^2 = (x + \frac{v_y}{\omega_c})^2 + (y - \frac{v_x}{\omega_c})^2 \qquad (3)$$

in the neighborhood of the point cusp, where (x,y) and (v_x, v_y) are the coordinate and the velocity of the particle, respectively, and $\omega_c = eB_z/Mc$. Using the radius R and the thermal Larmor radius ρ of a species, the total radius of the region of the species in the left point cusp is estimated by $\lambda = \sqrt{R^2 + \rho^2}$. We assume charge neutrality in the left point cusp, so that

$$R_e^2 + \rho_e^2 = R_i^2 + \rho_i^2 = \lambda^2$$

where suffixes denote the electron and the ion.

We have a strictly conserved angular momentum p_θ due to the axially symmetry of the system considered. We give the relation between p_θ and the cylindrical coordinates (R, Z) of the gyration center, in the neighborhood of the line and the point cusps. In the Cartesian coordinate, p_θ is written as

$$p_\theta = xp_y - yp_x \ ,$$

where momentum $\vec{p}$ is given by

$$p_x = M(v_x - \omega_c \, y/2),$$

$$p_y = M(v_y + \omega_c x/2).$$

For the sake of simplicity, we take a symmetric cusp ($B_r = rB_0/L$, $B_z = -2zB_0/L$), since it is easy to generalize the results to the case of Fig.2. Using the eq.(3), we have, in the neighborhood of the point cusp,

$$Z(R^2 - \rho^2) = - \frac{Lp_\theta}{M\omega_c{}^0} \ , \qquad (4)$$

$$\rho \equiv v_\perp/\omega_c, \quad \omega_c{}^0 = \frac{1}{2}|\omega_c(z=\pm L)| \quad = \frac{eB_0}{MC.}$$

In the neighborhood of the line cusp, we have

$$Z \ R^2 = - \frac{Lp_\theta}{M\omega_c{}^0} \ , \qquad (5)$$

because the velocity in the θ-direction, $v_\theta = p_\theta/Mr + \omega_c{}^0 rz/L$, vanishes there.

If a particle starting from Z = -L, with initial condition $R = R_{p1}$ and $\rho = \rho_{p1}$, reaches R = L, the z-coordinate Z_ℓ of its gyration center is given by

$$Z_\ell = \frac{-R_{p1}^2 + \rho_{p1}^2}{L} \quad , \tag{6}$$

using the relations (4) and (5). We see that Z_ℓ is negative (positive) when the particle starts from the left along an off-axis (axis-encircling) orbit, $R_{p1} > \rho_{p1}$ ($R_{p1} < \rho_{p1}$). For a fixed value of Z_ℓ there is a region of z($Z_\ell - \rho_\ell \leq z < Z_\ell + \rho_\ell$) accessible for the particle. The maximum and the minimum values of Z_ℓ determine the existence region of each species at R = L, such that

$$-\frac{R_{p1}^2}{L} - \rho_\ell < z < \frac{\rho_{p1}^2}{L} + \rho_\ell, \tag{7}$$

where ρ_ℓ is the Larmor radius at R = L. If we consider the case

$$\rho_{p1}^2/L << \rho_\ell,$$

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the gyration centers will be localized to $Z_\ell \simeq 0$, then the width of the existence region for each species become the order of ρ_ℓ in the line cusp. It is easy to treat the general cusp field (Fig.2), the results being as follows: the left-half width, $\Delta\ell_1$, of the plasma in the line cusp is given by

$$\Delta\ell_1 = \frac{B_{p_1}}{2B_\ell} \frac{R_{p_1}^2}{L} + \rho_\ell , \tag{8-a}$$

and the right-half width, $\Delta\ell_2$, of the plasma in the line cusp is also given by

$$\Delta\ell_2 = \frac{B_{p_1}}{2B_\ell} \frac{\rho_{p_1}^2}{L} + \rho_\ell , \tag{8-b}$$

where B_{p_1} and B_ℓ are the strength of the magnetic field in the left point cusp and the line cusp, respectively.

