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On the Electron Temperature Determination of a Relativistic Plasma by Laser Scattering

Kiyoshi Hayase* and Takayoshi Okuda*

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, JAPAN.

* Permanent address
Department of Electronics, Faculty of Engineering, Nagoya University

Abstract

The relativistic formula of laser scattering from a plasma is derived, and from which the spectra of the scattered light are calculated as functions of the electron temperature and the scattering angle. The spectrum deviates considerably from the Gaussian profile when the temperature is high. The relativistic effect must be taken into account in the temperature determination at the temperature higher than a few hundred eV, otherwise the temperature is overestimated in the blue side of the spectrum. Simple and accurate methods are presented for obtaining a correct temperature in a relativistic plasma.

1. Introduction

The laser scattering is one of important diagnoses in the experiment of a high temperature plasma. The spacial and temporal values of electron temperature and density are measured without perturbing the plasma. Many experimental and theoretical works have been devoted for the developement of this diagnosis.[1,2] The laser scattering technique using a ruby laser is applicable only to the plasma with the density higher than 10^{12} cm^{-3} due to the present technical levels of the detector and the output power of the ruby laser. In recent experiments of CTR, plasmas with very high temperature and high density are generated and confined. The electron temperature of a few keV is attained in Tokamak and theta-pinches. In those experiments, the laser scattering played an important role.[3,4] However, the theory of scattering was based on the nonrelativistic motion of electrons which scattered the incident light.[5] It is necessary at present to re-examine the conventional theory of scattering from which the electron temperature has been determined. In a relativistic theory, the velocity distribution function of electrons and the scattering formula are modified. As a result, the spectrum of the scattered light will be deformed from the classical one. Then, the correct temperature is not obtained by the classical theory if the temperature is very high. So that the relativistic formulation of the scattering theory is required. The first order theory was developed by R.A.Rappert[6] and J.Sheffield.[7]

The full relativistic theory was formulated by R.E. Pechacek et al..[8] However, there has been established no method for determining a correct temperature in a relativistic plasma.

In this paper, the spectra of the scattered light for different temperature and scattering angle are calculated from a full relativistic theory. The first order theory is also derived from the general formula and the temperature region where it holds good is shown. The practicable method of temperature determination is proposed. In these calculations, only the net Thomson scattering is treated, with which we meet usually in fusion experiments.

2. Spectral distribution of scattered light

In the present Tokamak and theta-pinch or other future fusion devices, the scattering parameter α is far less than unity. In this case, the collective process is neglected and the net Thomson scattering is observed. The spectral profile and the scattering intensity are only dependent on the velocity distribution function of electrons and the electron density, respectively. In the present calculation, the case $\alpha \ll 1$ is treated. It is also assumed that the plasma is enough transparent to the laser light and that the inhomogeneity of the plasma is neglected.

The plane incident monochromatic light is linearly polarized. The scattered light is observed in a plane perpendicular to the polarization vector of the incident light. The coordinate system is shown in Fig.1. The scattered light from an infinite plasma is[9)]

$$\vec{E}_s(R,t) = \frac{e^2}{mc^2}(1-\beta^2)^{1/2} \frac{[\vec{s}\times\{(\vec{s}-\vec{\beta})\times(\vec{E}_i+\vec{\beta}\times(\vec{i}\times\vec{E}_i)-\vec{\beta}(\vec{\beta}\cdot\vec{E}_i))\}]}{R(1-\vec{s}\cdot\vec{\beta})^3}$$

$$\times \cos(\vec{k}_s\cdot\vec{R}-\omega_s t-\vec{k}_i\cdot\vec{r}(0)) \qquad (1)$$

where $\vec{E}_s(R,t)$ is the scattered light at the distance R and the time t, e the electronic charge, m the electron mass, c the light velocity, $\vec{\beta}=\vec{v}/c$, $\vec{v}$ the electron velocity, $\vec{s}$ the unit vector directing from the scattering point to the observer, $\vec{E}_i$ the amplitude of the incident light, $\vec{i}$ the unit vector directing the propagation of the incident light, $\vec{k}_i$ and $\vec{k}_s$ the wave vectors of the incident and scattered lights respectively and ω_s the frequency of the scattered light. This formula can be expressed in a scalar form. If we take only the components which have the same direction with $\vec{E}_i$,

$$E_s(R,t) = \frac{e^2}{mc^2}(1-\beta^2)^{1/2} \frac{\{1-\beta_x(1+\cos\theta)-\beta_y\sin\theta-\beta_z^2(1+\cos\theta)}{R(1-\beta_x\cos\theta-\beta_y\sin\theta)^3}$$

$$\frac{+\beta_x^2\cos\theta+\beta_x\beta_y\sin\theta\}}{} E_i\cos(k_sR-\omega_s t-\vec{k}_i\cdot\vec{r}(0)) \qquad (2)$$

where θ is the scattering angle. The time-averaged power of the scattered light $P_s(R)$ is

$$P_s(R) = \frac{c}{4\pi}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|E_s(R,t)|^2 dt \qquad (3)$$

