INSTITUTE OF PLASMA PHYSICS

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RESEARCH REPORT

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AMPLIFICATION OF SOUND WAVES IN PARTIALLY IONIZED GASES

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Synopsis

The behavior of the sound wave in a positive column of grow discharge has been experimentally investigated.

When the sound wave propagates in the positive column, the amplification of the sound wave is observed. The growth rate of the amplitude and the phase velocity are increased with the discharge current. When the discharge tube is cooled down to the liquid nitrogen temperature, the growth rate increases three times as much as that in the room temperature.

The experimental results suggest that the sound wave is unstable in the grow discharge.

§1. Introduction

This paper reports an experimental study of the sound waves propagating in a weakly ionized gas. The behavior of the sound waves in the partially ionized gases is affected by the transfer of the energy among electrons, ions, and neutral atoms which does not take place in the regular gases. The observations of the spontaneously excited sound waves in gaseous discharges[1-3] have motivated some theoretical analyses of the unstable sound waves in such a medium.[4-7] It has been pointed out that the sound wave is unstable because the electrons nonuniformly heat the neutral atoms through elastic collisions. To confirm this mechanism, Fitaire and Mantei[8,9] conducted an experiment. Although they observed spatial growth of the externally excited wave, the observed growth rate was inconsistent with the theoretical prediction. The present experiment is attempted to make the qunatitative measurements of the growth rate and the phase velocity and comparison with the theoretical analyses.

We analyze the wave propagation in the next section in such a way that the final expressions are conveniently compared with the experimental results. Section 3 will be devoted for the description of the experimental results, and discussions will be given in Sec.4.

§2. Analysis of Wave Propagation

We first analyze the propagation of sound wave in a cylinder filled with weakly ionized gas (positive column). Basic equations which describe the sound wave are the momentum equation and the equation of continuity for neutral atoms,

$$M_n N_n \frac{\partial V_n}{\partial t} = -\nabla P_n + \frac{4}{3}\xi \nabla \cdot \nabla V_n \qquad (1)$$

and

$$\frac{\partial N_n}{\partial t} + \nabla \cdot (N_n \mathbb{V}_n) = 0 \qquad (2)$$

Since our main attention is directed to the effects resulting from energy transfer from the electrons to the neutral atoms through the elastic collisions, we employ the second moment equation in addition to eq.(1) and eq.(2),*

* The heat source term ηj^2 will be discussed in Appendix.

$$\frac{3}{2} N_n \frac{\partial T_n}{\partial t} + N_n T_n \nabla \cdot \mathbb{V}_n + \nabla \cdot q_n = \eta |j_o|^2 \qquad (3)$$

Here, N_n, V_n, P_n, T_n, M_n, q, and ξ are the particle density, the fluid velocity, the pressure, the temperature, the mass, the heat flux and the viscosity of the neutral gas respectively. In eq.(3) we have made following assumptions. The DC current due to the drift of the electrons heats the electrons by themselves. Then, all the thermal energy of the electrons is transferred into the neutral atoms before electrons hit the wall and loss their energy. Under this assumption, the heat source term for the neutral gas is $\eta|j_o|^2$, where η is the resistivity of the electron $\frac{3}{2}(m\nu_{en}/N_e e^2)$ and j_o the current density in the positive column.

Let us consider a plane wave mode in the cylinder for which only the axial components of v_n and q_n have to be retained. We linearize eqs.(1) - (3) and assume the first-order quantities are proportional to $\exp(ikz - i\omega t)$. With the aid of a relation $q_n = -\frac{3}{2} N_n \kappa \nabla T_n$, the first-order equations are

$$-i\omega\frac{3}{2}N_n t_n + ikT_n N_n v_n + \frac{3}{2}N_n \kappa k^2 t_n = \eta_o j_o^2 \frac{v_n}{(\frac{\omega}{k})}, \quad (4)$$

$$-i\omega n_n + iN_n k v_n = 0 \quad (5)$$

and

$$-i\omega N_n M_n v_n = -iN_n k t_n - ikT_n n_n - \frac{4}{3}\xi k^2 v_n \quad (6)$$

