INSTITUTE OF PLASMA PHYSICS

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Diffusion of Plasma Confined in Stellarator with $\varepsilon_t > \varepsilon_h$

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Abstract

Diffusions of plasma confined in a stellarator are discussed when the field modulation ε_t of toroidal curvature is larger than the modulation ε_h of helical field, which is usal case of the toroidal stellarator with medium aspect ratio. The dependence of the diffusion coefficient on the collision frequency ν in the very low frequency region is different in the cases of $\varepsilon_t > \varepsilon_h$ and $\varepsilon_t < \varepsilon_h$.

Diffusions of plasmas confined in stellarator has been discussed in details when the field modulation ε_t of the toroidal curvature is less than the modulation ε_h of helical field.[1)-5)] However, the situation is reverse in the case of toroidal stellarator with medium aspect ratio ($\varepsilon_t > 0.1$). The diffusion of the plasma in the more realistic case is considered here.

Denoting the coordinates in plasma cross section by r, θ and the angle along the minor toroidal axis by ψ, the field modulation of the toroidal stellarator field is described as follows

$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(\ell\theta - m\psi)] \tag{1}$$

$$\varepsilon_t = r/R \qquad \varepsilon_h = \varepsilon_h^\circ (r/a)^\ell \tag{2}$$

where R and a are the major and the minor radius of the plasma respectively. The banana motion trapped by the helical mirror is described using the longitudinal invariant J as follows[6)]

$$J(r,\ \theta,\ W,\ \mu) = m\oint v_{/\!/}\, d\ell$$

$$= (2m)^{1/2}\oint [W - e\phi - \mu B_0(1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(\ell\theta - m\psi)]^{1/2} d\ell$$

$$= 16R|q|[m\mu B_0 \varepsilon_h(r)]^{1/2}[E(\kappa) - (1 - \kappa^2)K(\kappa)] \tag{3}$$

where

$$\kappa^2(r,\theta,W,\mu) = \frac{W - \mu B_0[1 - \varepsilon_t(r)\cos\theta \quad - \varepsilon_h(r) \quad - e\phi(r)]}{2\mu B_0 \varepsilon_h(r)}$$

(4)

$$\frac{d\theta}{dt} = \frac{1}{eBr}\,\frac{\partial J}{\partial r}\Big/\frac{\partial J}{\partial W} \tag{5}$$

$$\frac{dr}{dt} = -\frac{1}{eBr}\,\frac{\partial J}{\partial \theta}\Big/\frac{\partial J}{\partial W} \tag{6}$$

where W and μ are the energy and the magnetic moment of the trapped particle, ϕ being the electrostatic potential. $q = 2\pi/\iota$, ι being the rotational transform angle. These equations are reduced to

$$r\frac{d\theta}{dt} = V_\perp\cos\theta + r\omega_h + r\omega_E \tag{7}$$

$$\frac{dr}{dt} = V_\perp \sin\theta \tag{8}$$

$$\frac{V_\perp}{r} = \varepsilon_t \frac{T}{eBr^2} = \varepsilon_t\omega_0 \tag{9}$$

$$\omega_h = \varepsilon_h 2\ell\left(\frac{E}{K} - \frac{1}{2}\right)\frac{T}{eBr^2} = \varepsilon_h 2\ell\left(\frac{E}{K} - \frac{1}{2}\right)\omega_0 \tag{10}$$

$$\omega_E = \frac{eE_r r}{T}\,\frac{T}{eBr^2} = \alpha\omega_0 \tag{11}$$

$$T = \frac{m}{2}V_\perp^2 \qquad \omega_0 = \frac{T}{eBr^2} \qquad \alpha = \frac{eE_r r}{T} \tag{12}$$

The first term is due to toroidal drift $V_\perp$ and the second and third terms are rotation of banana trapped by helical field due to the stellarator field and the radial electric field. The ratio of three terms are approximately $\varepsilon_t : \varepsilon_h : \alpha$. ω_0 is the rotation frequency of E_r/B when $eE_r r = T$ ($\alpha = 1$). When ε_h is larger than others ($\varepsilon_h >> \varepsilon_t$, α), these equations give the supper banana motion and the diffusion coefficients derived in the reference[1)-5)] are shown in Fig.1.