In the above calculation, the scattering from an infinite plasma is assumed. If we observe the scattering from a finite region, the finite transit time effect must be considered. This effect was at first pointed out by R. Pechacek et al.[8] and cannot be neglected in a relativistic plasma. The finite transit time effect is taken into accountby multiplying $(1-\vec{s}\cdot\vec{\beta})^{1/2}$ to Eq.1. Finally the scattered power is expressed as follows.

$$P_s(R) = \left(\frac{c}{8\pi}\right)\left(\frac{e^2}{mc^2}\right)^2 (1-\beta^2) \frac{\{1-\beta_x(1+\cos\theta)-\beta_y\sin\theta-\beta_z^2(1+\cos\theta)}{R^2(1-\beta_x\cos\theta-\beta_y\sin\theta)^5}$$

$$\frac{+\beta_x^2\cos\theta+\beta_x\beta_y\sin\theta\}^2}{} E_i^2 \qquad (4)$$

The frequencies and wave vectors of the scattered and incident lights are connected by the following conservation laws,

$$\omega_s = \omega_i + \vec{k}\cdot\vec{v} \qquad (5)$$

$$\vec{k}_s = \vec{k}_i + \vec{k} \qquad (6)$$

where ω_i is the frequency of the incident light, $\vec{k}$ the scattering vector and $\omega_{i,s} = ck_{i,s}$. The spectral profile of the scattered light is derived from the velocity distribution of electrons along the $\vec{k}$ vector. Then, it is convenient to transform the coordinate system so that one coordinate axis is directed along the $\vec{k}$ vector just as in Fig.2. The relations between the old and new coordinate systems are

$$\beta_x = -\beta_{k\parallel}\cos\gamma + \beta_{k\perp}\sin\gamma \qquad (7)$$

$$\beta_y = \beta_{k\parallel}\sin\gamma + \beta_{k\perp}\cos\gamma \qquad (8)$$

$$\beta_z = \beta_z \qquad (9)$$

where $\beta_{k\parallel}$ and $\beta_{k\perp}$ are the components of β parallel and perpendicular to the vector $\vec{k}$ and γ is the angle between $\vec{k}_i$ and $\vec{k}$. The value of γ is given by

$$\cos\gamma = (\omega_i - \omega_s \cos\theta)/ck \qquad (10)$$

$$\sin\gamma = \omega_s \sin\theta/ck \qquad (11)$$

The scattered power in the new coordinate system is

$$P_s(R) = \left(\frac{c}{8\pi}\right)\left(\frac{e^2}{mc^2}\right)^2 (1-\beta_{k\parallel}^2-\beta_{k\perp}^2-\beta_z^2) \frac{\{1+\beta_{k\parallel}\{(1+\cos\theta)\cos\gamma - \sin\theta\sin\gamma\} - \beta_{k\perp}\{(1+\cos\theta)\sin\gamma+\sin\theta\cos\gamma\} + \beta_{k\parallel}^2 \cos\gamma(\cos\theta\cos\gamma-\sin\theta\sin\gamma)+\beta_{k\perp}^2\sin\gamma \times(\cos\theta\sin\gamma+\sin\theta\cos\gamma)+\beta_{k\parallel}\beta_{k\perp}\{\sin\theta(\sin\gamma-\cos\gamma)-2\cos\theta\cos\gamma\sin\gamma\}\}^2}{R^2\,[1+\beta_{k\parallel}(\cos\theta\cos\gamma-\sin\theta\sin\gamma)-\beta_{k\perp}\times(\cos\theta\sin\gamma+\sin\theta\cos\gamma)]^5}\, E_i^2 \qquad (12)$$