Here, the capital letters denote unperturbed quantities and the small letters the perturbed ones. Note that the resistivity η is not a constant because the neutral particle density oscillates. The first-order heat source term, right-hand-side of eq.(4), is calculated with the aid of eq.(5). Here, η_o is $(\frac{3}{2}m_e\langle\sigma\rangle v_{th}N_n/N_e e^2)$. The determinant of eqs.(4)-(6) expresses a dispersion relation

$$\frac{\omega^2}{k^2} = C^2 + i\left[\frac{\eta_o j_o^2}{\rho_n\omega} - \frac{4}{3}\frac{\xi}{\rho_n}\omega - \frac{2}{5}\kappa\omega\right] , \quad (7)$$

where ρ_n is the mass density ($= M_n N_n$). This dispersion relation is just one that is obtained by Tsendin.[5)]

Since the spatial damping or growth of the wave is measured in the experiment, the frequency ω should be a real number and the wavenumber k should be complex. From eq.(7), if we assume that real part of wavenumber k_r is much larger than imaginary part k_i, the real and the imaginary part of k are

$$k_r = \frac{\omega C}{\sqrt{C^4 + \left(\frac{\eta_o j_o^2}{\rho_n\omega} - \frac{4\xi\omega}{3\rho_n} - \frac{2}{5}\kappa\omega\right)^2}} \quad (8)$$

$$K_i = \frac{1}{2}\frac{\frac{\omega}{c}\left(\frac{\eta_0 j_0^2}{\rho_n\omega} - \frac{4}{3}\frac{\xi}{\rho_n}\omega - \frac{2}{5}\kappa\omega\right)}{\sqrt{c^4 + \left(\frac{\eta_0 j_0^2}{\rho_n\omega} - \frac{4}{3}\frac{\xi\omega}{\rho_n} - \frac{2}{5}\kappa\omega\right)^2}} \tag{9}$$

and

$$\frac{K_i}{K_r} = \frac{1}{2}\frac{\eta_0 j_0^2}{\rho_n c^2\omega} - \frac{2}{3}\frac{\xi}{\rho_n}\frac{\omega}{c^2} - \frac{1}{5}\frac{\omega\kappa}{c^2},$$

or

$$\frac{K_i}{K_r} = \frac{1}{2}\left(\frac{E}{P}\right)\frac{j_0}{\gamma\omega} - \frac{2}{3}\frac{\xi}{\rho_n}\frac{\omega}{c^2} - \frac{1}{5}\frac{\omega k}{c^2}, \tag{10}$$

where the relations $\eta_o j_o = E$, and $c^2 = \frac{\gamma P}{\rho_n}$ have been used. Since the sound wave propagates in the cylinder of radius "a", we add the friction terms[4)]

$$\text{Friction term} = \sqrt{\frac{\xi}{2\omega\rho_n a^2}} + \sqrt{\frac{\kappa}{2\omega\rho_n C_p}} \tag{10'}$$

to the right hand side of eq. (10), where c_p is the specific heat constant per unit mass at constant pressure. When ω is small the effect of friction between the gas and the wall dominates the other effect since the dissipation terms are proportional to ω and the friction term is to $\frac{1}{\sqrt{\omega}}$.

Joule heating of the gas by DC discharge current affects the feature of the propagation of sound waves in the following two ways. One is the transfer of the energy of the electrons to the neutral particles which occurs selectively at the compressed phase of the wave. This mechanism is already taken into account in eq.(4) and results in amplification of the wave [eq.(10)]. The other is a change in sound speed caused by overall heating of neutral gas. The increment of the temperature is calculated from eq.(3). If the length of the positive column is much longer than its radius, the variation of the zeroth-order quantities along Z direction can be ignored. With aid of the relation $q_n = -\frac{3}{2} N_n \kappa \nabla T_n$, and from the assumption that κ is proportional to $\frac{T}{M\nu_{nn}}$, eq.(3) is written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T^2}{\partial r}\right) = -\frac{4\eta_0 j_0^2 M_n \nu_{nn}}{3N_n} \tag{11}$$

in the cylindrical coordinates. This equation is integrated with the boundary condition $T = T_s$ at the wall $r = a$, and T^2 is given as

