INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

RESEARCH REPORT

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A THEORETICAL PREDICTION OF THE OBSERVED PLASMA HEATING IN THE COMPRESSION EXPERIMENT OF ITO'S GROUP AT OSAKA UNIVERSITY

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

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Abstract

A simple, model-independent description of the compression experiment of Itô's group at Osaka University predicts the observed plasma heating and final plasma β to within a factor of two.

Introduction

In the fast compression experiments of the group of Itô at Osaka University, remarkable plasma temperatures and densities have been produced. The purpose of this report is to demonstrate that the temperatures produced in the experiment can be theoretically predicted on the basis of a very simple thermodynamic computation. The computation does not require the use of any specific heating model, and so is valid for many experiments.

Let us first examine the experiment. Streams of hot deuterium plasma from two θ-pinch guns flow axially along a guide field into a powerful θ-pinch coil. The plasma is then compressed radially by a magnetic field with a rise-time of 1.4 μsec and a maximum field strength of 25 kG. The plasma does not seem to be compressed adiabatically, because the magnetic field penetrates the plasma column. Also, the plasma column does not decrease in radius as much as expected if adiabatic compression were present. The experimental results for three conditions of operation,[1] C_1, C_2, C_3, are summarised below in Table 1.

In addition, the theoretical predictions for the energy content and β of the plasma are compared with the results. The arrangement is observed to be excellent!

The theory is remarkably simple.[2] It assumes that a magnetic field is imposed suddenly on the surface of a

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1. Prof. Hiroshi Itô, private communication.
2. Observed by me first in an unpublished report by Prof. George Schmidt of Stevens Institute of Technology.

plasma. This means in a time short compared to its normal penetration time. It also assumes that the magnetic field be allowed subsequently to penetrate the plasma by any process whatever. Then, as a result, Poynting's vector shows that an energy flow through the surface of the plasma must have occurred at such a rate that the average energy content per cm^3 in the plasma must be $\frac{B_o^2}{4\pi}$, where B_o is the field strength of the final magnetic field in the plasma. Since the magnetic energy stored per cm^3 in the magnetic field in the plasma is $\frac{1}{2}\frac{B_o^2}{4\pi}$, the difference between this magnetic energy and the input energy must appear as heat in the plasma. In terms of mathematics, we find that,

$$\frac{B_o^2}{4\pi} - \frac{1}{2}\frac{B_o^2}{4\pi} = \frac{B_o^2}{8\pi} = \frac{3}{2}NK(T_e + T_i) \quad .$$

Note that we have implicitly assumed that the rate of heat loss is slow compared to the time scale of the experiment. This is the remarkably simple expression used to predict the theory value in Table 1. In addition, since the theory predicts that the thermal and the magnetic energy in the plasma are comparable the theory predicts a final value for $\beta \approx 0.5$.

Since the applied magnetic field penetrates anomalously rapidly, this suggests that the heating process is some sort of anomalous heating or turbulent process. This question does not affect our calculations, however.

The detailed calculations for the theory described above will be given later in the paper. Now we give a simple analogy to demonstrate the basic concepts of the theory. Consider the simple resister-capacitor circuit shown in Fig.1. Suppose a voltage V_o be suddenly impressed across the input terminals. The rate of power flow into the circuit shown in Fig.1 is given by,

$$V_o i = V_o \frac{dq}{dt} \quad ,$$

where i is the current flow into the system, and is equal to the rate of charge flow, $\frac{dq}{dt}$. The total amount of work W that flows into the system is given by

$$W = \int_0^\infty V i \, dt = V_o \int_0^\infty \frac{dq}{dt} dt = V_o q_o$$

where q_o is the total amount of charge that has flowed into the system. But we know that q_o has flowed into the system when the capacitor C is fully charged, so,

$$q_o = C V_o$$

Hence, the value of W is,

$$W = C V_o^2 \quad .$$

However, we know that the amount of work stored in a fully charged capacitor is $\frac{1}{2} C V_o^2$. Hence the difference must have been dissipated as heat,

$$CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2 = \text{HEAT} \ .$$

This result is most remarkable in that the value of R nowhere appears in the calculation. It can be a constant, an arbitrary function of V_o, or an arbitrary function of time. The form of R does not alter the final total amount of heat produced. In the case of magnetic energy flow, a quite similar argument holds, and the amount of heat produced is independent of the heating mechanism or the rate of heating.