The frequency spectrum of the scattered light is given by

$$P_s(R)\,d\omega_s\,d\Omega = \int d\vec{v}\,P(R)\,f(\beta)\,\delta\left(v_{k\parallel} - \frac{\omega_s-\omega_i}{k}\right) d\Omega \qquad (13)$$

where Ω is the solid angle, $f(\beta)$ the velocity distribution function of electrons and δ the delta function. If the Maxwellian distribution is assumed, it is written in the relativistic form,[10)]

$$f(\beta) = \frac{(mc^2/\kappa T)\exp-(mc^2/\kappa T)}{4\pi mc\kappa T(1-\beta^2)^{5/2} K_2(mc^2/\kappa T)} \times \exp-\left\{\frac{mc^2}{\kappa T}\left(\frac{1}{(1-\beta^2)^{1/2}}-1\right)\right\} \qquad (14)$$

where K is the Boltzmann constant, K_2 the modified Bessel function of second kind and of second order and T the electron temperature. The frequency spectrum of the scattered light is expressed in a normalized wavelength shift by using the following relations

$$c|k| = (\omega_s^2+\omega_i^2 -2\omega_s\omega_i\cos\theta)^{1/2} \qquad (15)$$

$$v_{k/\!/} = (\omega_s-\omega_i)/|k| \qquad (16)$$

$$dv_{k/\!/} = \frac{(\omega_i^2+\omega_i\omega_s)(1-\cos\theta)}{k^2 c^2}\cdot\frac{d\omega_s}{|k|} \qquad (17)$$

$$\omega_{s,i} = 2\pi c/\lambda_{s,i} \qquad (18)$$

$$d\omega_s = 2\pi c d\lambda_s/\lambda_s^2 \qquad (19)$$

$$x = (\lambda_s-\lambda_i)/\lambda_i \qquad (20)$$

$$dx = d\lambda_s/\lambda_i \qquad (21)$$

The spectral profile $P_s(R)dxd\Omega$ is calculated by using Eqs.(10)-(21). The result is very complicated. However, the profile is expressed in a simpler form at θ= 90° and 180°. The spectra for θ = 90° and λ_i = 6943 A° are illustrated as a function of electron temperature in Fig. 3, as an example. The intensity is normalized to unity at the incident wavelength. As the electron temperature rises, the symmetry of the profile is destroyed and the peak shifts to the blue side. This feature is resulted from the fact that the scattering cross-section increases in a shorter wavelength side. The shift of the peak is larger for larger scattering angle and higher electron temperature. The asymmetry is noticable above a few hundred eV. This means that the conventional method of temperature estimation from the spectrum does not hold good.

The reduction of a formula in a first order approximation is easy. It is written as

$$(1-3.5x)\exp-\frac{10^6 x^2}{7.84T\sin^2(\theta/2)}\,dx\,d\Omega \qquad (22)$$

where the numerical constants are omitted. It is a simple form. The term of the exponential function is the same with that from the nonrelativistic theory. The factor (1-3.5x) is added only. The first order theory is approximately correct below 1 keV. The spectra from the classical, first order relativistic and full relativistic theories are compared at $\theta = 90°$ in Fig.4. The first order approximation is inadequate at T = 4 keV.

The peak of the spectrum is deviated from the incident wavelength. The peak of the shift is calculated by differentiating Eq.(22). We get

$$x = -2.74\times10^{-6}T\sin^2(\theta/2) \qquad (23)$$

or

$$\Delta\lambda = -0.19T\sin^2(\theta/2) \qquad (24)$$

where $\Delta\lambda$ is in unit of Å. It is as large as 100 Å at $\theta = 90°$ and T = 1keV. These formulas are correct below about 10 keV.

3. Temperature determination

We consider here the method of the temperature determination by utilizing the above calculations. The formula in the first or second order approximation will be effectively used below 10 keV which we meet with in the present Tokamak and theta-pinch devices. However, the functional form of the spectrum is so complicated that the temperature cannot be determined uniquely from the spectral broadening just as in a conventional way. In the region where the first order approximation is correct, the equivalent classical form of the spectrum is written by deviding each experimental spectral intensity by coefficient (1-3.5x). Then we get a Gaussian profile. The temperature is estimated from the half width of the broadening. This is a simple and correct method of temperature determination of a relativistic plasma. The method can easily be applied in a data processing by a computer. The electron density is estimated from the spectral broadening and the scattering intensity at the central wavelength just as by a conventional method. In a higher temperature region where the first order approximation is inadequate, the shift of the spectral peak is large enough to be distinguished. The wavelength shift of the peak is uniquely related to the temperature. Therefore, the electron temperature is determined from the shift of the peak from the incident wavelength. However, this method is useless in a lower temperature region where the shift of the peak is too small to be measured. The wavelength shift of the spectral peak at $\theta = 90°$ is shown against the temperature in Fig.5.