$$T^2 = T_s^2 - \frac{\eta_0 j_0^2 M_n \nu_{nn}}{3N_n}(r^2 - a^2) . \tag{12}$$

We have assumed the current density j_0 is uniform in the positive column. Increment of the temperature changes the sound speed as given by

$$C^4 = C_0^4 + \frac{\eta_0 j_0^2 \nu_{nn} \gamma^2 a^2}{6 N_n M_n} , \tag{13}$$

where C_0 is the sound speed without discharge, and γ the adiabatic constant. With the aid of $E = \eta_0 j_0$, above equation is written as

$$\left(\frac{C}{C_0}\right)^2 = \left(1 + \left(\frac{E}{P}\right) \dot{J}_0 \frac{\gamma \nu_{nn} a^2}{6 C_0^2}\right)^{\frac{1}{2}} \qquad (14)$$

§3. Experiments

Experiments are carried out in the discharge tubes filled with gas (He, Ar, or Ne) at the pressure of several Torr. The sound wave is launched by a horn type·loud speaker set at the end of the tube and is received by a microphone. One of the set-up employed in the present experiments is shown in Fig.1. The plasma is produced by DC discharge in the region between the ladder cathode and the mesh anode. The length of the plasma is variable. The sound system is set carefully in such a way that the signal transmitted through the tube wall is eliminated. The frequency studied is in a range between 3 kHz and 8 kHz. The typical discharge parameters are as follows; the electron temperature T_e = (2 - 3) eV; the electron density $n_e = 10^9 - 10^{10}$ cm^{-3}, the neutral gas pressure P_n = several Torr. The electron temperature is almost independent of the discharge current while the density is linearly proportional to the discharge current. The values of E/P are 1.5 ∿ 2 V/cm Torr for argon and neon and 4.5 ∿ 5 V/cm Torr for helium, where E is the static electric field in the plasma.

3.1 Sound Velocity

The dispersion relation (the relation between the frequency and the axial wavenumber k) is measured before the discharge is turned on. The experiments show that the dominant mode of wave propagation is well described by

$$k^2 + K^2 = \frac{\omega^2}{c^2} , \qquad (15)$$

where K = 3.83/a and, a, is the radius of the tube.

The variation of the phase velocity due to the discharge is measured by employing the interferometer. The procedure of the experiment is as follows: The phase delay of the received signal is measured first, when the discharge is run. Then, the length of the plasma is changed by moving the anode position and the change of the phase $\Delta\phi$ from the original phase is measured as a function of the increment of the plasma length ΔL. A typical result is plotted in Fig.2 which shows that the plasma increase the phase velocity. A measurement in the case when the direction of the electron drift is reversed shows no difference from the data shown in Fig.2. The phase velocity V_g in the tube filled with the plasma is obtained by employing the relation

$$V_g = V_{go} \left(1 + \frac{\Delta\varphi}{\Delta L} \frac{V_{go}}{\omega} \right)^{-1} , \qquad (16)$$

where V_{go} is the phase velocity in the tube when no plasma is in the tube. The dependence of V_g/V_{go} on the discharge current obtained by above procedure is shown in Fig.3. The relation (14), shown by the solid curve, fits the experimental points very well. Therefore, the assumption that "almost all energy dissipated into the electron thermal energy is transferred to the neutral atoms" seems correct.

3.2 Growth Rate

When no plasma is in the tube, the wave damps because of the viscosity and the heat conduction. If the plasma is produced by the small discharge current, then the damping is observed to be reduced. When the discharge current is increased, the wave starts growing in space.

As shown in Fig.4, the growth rate is linearly proportional to the discharge current. It is also found that the value of k_i is independent of the frequency. There is a little difference in the growth rate between the case when the wave propagation direction is parallel to the direction of the electron drift and the case when they are anti-parallel. The growth rates of the waves in argon, helium, and neon are measured. The growth rate in helium is about twice as much as that in argon and neon. However, the quantity $(k_i/k_r)(P/E)$ is independent of the kind of gas (Fig.5). The scattering of the experimental points in Fig.5 mainly results from poor reproducibility of the discharge. The solid line which represents eq.(10) fits the experimental result with a difference of factor 2.

The experiments performed in two different diameter tubes show that k_i/k_r is in a order of magnitude $\sim 10^{-3}$ in the discharge tube of 4 cm diameter. On the other hand, no observable amplification of the wave is found in the

10 cm diameter tube, because the amplification terms of eq.(10) is small compared with the dissipation ones.