When $\varepsilon_t >> \varepsilon_h$, α, the toroidal drift is dominant and the step length Δ is $V_\perp/\nu_{eff} = V_\perp \varepsilon_h/\nu$ and the diffusion coefficient is

$$D = \sqrt{\varepsilon_h}(V_\perp/\nu_{eff})^2\nu_{eff} = \varepsilon_h^{3/2}\frac{V_\perp^2}{\nu} = \varepsilon_t^2\varepsilon_h^{3/2}\frac{\omega_0}{\nu}\frac{T}{eB}. \qquad (13)$$

ν_{eff} is the effective collision frequency and ν is the collision frequency. The step length Δ is larger than r, when $\nu < V_\perp \varepsilon_h/r = \varepsilon_t\varepsilon_h\omega_0$, and the trapped banana is immediately escape from plasma region. The confinement time τ in this case is determined by the rate of diffusion in velocity space. The diffusion equation of velocity space is given as follows using the spherical coordinates (v, γ, δ),[7)]

$$\frac{1}{v^2\sin\gamma}D_\perp\frac{\partial}{\partial\gamma}(\sin\gamma\frac{\partial f}{\partial\gamma}) + \frac{1}{v^2}\frac{\partial}{\partial v}[v^2(D_{/\!/}\frac{\partial f}{\partial v} - Af)] = \frac{\partial f}{\partial t} \qquad (14)$$

where D and A are the diffusion tensor and the dynamic friction coefficient and satisfy the relation $mvD_{/\!/}/T + A = 0$.

If the distribution function f is set by

$$f = \Theta(\gamma) e^{-\frac{mv^2}{2T}} e^{-\frac{t}{\tau}} \tag{15}$$

we have

$$\frac{1}{\sin\gamma} \frac{\partial}{\partial\gamma}(\sin\gamma \frac{\partial\Theta}{\partial\gamma}) = -\frac{v^2}{D_{\perp}\tau} \Theta. \tag{16}$$

Boundary condition in velocity space is similar to the mirror field but in opposite way, that is

$$\Theta = 0 \text{ at } \cos\gamma = (1 - \varepsilon_h)^{1/2} \simeq 1 - \frac{1}{2}\varepsilon_h$$

The approximate solution is $\Theta \propto \cos\gamma$ and we have

$$\frac{1}{\tau(v)} = \frac{2D_{\perp}}{v^2} = 2\nu(1 - \frac{T}{2mv^2}) = 2\nu(1 - \frac{\langle v^2\rangle}{6v^2}) \simeq 2\nu .$$

If the diffusion coefficient is formally defined by $D = r^2/5.8\tau$, then it is given by

$$D = \frac{2\nu}{5.8} r^2 \sim \frac{r^2}{3}\nu. \tag{17}$$

This result is similar to the diffusion due to the ripple of toroidal field derived by heuristic discussion of Stringer.[8] When $\nu > \varepsilon_t \varepsilon_h \omega_0$, $D = r^2\nu/3$ gives the upper limit of diffusion coefficient. The diffusion coefficient does not depend on the collision frequency very much and is equal to

$$D_h = \frac{\varepsilon_t \varepsilon_h}{3} \frac{T}{eB} \tag{18}$$

Over all dependence of the diffusion coefficients is shown in Fig.2.

When the plasma potential is order of the plasma temperature($\alpha \sim 1$), ω_E term in equation (7) is dominant and the diffusion coefficient does not depend on the ratio of ε_t and ε_h. The diffusion coefficients are[5]

$$D = \varepsilon_t^2 \varepsilon_h^{3/2} \frac{\omega_0}{\nu} \frac{T}{eB} \qquad \text{for } \nu > \varepsilon_h \omega_E \tag{19}$$

$$D = \varepsilon_t^2 \left(\frac{\nu}{\omega_E}\right)^{1/2} \frac{1}{\alpha} \frac{T}{eB} \qquad \text{for } \nu < \varepsilon_h \omega_E. \tag{20}$$