The most experimental data have been interpreted by the nonrelativistic theory. Hence, the spectral profile should be deviated from the Gaussian distribution in a higher temperature region even if the velocity distribution of electrons is Maxwellian. Then, how much does the temperature differ when we do not take into account the relativistic effect, that is, we assume the Gaussian profile of the spectra. As is expected from Fig.3, the temperature may be overestimated in the blue side of the spectrum and underestimated in the red side if we assume that the experimental profile is Gaussian. The temperature is estimated from the most fitting Gaussian profile, in which only the middle region of the broadening $(1/10)|-\Delta\lambda_{1/10}| < |-\Delta\lambda_s| < |-\Delta\lambda_{1/10}|$ is considered, where $-\Delta\lambda_{1/10}$ is the wavelength at which the intensity of the spectrum is one tenth of the peak. The differences of the temperature by this method from the correct one are shown in Fig.6. As the spectra are usually observed in the blue or shorter wavelength side, the temperature is overestimated by 10, 15 and 20 % at 0.5, 1.0 and 2.0 keV, respectively. The error cannot be neglected. In a very high temperature region where the shift of the spectral peak is extremely large, the fitting to the Gaussian distribution is meaningless.

4. Conclusion

As shown in the previous section, the electron temperature estimated on the basis of the classical theory is appreciably higher than the one from the relativistic theory in the blue side of the spectrum. The difference becomes larger as the temperature rises. So that the relativistic effect must be taken into account above a few hundred eV. The method of temperature determination described here is simple and accurate.

The present calculation is based on the isotropic and Maxwellian distribution of electron velocity. However, we often meet with the anisotropic and non Maxwellian distribution. In such a case, the calculation and the result are much more complicated.

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Figure Captions

Fig.1 Coordinate system.

Fig.2 Old and new coordinate systems and vector relation.

Fig.3 Spectra of the scattered light of a ruby laser at $\theta = 90°$. The scattering intensity is normalized to unity at the incident wavelength.

Fig.4 Comparison of the spectra at $\theta = 90°$ and T = 1 and 4 keV calculated from nonrelativistic (—·—) first-order relativistic (------) and full relativistic (——) theories. The intensity is normalized to unity at the incident wavelength.

Fig.5 Shift of the spectral peak $\Delta\lambda_{peak}$ against the electron temperature at $\theta = 90°$.

Fig.6 Comparison of the correct and approximate temperatures calculated from the full relativistic theory and the most fitting Gaussian profile based on the nonrelativistic theory, respectively.