If a particle starting from Z = -L with the initial condition $R = R_{p_1}$ and $\rho = \rho_{p_1}$ reaches Z = +L, by using (4) and (5) we have,

$$R_{p_1}^2 - \rho_{p_1}^2 = -R_{p_2}^2 + \rho_{p_2}^2 , \tag{9}$$

where the suffix 2 represents the value at Z = +L. When the particle starts from Z = -L in the off-axis condition, $R_{p_1} > \rho_{p_1}$, it reaches Z = +L along an encircling orbit, $R_{p_2} < \rho_{p_2}$,[7] so that the radius, Δr, of the existence region at Z = +L is $\Delta r \leq 2\,\rho_{p_2}$. When the particle starts from Z = -L encircling the axis, it reaches Z = +L in the off-axis condition, the maximum radius being estimated as

$$\Delta r < \mathrm{Max}(R_{p_2}) + \rho_{p_2} \sim (1+\sqrt{2})\,\rho_{p_2} . \tag{10}$$

Here the eq.(9) and the relation $\rho_{p_1} \sim \rho_{p_2}$ are used. For the general cusp field (Fig.2), the generalized relation of eq.(9) is written as

$$B_{p_1}(R_{p_1}^2 - \rho_{p_1}^2) = -B_{p_2}(R_{p_2}^2 - \rho_{p_2}^2), \qquad (9')$$

which directly gives the radius as

$$\Delta r \lesssim (1 + \sqrt{1 + \frac{B_{p_2}}{B_{p_1}}})\rho_{p_2} \quad . \qquad (10')$$

According to the relation (10), we may conclude that the radius of the plasma in the right point cusp should be of the order of the ion Larmor radius. This is apparently inconsistent with the experimental result that the ratio $\Delta r/\rho_{p_2 i}$ is of order $2\sqrt{L/\rho_{\ell i}}$. In the next section this inconsistency will be resolved.

§4. "Counterflow of electrons"

According to the result of the previous section, the width (radius) of each species should be the order of each Larmor radius in the line (point) cusp. We may say that electrons are difficult to enter in the right region. Most of electrons starting from the left point cusp in the off-axis condition go towards the line cusp, because of their small Larmor radius. But only a small part of electrons reach the right point cusp, when the distances of their gyration centers from the z-axis are less than several times the Larmor radius.

Although a considerable number of ions enter once the right region, they are pulled back because of the charge separation between ions and electrons, so that it should also be difficult for ions to enter deeply into the right region near the line cusp. We can hardly understand the experimental result that the bright region of the plasma is almost symmetric between the left and the right region.

The experimental result implies that electrons drift effectively across the magnetic field in the right region. There may be a consistent explanation for this effective crossing. The electrons leaking through the line cusp will encounter the conductive wall of the vacuum chamber. There occurs a certain effective crossing mechanism and electrons flow again backwards into the ion rich region. Ions move to cancel the charge separation caused by the electrons' counter-flow, because of their larger Larmor radius than the electrons. As electrons flow back almost along the magnetic line of force, the boundary of the plasma becomes nearly identical with a line of force, as shown by the dotted line in Fig.3.

If we can prevent electrons from counterflowing, the existence region of electrons will be determined by the single particle theory in the previous section. Ions can hardly go out of the electron region, since otherwise they produce charge separation. The small deviation of the ion region from the electron region makes the sheath, whose width will be less than the ion Larmor radius. Then the plasma in the right point cusp will become so slender for the rf field to be applicable.

This situation seems to be attained if the electron counter-flow is successfully suppressed by, for example, making use of insulating material for the wall of the vacuum chamber.

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Figure Captions

Fig.1 A linear system composed of a certain axially symmetric system and two cusp geometries. The rf electric field is applied to the line and the point cusps.

Fig.2 The general cusp field with the symmetry z-axis.

Fig.3 The dotted and the hatched regions denote the ion and the electron regions respectively. The dotted line is the plasma boundary when the counterflow of electrons exists, while the broken line is the plasma boundary when the counterflow is suppressed.