If ν is less than $\nu_r' \equiv 3\,\varepsilon_t \varepsilon_h^{1/2} \omega_0$ while $\nu > \nu_E \equiv \varepsilon_h \omega_E$, the diffusion coefficient of (20) is larger than those of (17), (18). The relations (20) and (19) hold under the condition of $\varepsilon_h \omega_E = \varepsilon_h \alpha \omega_0 > 3\,\varepsilon_t \varepsilon_h^{1/2} \omega_0$, that is,

$$\alpha > 3\,\varepsilon_t / \varepsilon_h^{1/2} \tag{21}$$

Therefore, maximum diffusion is limited by $D_h \equiv (\varepsilon_h \varepsilon_t/3)(T/eB)$ or $D_E = (1/\alpha)\varepsilon_t^2 \varepsilon_h^{1/2}(T/eB)$. These relations are also plotted in Fig.1 and 2. When $\varepsilon_t = 0.1$, $\varepsilon_h = 0.04$, $\alpha = 1$, the diffusion coefficient D_h is $\varepsilon_t \varepsilon_h\ 16/3\ D_B \simeq 0.021\ D_B$. Where D_B is the Bohm diffusion coefficient. In this case, the condition of (21) is not satisfied. In conclusion, diffusions of the plasma confined in stellarator with $\varepsilon_t > \varepsilon_h$ is considered. The formula (13) of D in the region of $1/\nu$ dependence is same in the three cases of $\varepsilon_h \gg \varepsilon_t$, α; $\varepsilon_t \gg \varepsilon_h$, α; $\alpha \gg \varepsilon_t$, ε_h. However, the dependence of D on ν in the very low frequency region is different in three cases and must be treated carefully.

Reference

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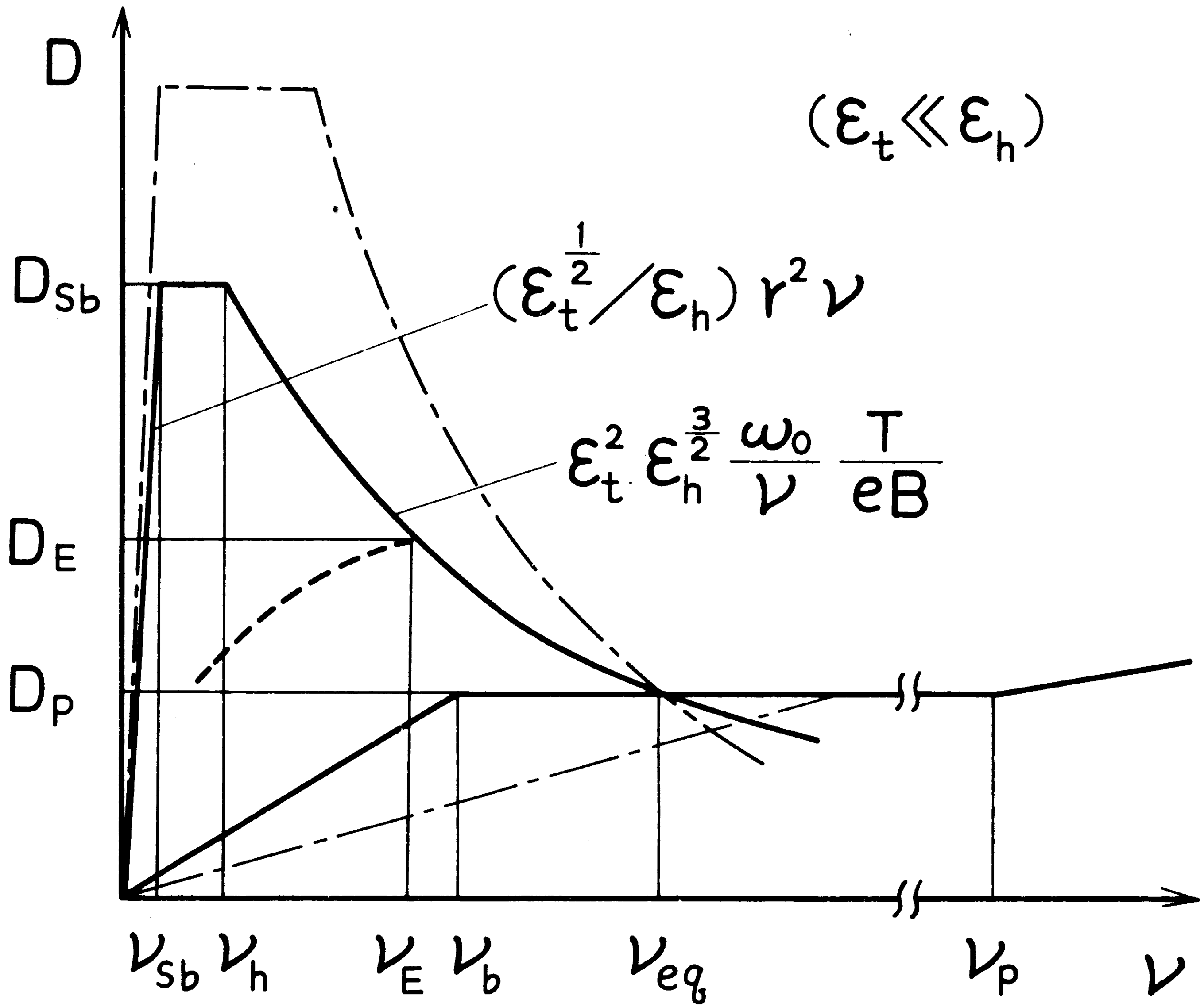
Figure Caption