3.3 Effect of the Temperature of Neutral Gas

When the discharge tube, i.e., the neutral gas, is cooled, an enhancement of the growth rate is observed. The experiments are carried out using a U shaped discharge tube which is cooled by liquid nitrogen (Fig.6). The growth rate measured when the tube is cooled is compared to that when no cooling in Fig.7. It is shown that the value of k_i/k_r under cooling is 2 $\sim$ 3 times greater than that of without cooling.

According to eq.(10), k_i/k_r is proportional to p^{-1} so that to T^{-1}. Therefore k_i/k_r at 77°K is predicted to be 3.8 times as much as that at room temperature. The disagreement between the experimental value and the prediction of eq.(10) would be due to the fact that the temperature of the gas is higher than 77°K since the discharge current heats the gas. Indeed, the heating of the gas is found by monitoring the gas pressure by oil manometer.

§4. Discussions

First of all we are interested in the dependence of the growth rate on the tube diameter. The experiments, described above, have confirmed that the growth rate is proportional to $(E/p)\cdot j$. According to the classical theory for the positive column,[10] the value of E/p is a function of the radius of the discharge tube. The value of E/p is large when the radius is small. If we regard the discharge tube as a resonant cavity, then the spontaneous excitation of the sound wave occurs when the amplification overcomes the damping induced by the viscosity, the heat conduction and friction to the wall. This instability condition is easily satisfied when the radius of the tube is small.

§5. Conclusion

In conclusion, (1) the amplification of the sound waves is observed experimentally. A theory, which takes into account the mechanism that the electron thermal energy is selectively fed to the neutral gas, accounts for the observed growth rate. (2) The growth rate in the lower temperature gas is observed to be greater than that in the higher temperature gas. (3) The phase velocity of the sound wave is observed. The increase of the sound speed can be explained by overall heating of neutral atoms by the discharge currents.

APPENDIX

In this appendix we discuss the source term of the form $\eta_o j^2$ which is appeared in eq.(3). The energy of directed electron drift due to external field E is dissipated into thermal energy through elastic collisions between the electrons and the neutral atoms. The corresponding heating rate is $N_e eEv_d$, where v_d is the drift velocity of the electrons.

Through the elastic collisions, the electron thermal energy is transferred to the neutral atoms by the rate represented by $\frac{1}{2}\mu N_e m_e v_e^2 (v_e/\lambda_e)$. Here v_e, λ_e, μ denotes the thermal velocity and mean-free-path of the electrons and mass ratio between an electron and a neutral atom, respectively. The conservation of energy for electrons is

$$\frac{d}{dt}\left(\frac{N_e m_e v_e^2}{2}\right) = N_e e E v_d - \mu\left(\frac{N_e m_e v_e^2}{2}\right)\left(\frac{v_e}{\lambda_e}\right) \qquad (A1)$$

The heat loss directly dissipated to the wall is neglected. Since sound frequency is very low compared to v_e/λ_e, a stationary state is reached. Neglecting the time derivative, eq.(A1) is written as

$$\eta_o \dot{j}_o^2 = \mu\left(\frac{N_e m_e v_e^2}{2}\right) \nu_{en} = \frac{3}{2}\mu \nu_{en} P_e \qquad (A2)$$

Here, the relation $E = \eta_o j$ is used. If we substitute $\eta_o j_o^2$ in eq.(A2) into eq.(10), the growth rate is written as

$$\frac{k_i}{k_r} = \frac{\eta_o \dot{j}_o^2}{2\rho_n c^2 \omega} = \frac{3}{4}\left(\frac{\nu_{en}}{\omega}\right)\left(\frac{P_e}{P_n}\right)\mu \qquad (A3)$$

which are the expressions obtained by Tsendin[5] and Ingard.[4]

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FIGURE CAPTIONS

Fig.1. Schematic diagram of the experimental arrangement of 10 cm diameter discharge tube.

Fig.2. The change of the phase $\Delta\phi$ from the original phase are shown as a function of plasma length ΔL.