Calculation of Magnetic Power Flow

Consider a long cylinder of plasma, as shown in Fig.2. The power flow P through the cylindrical surface is given by $P = -\int \vec{E} \times \vec{H} \cdot d\vec{A}$, where $\vec{E}$ and $\vec{H}$ are the surface electrostatic and magnetic fields respectively, and $d\vec{A}$ is a unit of surface area. The minus sign is to insure an inward flow of power, as the vector $d\vec{A}$ by convention points outward. Next, by a simple vector law, we note that,

$$\int \vec{E} \times \vec{H} \cdot d\vec{A} = \int \vec{E} \cdot \vec{H} \times d\vec{A} \ .$$

Next, note that $d\vec{A} = d\vec{s} \times d\vec{\ell}$, so

$$\int \vec{E} \cdot \vec{H} \times d\vec{A} = \int \vec{E} \cdot \vec{H} \times (d\vec{S} \times d\vec{\ell}) \ .$$

This can be expanded by a simple vector law, as shown below,

$$\int \vec{E}\cdot\vec{H}\times(d\vec{S}\times d\vec{\ell}) = \int \vec{E}\left\{(\vec{H}\cdot d\vec{\ell})d\vec{S} - (\vec{H}\cdot d\vec{S})d\vec{\ell}\right\} .$$

Next, we assume by the symmetric arrangement that $\vec{H}\cdot d\vec{s} = 0$. Also, we assume that $\vec{H}$ is a constant $\vec{H}_o$ in time, and is uniform along $\vec{\ell}$. Under these assumptions, we find that,

$$\int \vec{E}\cdot\vec{H}\times(d\vec{S}\times d\vec{\ell}) = \vec{H}_0\cdot\vec{\ell}\int \vec{E}\cdot d\vec{S} .$$

But the integral of $\int \vec{E}\cdot d\vec{s}$ can, by Stoke's theorem, be expressed as shown below,

$$\int \vec{E}\cdot d\vec{S} = \int \nabla\times\vec{E}\cdot d\vec{a}$$

Note that $d\vec{a}$ is an area element of a cross section of the plasma column, not the side!

But, by Maxwell's equations, we find that,

$$\int \nabla\times\vec{E}\cdot d\vec{a} = \int -\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a} ,$$

where we note that the value of B corresponds to flux flowing inside the plasma volume, not flowing along the plasma surface. Thus, we find P to be,

$$P = \vec{H}_0\cdot\vec{\ell}\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$$

The total work W entering the plasma through the sides is given by,

$$W = \int P\,dt = \int \vec{H}_0 \cdot \vec{\ell} \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}\,dt = \vec{H}_0 \cdot \vec{\ell} \int \vec{B}_0 \cdot d\vec{a}$$

where $\vec{B}_o$ is the final state of magnetization inside the plasma. We assume $\vec{B}_o$ is everywhere a constant, that $\vec{B}_o = \mu_o \vec{H}_o$, and that $\vec{B}_o$ is parallel to $d\vec{a}$. We then find that $W = (\vec{H}_o, \vec{\ell})\vec{B}_o \cdot \int d\vec{a} = H_o B_o \ell a$, where a is the scalar surface area of the plane cross section. Since ℓa is now the volume of the plasma cylinder being considered, the work input per unit volume on the average is $H_o B_o = \frac{B_o^2}{\mu_o}$. Since the work going into the magnetic field is known to be $\frac{1}{2} \frac{B_o^2}{\mu_o}$, the difference between the above value and this quantity must appear as heat,

$$\frac{B_o^2}{\mu_o} - \frac{1}{2}\frac{B_o^2}{\mu_o} = \frac{1}{2}\frac{B_o^2}{\mu_o} = \frac{3}{2} NK (T_e + T_i)$$

as discussed before. The above is an M.K.S. calculation. The appropriate C.G.S. calculation results in 4π replacing μ_o.

A final note is that the equation above seems to fit the turbulent heating results of the group of Kawabe et al., at Nagoya, if for $\vec{B}_o$ we use an appropriate average value for the current-produced poloidal magnetic field appearing in the experiment. It is possible that the final work content of many experiments may be predicted in a similar fashion.