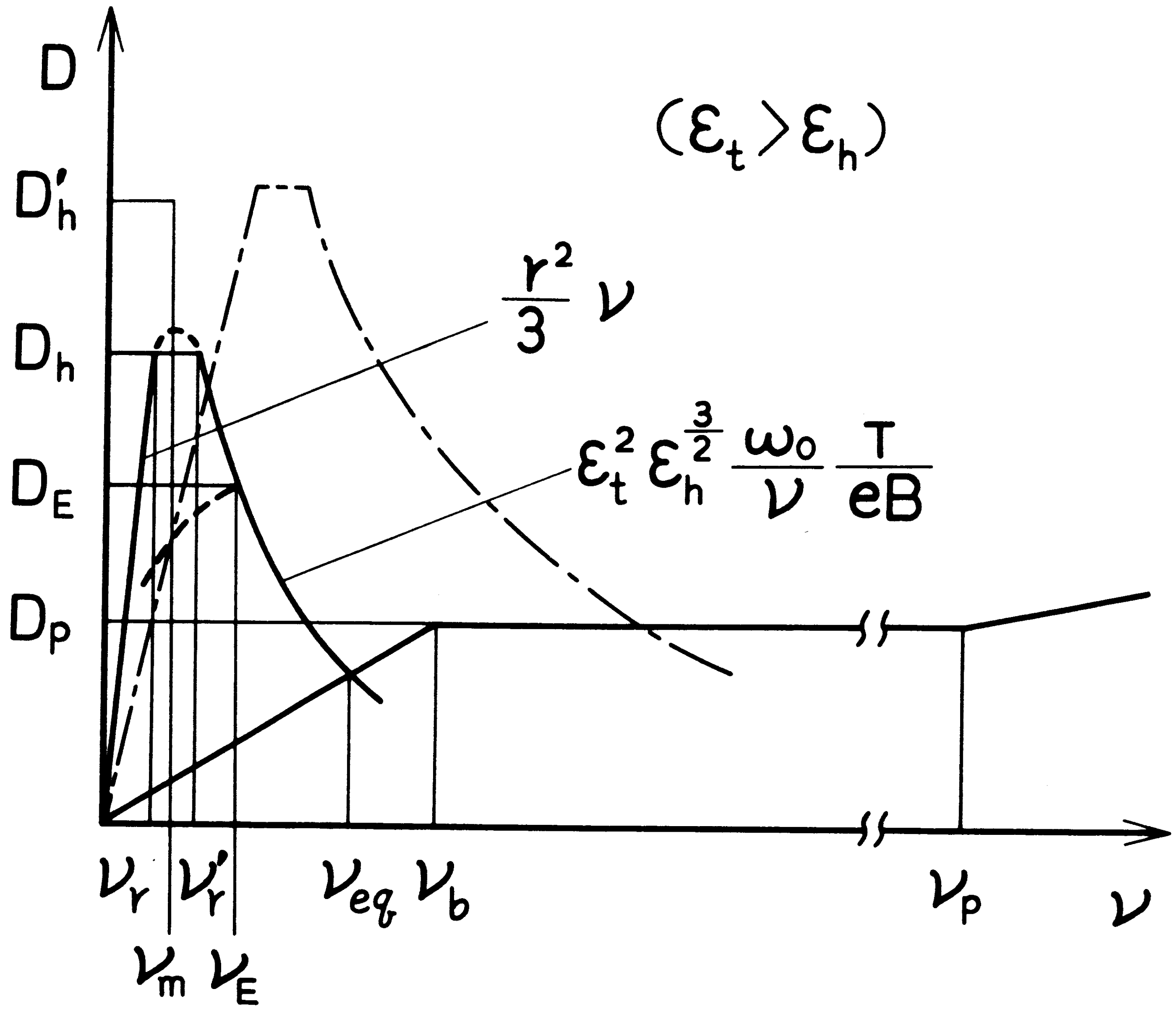
Fig.1. Dependence of diffusion coefficients of plasma confined in stellarator with $\varepsilon_t < \varepsilon_h$. $D_p = q^2\rho_e^2\nu_0$, $\nu_p = v_{th}/qR$, $\nu_b = \varepsilon_t^{3/2}\nu_p$, $D_{sb} = (\varepsilon_t^2/\varepsilon_h^{1/2})(T/eB)$, $\nu_h = \varepsilon_h^2\omega_0$, $\nu_{sb} = (\varepsilon_t/\varepsilon_h)^{3/2}\nu_h$, $\nu_{eq} = (\varepsilon_h/\varepsilon_t)^{3/2}\nu_b = \varepsilon_h^{3/2}\nu_p$, $D_E = \varepsilon_t^2\varepsilon_h^{1/2}(1/\alpha)T/eB$, $\nu_E = \varepsilon_h\omega_E = \varepsilon_h\alpha\omega_0$. —·— line shows the diffusion coefficients of stellarator with $\varepsilon_t > \varepsilon_h$ while keeping ε_h constant.