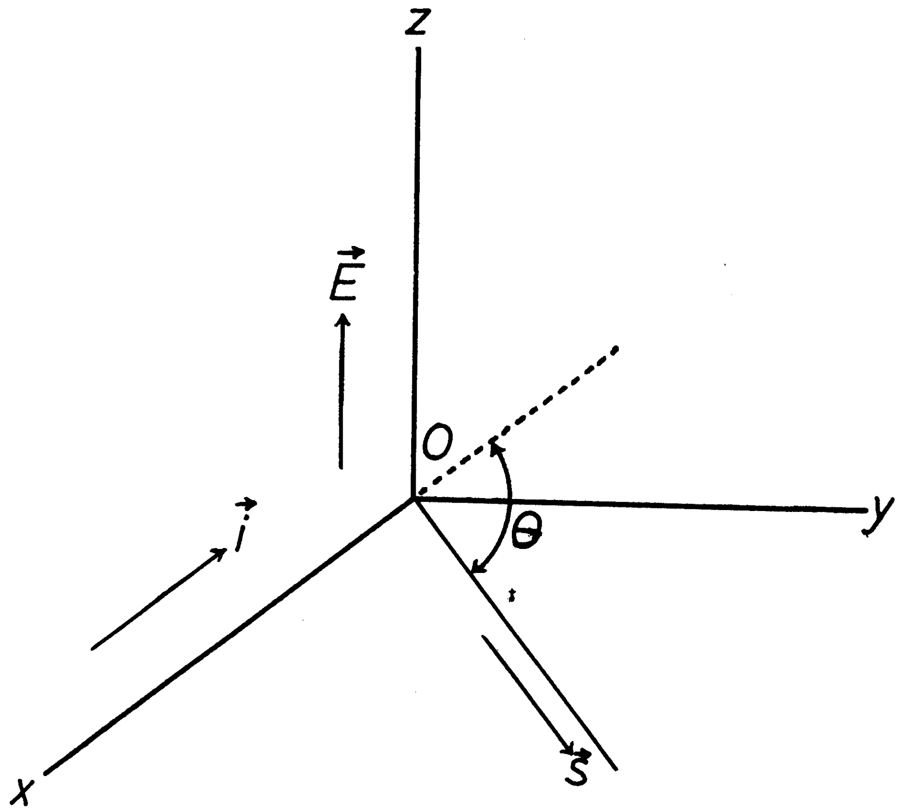
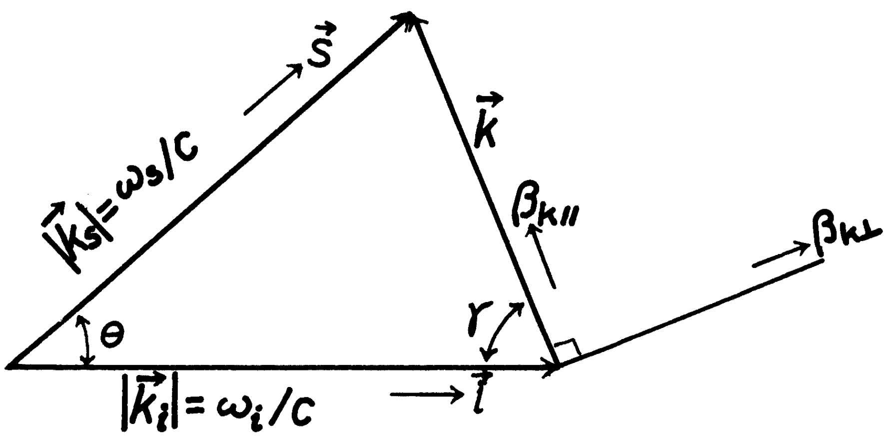
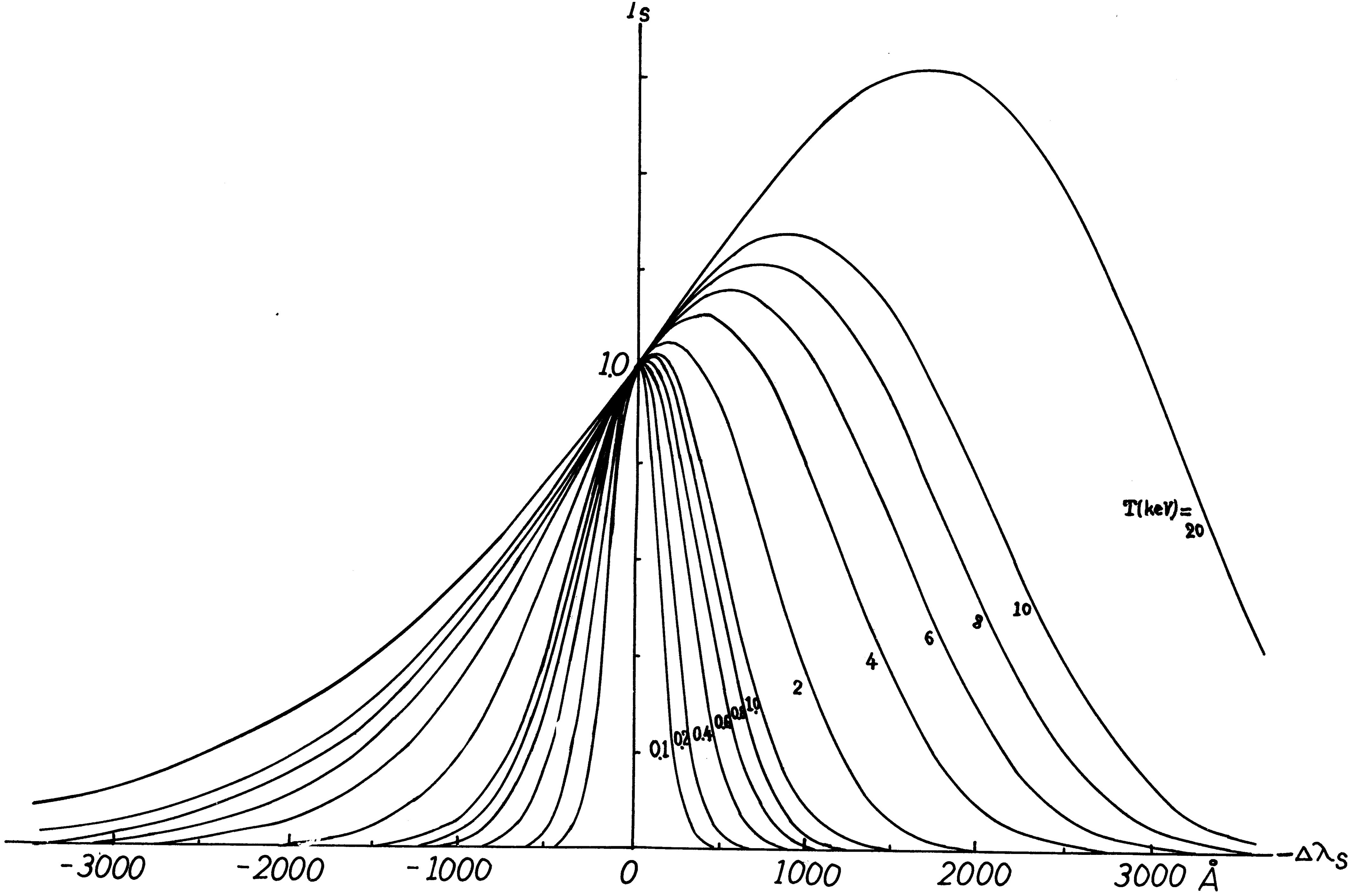