Fig.3. The dependence of V_g/V_{go} on the discharge current. Here, V_g and V_{go} are the phase velocity in the tube with and without discharge respectively. The solid line denotes the relation (14).

Fig.4. Dependence of the growth rate on the discharge current. The circles denote the growth rate parallel to the electron drift velocity, the triangles antiparallel to the electron drift.

Fig.5. Dependence of $\frac{k_i}{k_r}(\frac{E}{p})^{-1}$ on the discharge current. k_r and k_i are the real and imaginary part of the wave-number, P and E are the neutral gas pressure and the electric field in the positive column respectively. The solid line denotes eq.(10). The figure shows that the value of $(k_i/k_r)(P/E)$ is independent of the gas species.

Fig.6. The U shaped discharge tube which is cooled by liquid nitrogen.

Fig.7. The growth rate measured when the tube is cooled (77°K) and when it is not cooled (300°K).

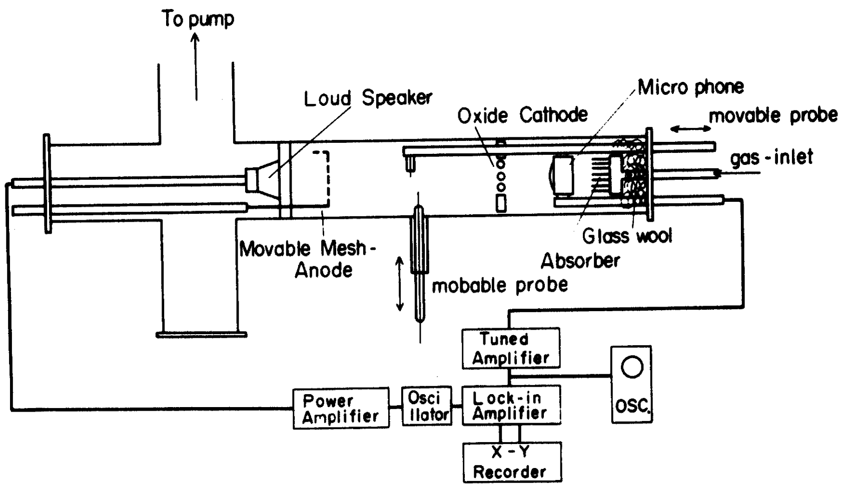
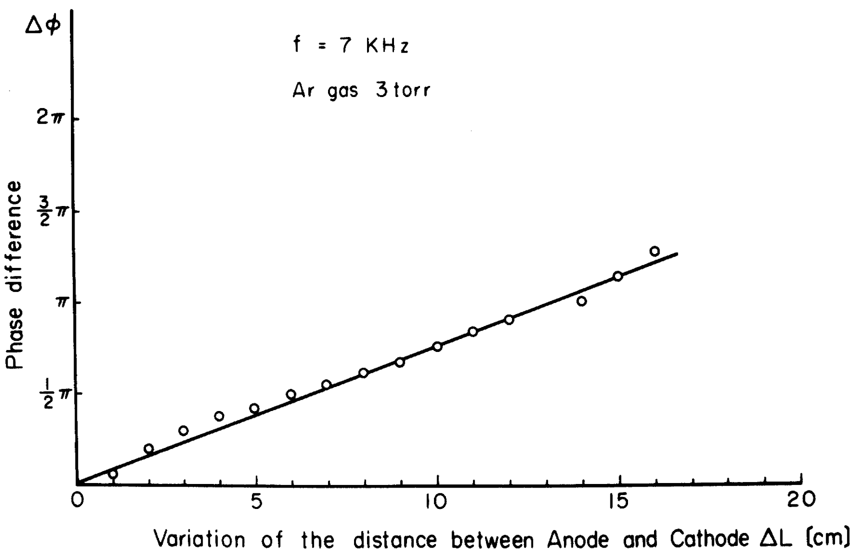
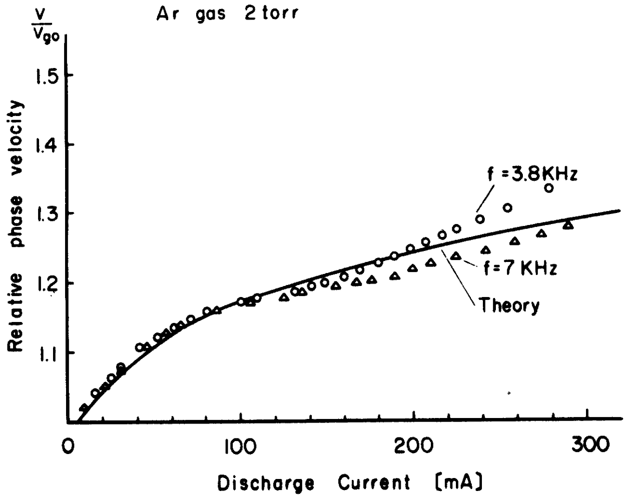