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		EXPERIMENT					THEORY	
	Condition	$n (cm^{-3})$	T_e (eV)	T_i (eV)	β	$\frac{3}{2}NK(T_e+T_i)$ ergs/cm^3	$\frac{3}{2}NK(T_e+T_i)$ ergs/cm^3	β
C-1	initial	3×10^{14}	12	100	1			
	Final	9×10^{14}	3.5×10^3	4.0×10^3	0.5	1.62×10^7	2.48×10^7	$\gtrsim 0.5$
C-2	Initial	2.5×10^{14}	5	10	0.1			
	Final	2.0×10^{15}	3.2×10^3	3.0×10^3	1	2.96×10^7	2.48×10^7	$\gtrsim 0.5$
C-3	Initial	8×10^{13}	1	≤ 5	<0.1			
	Final	1.5×10^{15}	2.7×10^3	2.0×10^3	0.5	1.68×10^7	2.48×10^7	$\gtrsim 0.5$

Table 1

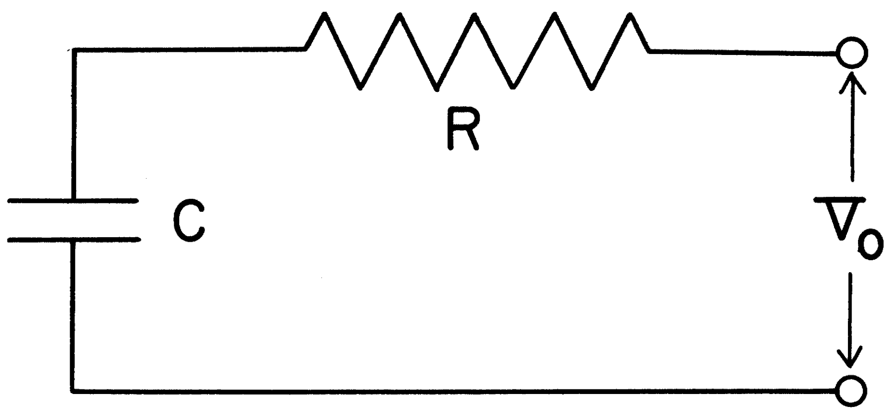

Fig.1. Resistor-capacitor circuit.

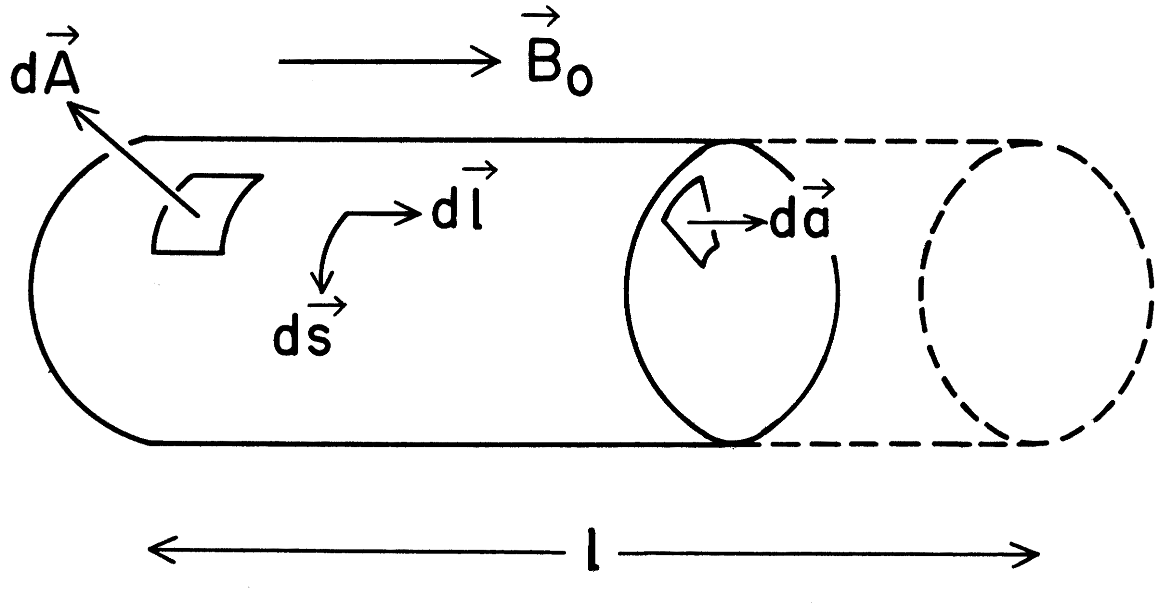

Fig.2. Cylindrical coordinates.