Fig.2. Dependence of diffusion coefficients of plasma confined in stellarator with $\varepsilon_h < \varepsilon_t$. D_p, ν_p, ν_b, D_E, ν_E are same to those of Fig.1. $D_h = (\varepsilon_t\varepsilon_h/3)(T/eB)$, $D_h' = (\varepsilon_t\varepsilon_h^{3/4}/\sqrt{3})(T/eB)$, $\nu_r = \varepsilon_t\varepsilon_h\omega_0$, $\nu_m' = \sqrt{3}\,\varepsilon_t\varepsilon_h^{3/4}\omega_0$. $\nu_r' = 3\,\varepsilon_t\varepsilon_h^{1/2}\omega_0$, $\nu_{eq}' = (\varepsilon_h/\varepsilon_t)^{3/4}\nu_b$. —··· line shows the diffusion coefficients of stellarator with $\varepsilon_h > \varepsilon_t$ while keeping ε_t constant.



D
$(\varepsilon_t \ll \varepsilon_h)$
D_{Sb}
$(\varepsilon_t^{\frac{1}{2}}/\varepsilon_h)\, r^2 \nu$
$\varepsilon_t^2 \varepsilon_h^{\frac{3}{2}} \frac{\omega_0}{\nu} \frac{T}{eB}$
D_E
D_P
ν_{Sb}
ν_h
ν_E
ν_b
ν_{eq}
ν_p
ν

Fig. 1



D
$(\varepsilon_t > \varepsilon_h)$
D'_h
D_h
D_E
D_p
$\frac{r^2}{3}\nu$
$\varepsilon_t^2 \varepsilon_h^{\frac{3}{2}} \frac{\omega_0}{\nu} \frac{T}{eB}$
ν_r
ν'_r
ν_m
ν_E
ν_{eq}
ν_b
ν_p
ν

Fig. 2