To pump
Loud Speaker
Oxide Cathode
Micro phone
movable probe
gas - inlet
Glass wool
Absorber
Movable Mesh-
Anode
mobable probe
Tuned
Amplifier
Power
Amplifier
Osci
llator
Lock-in
Amplifier
OSC.
X - Y
Recorder

Fig. 1



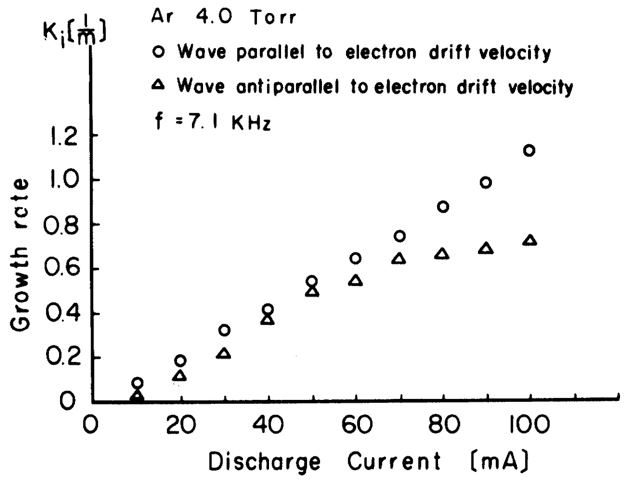
Δφ
f = 7 KHz
Ar gas 3 torr
2π
3/2 π
π
1/2 π
Phase difference
0
5
10
15
20
Variation of the distance between Anode and Cathode ΔL (cm)

Fig. 2



Ar gas 2 torr
v/Vgo
Relative phase velocity
1.5
1.4
1.3
1.2
1.1
f =3.8KHz
f=7 KHz
Theory
0
100
200
300
Discharge Current [mA]

Fig. 3



Ar 4.0 Torr
o Wave parallel to electron drift velocity
△ Wave antiparallel to electron drift velocity
f = 7.1 KHz
$K_i[\frac{1}{m}]$
Growth rate
1.2
1.0
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
Discharge Current [mA]