Fig.1



$\vec{s}$
$|\vec{k}_s| = \omega_s/c$
$\vec{k}$
$\beta_{k\parallel}$
$\beta_{k\perp}$
θ
γ
$|\vec{k}_i| = \omega_i/c$
$\vec{i}$

Fig. 2



I_S
1.0
T(keV)=
20
10
8
6
4
2
0.1
0.2
0.4
0.6
0.8
1.0
-3000
-2000
-1000
0
1000
2000
3000 Å
$-\Delta\lambda_S$

Fig. 3

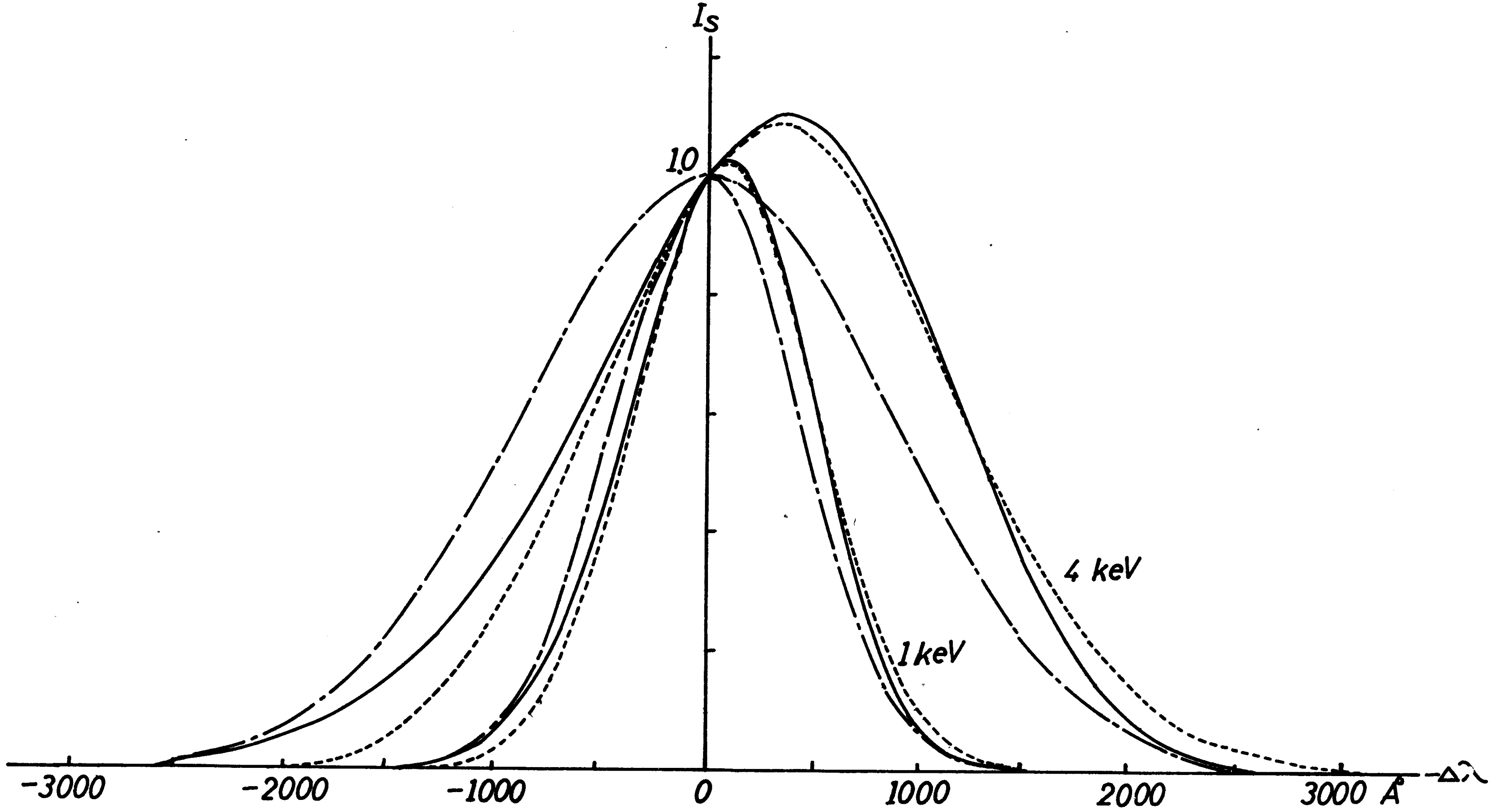


Fig.4

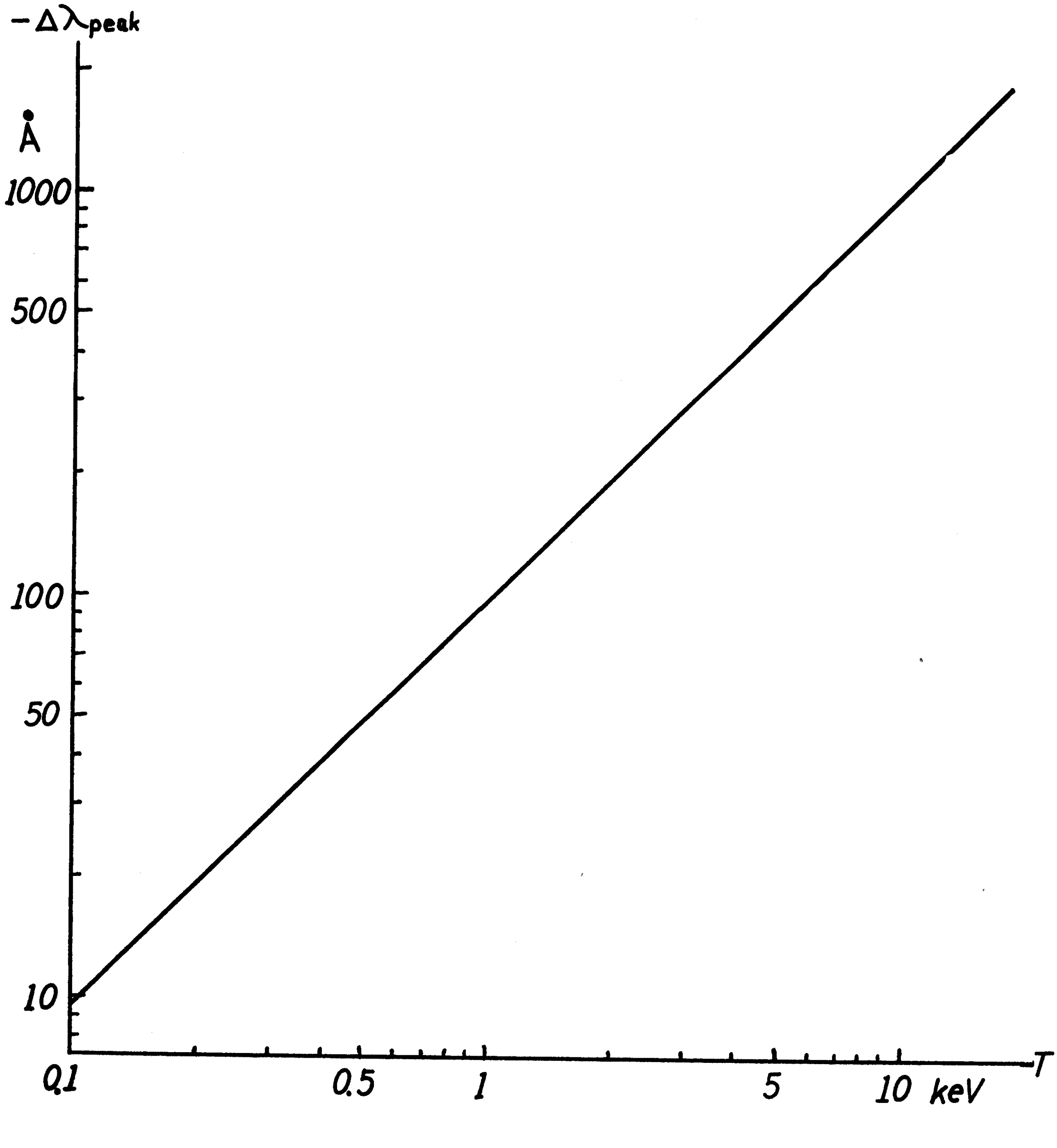


Fig.5

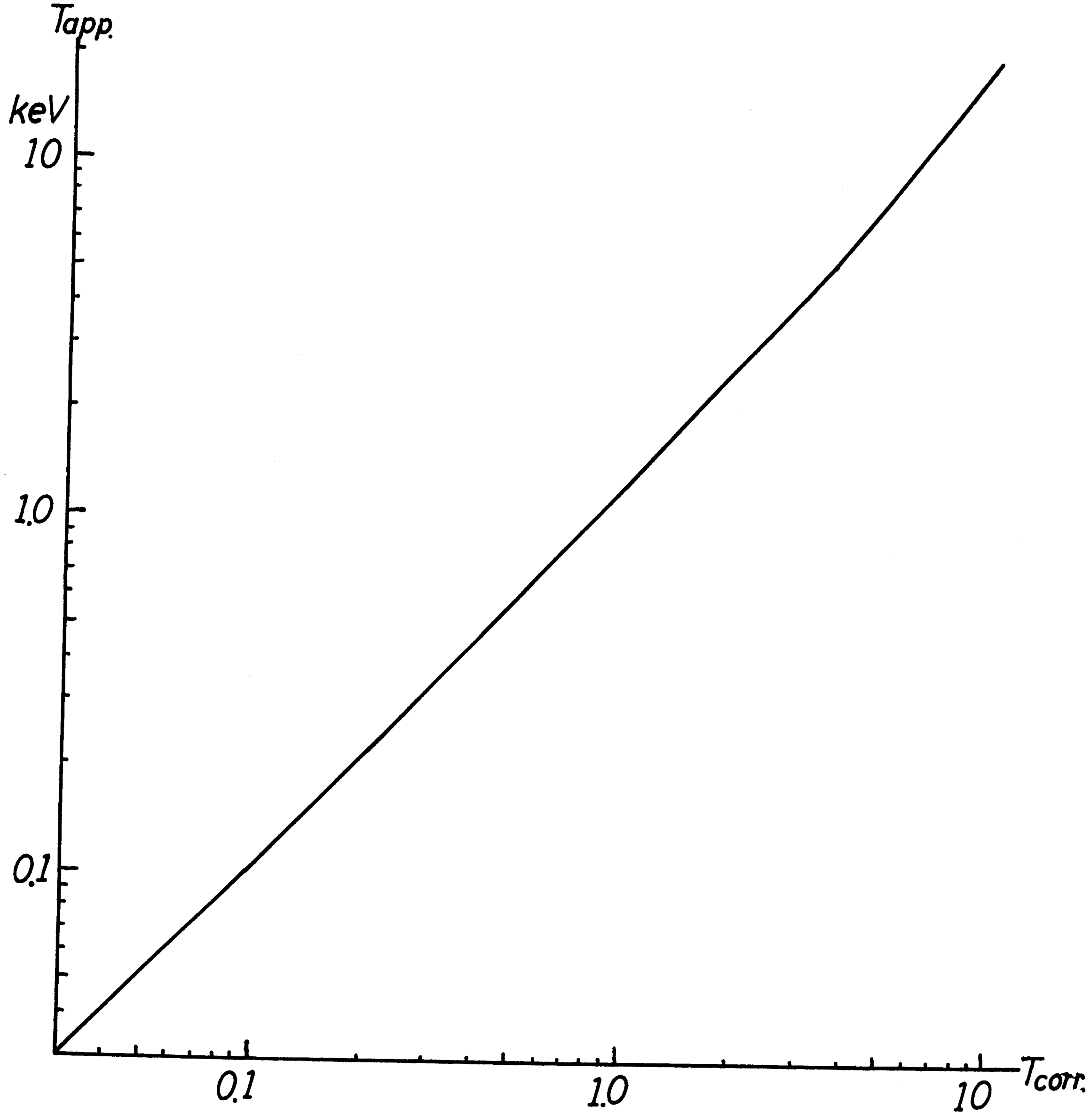


Fig.6