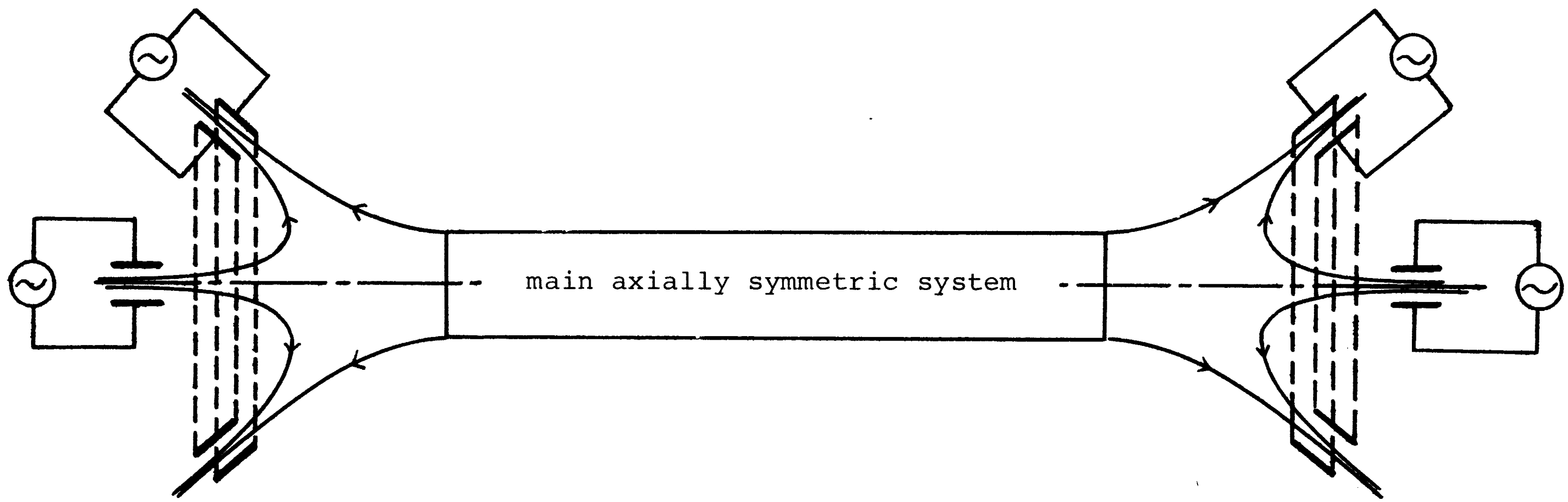

FIG. 1

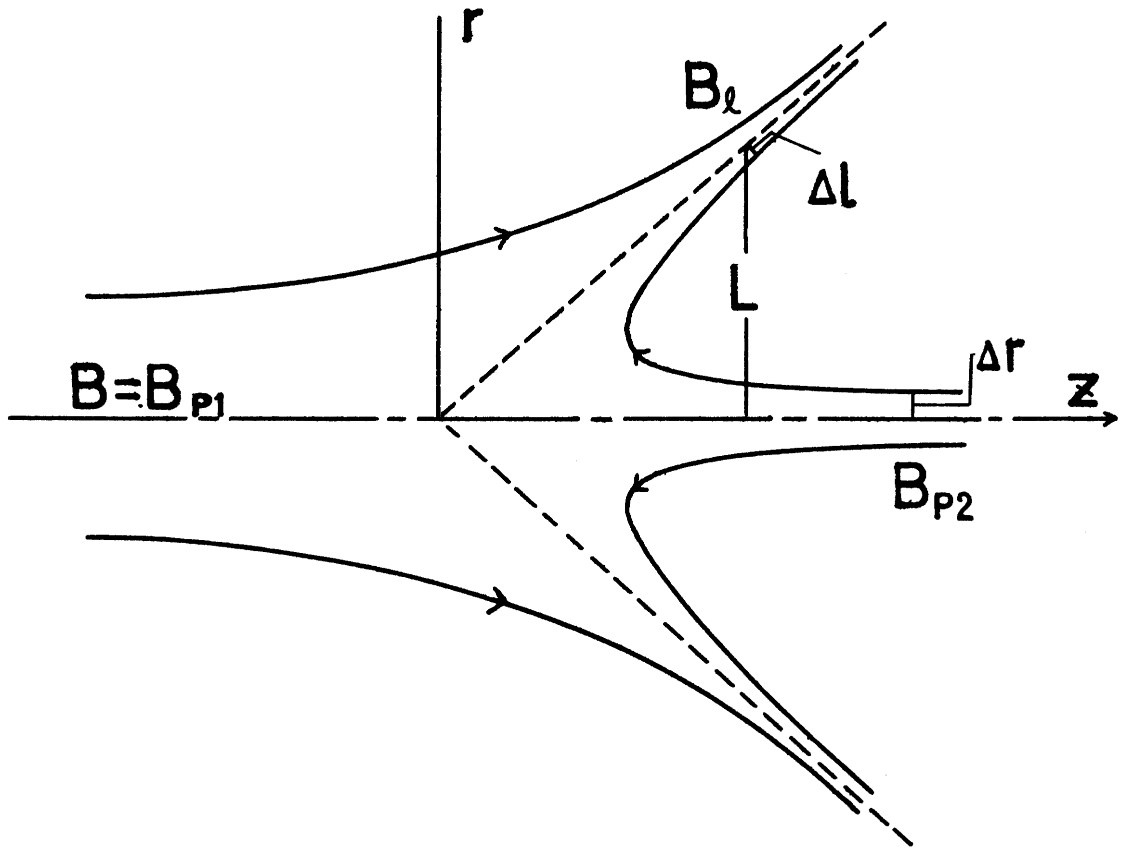

FIG. 2

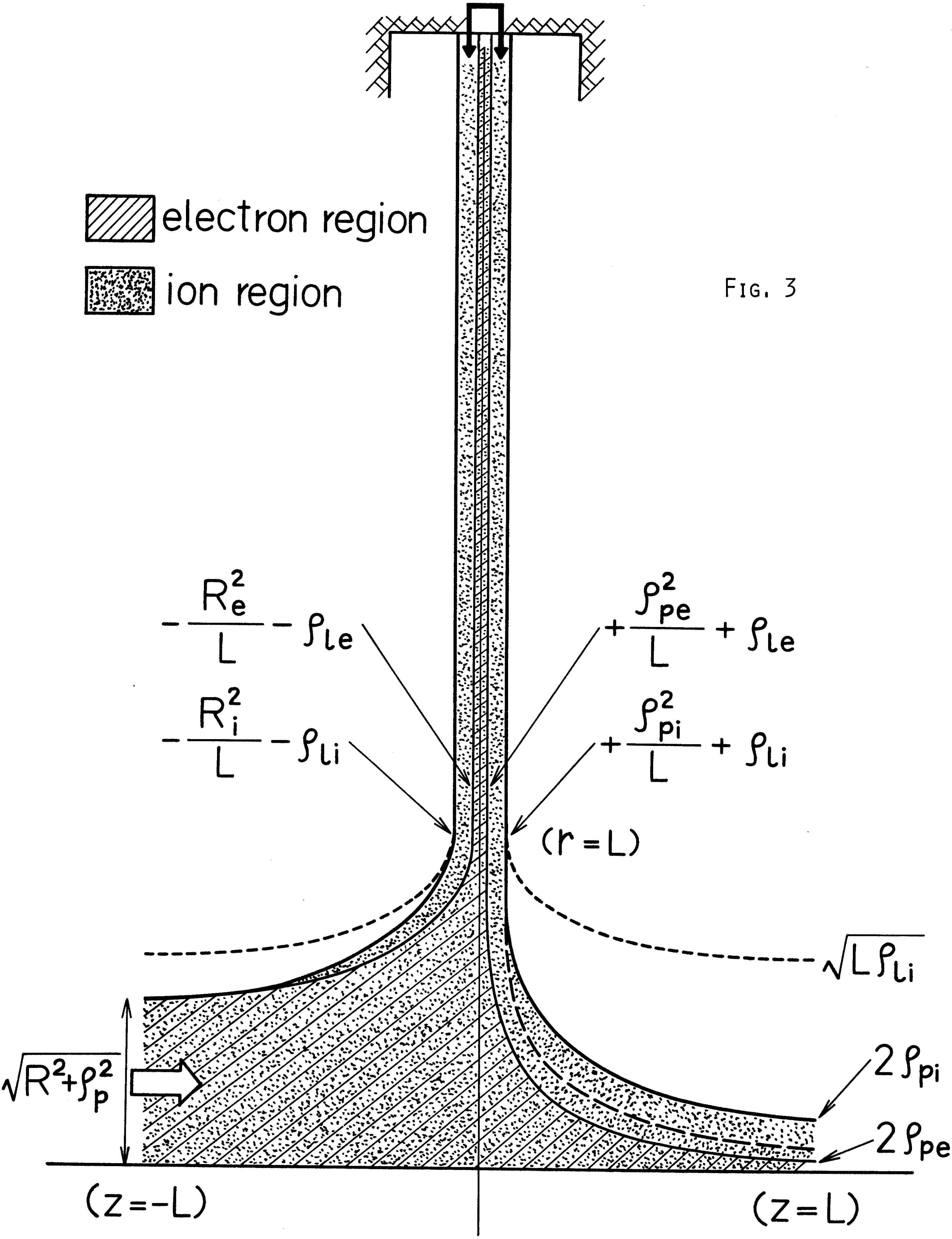
electron region
ion region
$-\frac{R_e^2}{L} - \rho_{Le}$
$-\frac{R_i^2}{L} - \rho_{Li}$
$+\frac{\rho_{pe}^2}{L} + \rho_{Le}$
$+\frac{\rho_{pi}^2}{L} + \rho_{Li}$
(r = L)
$\sqrt{L\rho_{Li}}$
$\sqrt{R^2+\rho_p^2}$
$2\rho_{pi}$
$2\rho_{pe}$
(z = −L)
(z = L)

FIG. 3