Fig. 4

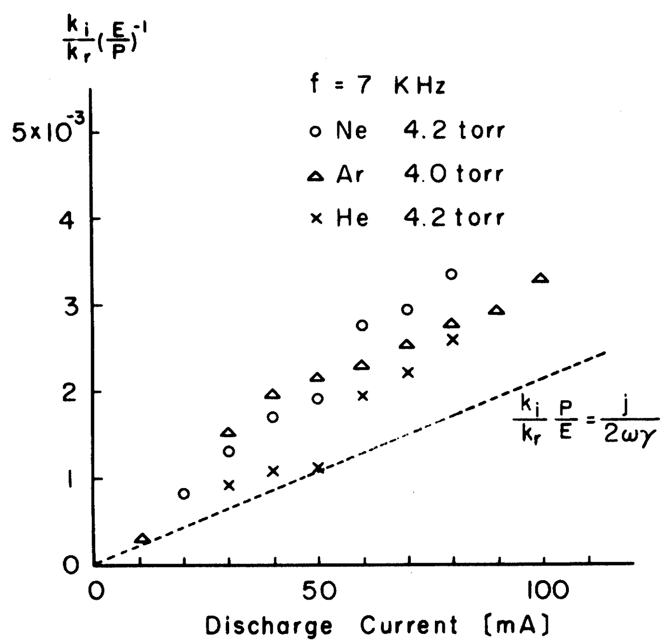
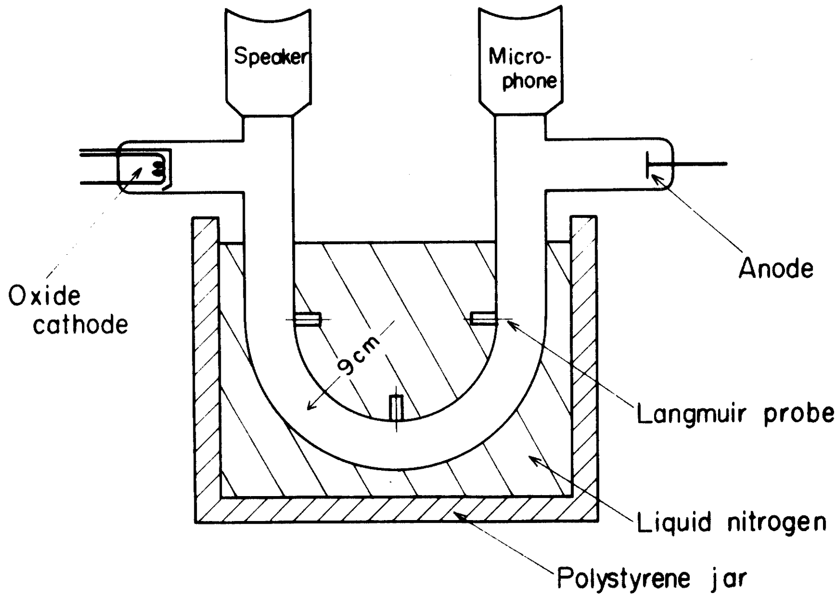
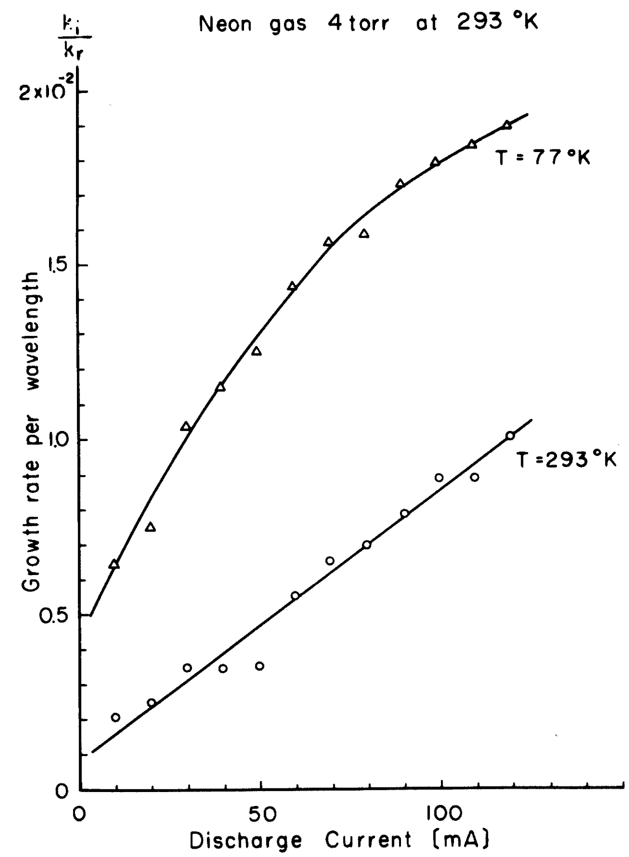

$\frac{k_i}{k_r}(\frac{E}{P})^{-1}$
f = 7 KHz
o Ne 4.2 torr
△ Ar 4.0 torr
× He 4.2 torr
5×10^{-3}
4
3
2
1
0
0
50
100
Discharge Current [mA]
$\frac{k_i}{k_r}\frac{P}{E}=\frac{j}{2\omega\gamma}$

Fig. 5



Speaker
Micro-
phone
Oxide
cathode
Anode
9cm
Langmuir probe
Liquid nitrogen
Polystyrene jar

Fig. 6



Neon gas 4 torr at 293 °K
$\frac{k_i}{k_r}$
2×10⁻²
1.5
1.0
0.5
0
Growth rate per wavelength
T = 77 °K
T = 293 °K
0
50
100
Discharge Current (mA)

Fig. 7